Statistical Inference Project Part 1

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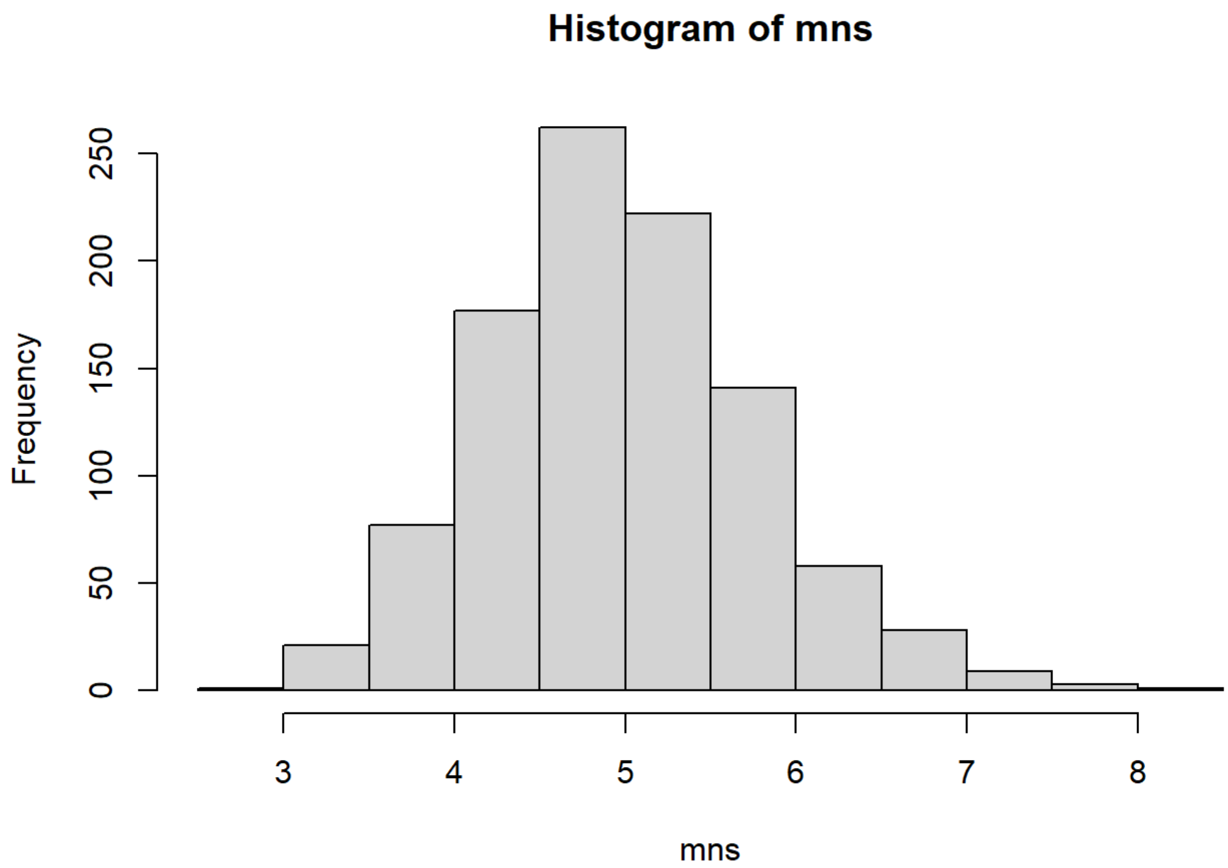
Purpose

In this project you will investigate the exponential distribution in R and compare it with the Central Limit Theorem. The exponential distribution can be simulated in R with rexp(n, lambda) where lambda is the rate parameter. The mean of exponential distribution is 1/lambda and the standard deviation is also 1/lambda. Set lambda = 0.2 for all of the simulations. You will investigate the distribution of averages of 40 exponentials. Note that you will need to do a thousand simulations.

Simulations

```
set.seed(42) # seed for reproductibility
lambda <- 0.2
n <- 40 # number of exponentials
sim <- 1000 # number of simulations

mns = NULL
for (i in 1 : 1000) mns = c(mns, mean(rexp(n, lambda)))
hist(mns)
```



As can be seen in the histogram, this distribution is close to normal distribution.

Sample Mean versus Theoretical Mean

The mean μ of a exponential distribution of rate λ is $\mu = \frac{1}{\lambda}$, therefore

```
mu = 1/lambda
mu
```

```
## [1] 5
```

Since the sample mean of the 1000 simulations, we have shown that expected mean is close to sample mean.

```
mean(mns)
```

```
## [1] 4.986508
```

Sample Variance versus Theoretical Variance

The variance Var of a exponential distribution of rate λ is $Var = \sigma^2 = \frac{\mu^2}{n}$, therefore

```
Var = mu^2/n
Var
```

```
## [1] 0.625
```

Since the sample variance of the 1000 simulations, we have shown that expected variance is close to sample variance.

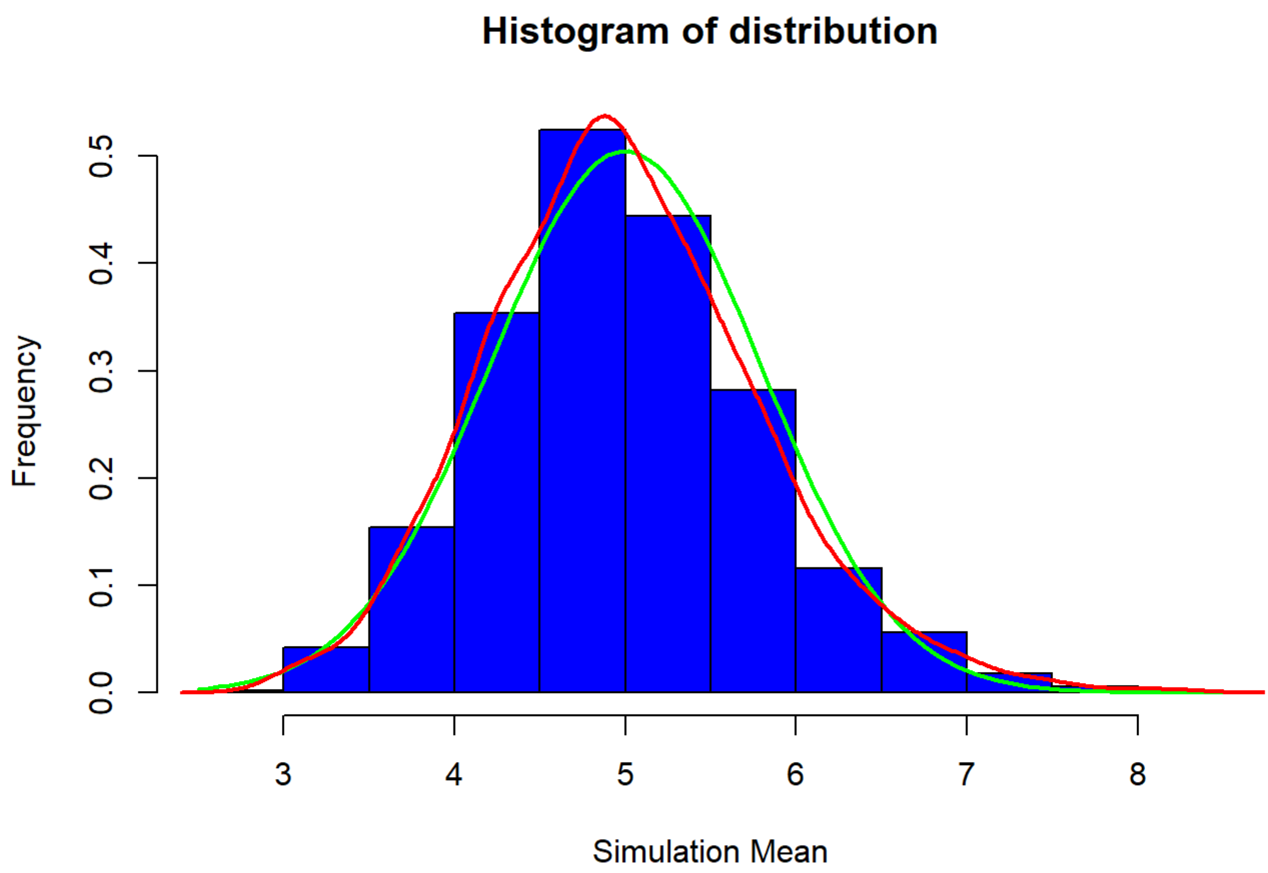
```
var(mns)
```

```
## [1] 0.6344405
```

Distribution

Comparison of a normal distribution with mean $\mu = \lambda$ and standard deviation = $ $ with the simulations density plot. In green, we have the normal distribution and in red the density of the simulations histogram.

```
hist(mns, freq = FALSE, col="blue", xlab = "Simulation Mean", ylab = "Frequency", main = "Histogram of distribution")
curve(dnorm(x, mean=lambda^-1, sd=(lambda*sqrt(n))^-1), add=TRUE, col='green', lwd=2)
lines(density(mns),col="red",lwd=2)
```



As a result, we have that the density of the simulations approaches the normal curve with the theoretical values.