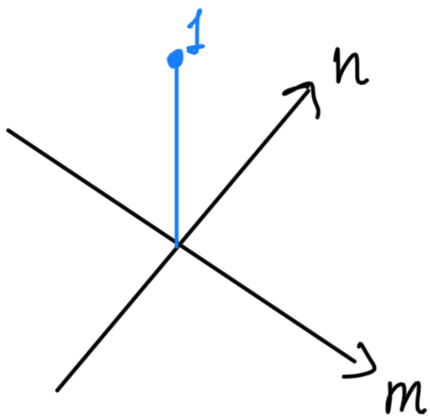
Lab 4 Calculations

Part II

- Representation of a 2D signal $x[m,n]$ in terms of shifted impulses $\delta[m,n]$:

we have $\delta[m,n] = \begin{cases} 1, & \text{if } m=0, n=0 \\ 0, & \text{otherwise} \end{cases}$



$$x[m,n] = \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} x[k,l]\,\delta[m-k, n-l]$$

(Sifting Property)

$x[k,l]$ are the weights of the shifted impulses.

- Let $y[m,n]$ be a LTI system such that:

$$\delta[m,n] \longrightarrow \boxed{y[m,n]} \longrightarrow h[m,n]$$

① Due to superposition, we have:

$$x[m,n] \longrightarrow \boxed{y[m,n]} \longrightarrow \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} x[k,l]\, h_{k,l}[m,n]$$

② Due to time Invarience, we have :

$$h_{k,l}[m,n] = h_{0,0}[m-k, n-l]$$
$$= h[m-k, n-l]$$

Finally, we have

$$\boxed{\begin{aligned} y[m,n] &= \sum_{k=-\infty}^{+\infty} \sum_{l=-\infty}^{+\infty} x[k,l] \cdot h[m-k, n-l] \\ &= x[m,n] ** h[m,n] \end{aligned}}$$

Part III :

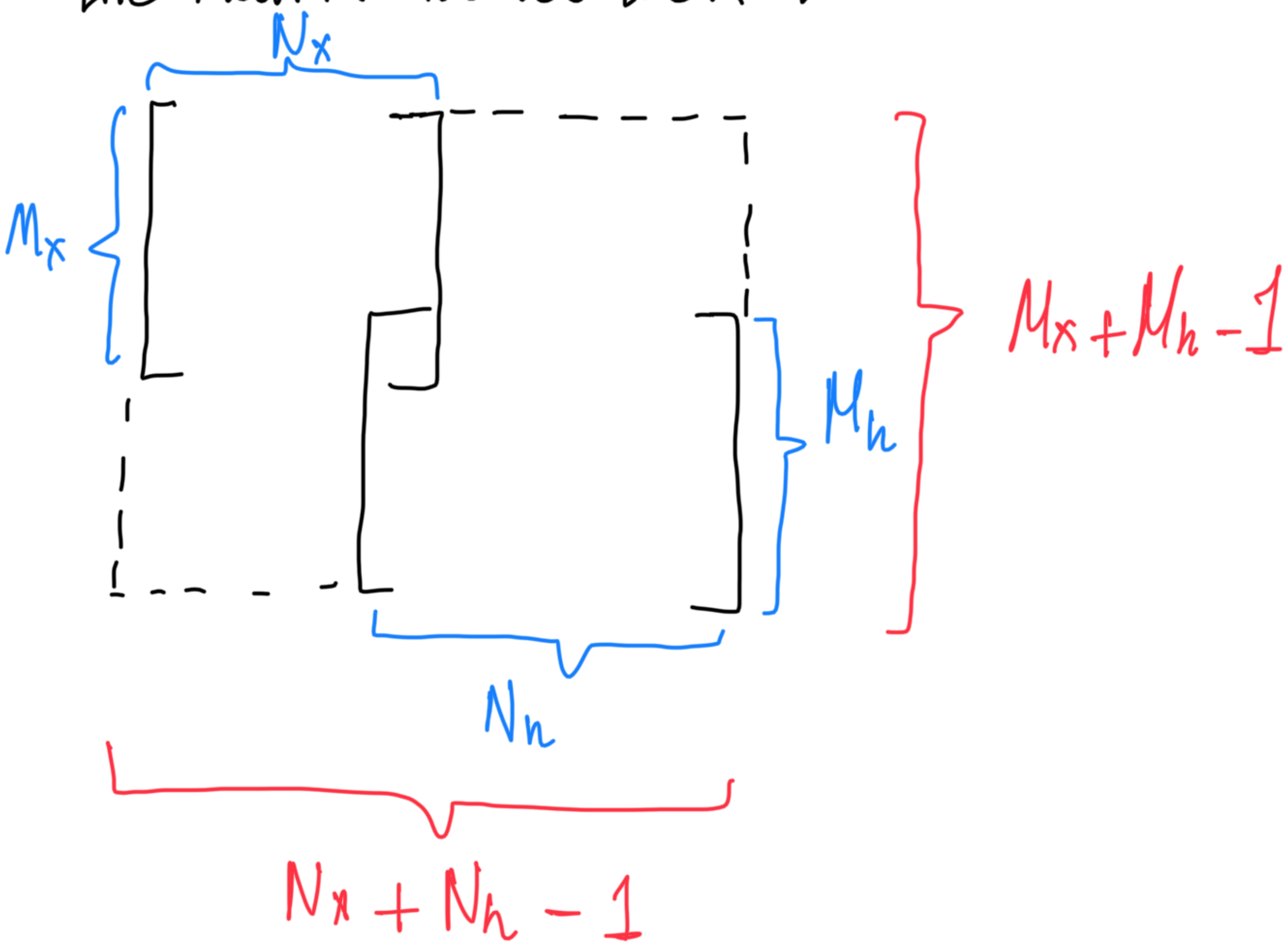
$$y[m,n] = \sum_{k=0}^{M_h-1} \sum_{l=0}^{N_h-1} \underbrace{h[k,l]}_{\text{Matrix of size } M_h \times N_h} \cdot \underbrace{x[m-k, n-l]}_{\text{Matrix of size } M_x \times N_x}$$

$x[m,n] \neq 0$ when $\{ 0 \leq m \leq M_x - 1$

$$0 \leq n \leq N_x - 1$$

$$h[m,n] \neq 0 \text{ when } \begin{cases} 0 \leq m \leq M_h - 1 \\ 0 \leq n \leq M_h - 1 \end{cases}$$

Then, for an arbitrary output $y[m,n]$, the matrix should look like



$$\boxed{\begin{aligned} M_y &= M_x + M_h \\ N_y &= N_x + N_h \end{aligned}}$$

The convolution is the sum of weighted and shifted impulse response. The output y would have value if and only if x and h have non-zero values at locations $\underline{k}$ and $\underline{l}$.

Then, the output matrix (y) has non-zero values when the corresponding positions of matrices (X) and (h) have non-zero values

Symbolically, $y[m,n] \neq 0$ When $\begin{cases} 0 \leq m \leq M_y - 1 \\ 0 \leq n \leq N_y - 1 \end{cases}$