Subject: Part 1 Date: ....../....../......

$$y[n] = \sum_{l=1}^{N} a[l]\, y[n-l] + \sum_{k=0}^{M} b[k]\, x[n-k] \; ; \quad \begin{matrix} x[n]=0 \\ y[n]=0 \end{matrix} \text{ for } n<0$$

① $$y[0] = \sum_{l=1}^{N} a[l]\, y[-l] + \sum_{k=0}^{M} b[k]\, x[-k]$$

$$\sum_{l=1}^{N} a[l]\, y[-l] = a[1]\underbrace{y[-1]}_{0} + a[2]\underbrace{y[-2]}_{0} + \dots + a[N]\underbrace{y[-N]}_{0} = 0$$

$$\sum_{k=0}^{M} b[k]\, x[-k] = b[0]\,x[0] + b[1]\underbrace{x[-1]}_{0} + \dots + b[M]\underbrace{x[-M]}_{0} = b[0]\,x[0]$$

$$\boxed{y[0] = b[0]\,x[0]}$$

② $$y[1] = \sum_{l=1}^{N} a[l]\, y[1-l] + \sum_{k=0}^{M} b[k]\, x[1-k]$$

$$\sum_{l=1}^{N} a[l]\, y[1-l] = a[1]\,y[0] + a[2]\underbrace{y[-1]}_{0} + \dots + a[N]\underbrace{y[-(N-1)]}_{0} = a[1]\,y[0]$$

$$\sum_{k=0}^{M} b[k]\, x[1-k] = b[0]\,x[1] + b[1]\,x[0] + b[2]\underbrace{x[-1]}_{0} + \dots + b[M]\underbrace{x[-(M-1)]}_{0}$$

$$= b[0]\,x[1] + b[1]\,x[0]$$

$$\boxed{y[1] = a[1]\,y[0] + b[0]\,x[1] + b[1]\,x[0]}$$

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$$y[n] = \sum_{l=1}^{N} a[l]\, y[n-l] + \sum_{k=0}^{M} b[k]\, x[n-k]$$

Taking the z-transform of $y[n]$

$$Y(z) = \sum_{l=1}^{N} a[l]\, Y(z) \cdot z^{-l} + \sum_{k=0}^{M} b[k] \cdot X(z) \cdot z^{-k}$$

$$Y(z)\left[1 + \sum_{l=1}^{N} -a[l]\, z^{-l}\right] = \sum_{k=0}^{M} b[k] \cdot X(z) \cdot z^{-k}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b[k]\, z^{-k}}{1 + \sum_{l=1}^{N} -a[l]\, z^{-l}} = \frac{\sum_{p=0}^{P} C_n[p]\, z^{-p}}{\sum_{q=0}^{Q} C_d[q]\, z^{-q}}$$

Matching the terms, we get:

$\boxed{P = M}$, $\boxed{Q = N}$, $C_n[p] = b[k]$, $C_d[p] = \begin{cases} 1, & q = 0 \\ -a[l], & \text{otherwise} \end{cases}$

Subject : Part 2 Date : ....../....../......

a) $a[l]=0$ for $1 \leq l \leq N$. $b[k]=e^{-k}$ for $0 \leq k \leq M-1$
else, 0

$D_4 = mod(21903608, 4) = 0$
$M = 5 + D_4 = 5$

b) $h[0] = b[0] = e^{0} = 1$

$h[1] = b[1] = e^{-1} = 0.3678$
$h[2] = b[2] = e^{-2} = 0.1353$
$h[3] = b[3] = e^{-3} = 0.0498$

$h[4] = b[4] = e^{-4} = 0.0183$

$\boxed{h[n] = b[n]}$

c) $h[n] = 0$ for $n > 4$ $\Rightarrow$ The filter is FIR

The length of the impulse response is $M = 5$

d) $y[n] = \sum_{k=0}^{M=5-1} e^{-k} x[n-k]$

Z Trans.

$Y(z) = \sum_{k=0}^{4} e^{-k} X(z) z^{-k} \Rightarrow Y(z) = X(z) \sum_{k=0}^{4} e^{-k} z^{-k}$

$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{4} e^{-k} z^{-k}$

$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \sum_{k=0}^{4} e^{-k} e^{-jwk} = \sum_{k=0}^{4} e^{-k(1+jw)}$

$$\boxed{H(e^{jw}) = \frac{1 - e^{-5(1+jw)}}{1 - e^{-(1+jw)}}}$$

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e) It's a low pass Filter with $w_c = 1.123$ rad/s and the bandwidth is 2.220 rad/s

Subject: Part 3 Date: ....../....../......

ID = 21903608

$n_1 = 4 \quad n_2 = 3 \quad n_3 = 11 \quad n_4 = 2 \quad n_5 = 5 \quad n_6 = 8 \quad n_7 = 2 \quad n_8 = 10$

$$Z_1 = \frac{n_2 + jn_3}{\sqrt{n_2^2 + n_3^2}} = \frac{3 + 11j}{\sqrt{130}}$$

$$P_1 = \frac{n_1 + jn_5}{\sqrt{1 + n_1^2 + n_5^2}} = \frac{4 + 5j}{\sqrt{42}} \qquad P_2 = \frac{n_8 + jn_6}{\sqrt{1 + n_8^2 + n_6^2}} = \frac{10 + 8j}{\sqrt{165}}$$

a) $$H(z) = \frac{z - Z_1}{(z - P_1)(z - P_2)} \Rightarrow H(z) = A \frac{(1 - (Z_1) z^{-1})}{(1 - (P_1) z^{-1})(1 - (P_2) z^{-1})}$$

b) $$H(z) = \frac{Y(z)}{X(z)}$$

$$Y(z)(1 - P_1 \cdot z^{-1})(1 - P_2 \cdot z^{-1}) = A \cdot X(z)(1 - Z_1 z^{-1})$$

$$\rightarrow Y(z) - Y(z) \cdot (P_1 + P_2) z^{-1} + Y(z) \cdot (P_1 \cdot P_2) z^{-2} = A X(z)(1 - Z_1 z^{-1})$$

$z^{-1}\{\}$

$$y[n] - y[n-1] \cdot (P_1 + P_2) + y[n-2] \cdot (P_1 P_2) = \overset{1}{A} x[n] - \overset{1}{A} Z_1 x[n-1]$$

$$y[n] = y[n-1](P_1 + P_2) - y[n-2](P_1 P_2) + x[n] - Z_1 x[n-1]$$

$$\boxed{\begin{array}{l} y[n] = \sum_{l=1}^{2} a[l]\, y[n-l] + \sum_{k=0}^{1} b[k]\, x[n-k] \\ a[1] = P_1 + P_2 \quad b[0] = 1 \\ a[2] = -P_1 P_2 \quad b[1] = -Z_1 \end{array}}$$

Scanned with CamScanner

c) $H(z) = \dfrac{1 - z_1 z^{-1}}{(1-P_1 z^{-1})(1-P_2 z^{-1})} = \dfrac{A}{(1-P_1 z^{-1})} + \dfrac{B}{(1-P_2 z^{-1})}$

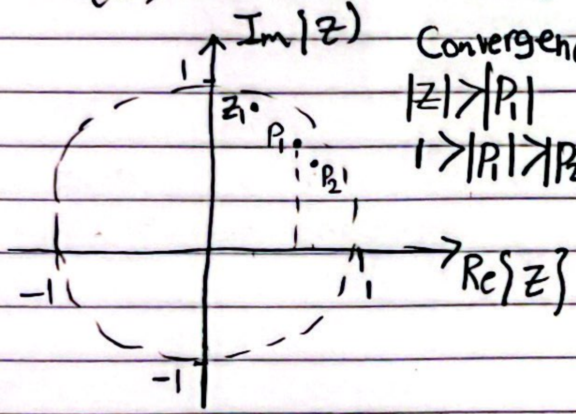
$= \left(\dfrac{1 - z_1 P_1^{-1}}{1 - P_2 P_1^{-1}}\right) \dfrac{1}{(1-P_1 z^{-1})} + \left(\dfrac{1 - z_1 P_2^{-1}}{1 - P_1 P_2^{-1}}\right) \dfrac{1}{(1-P_2 z^{-1})}$

$h[n] = \left(\dfrac{1 - z_1 P_1^{-1}}{1 - P_2 P_1^{-1}}\right)(P_1)^n u[n] + \left(\dfrac{1 - z_1 P_2^{-1}}{1 - P_1 P_2^{-1}}\right)(P_2)^n u[n]$

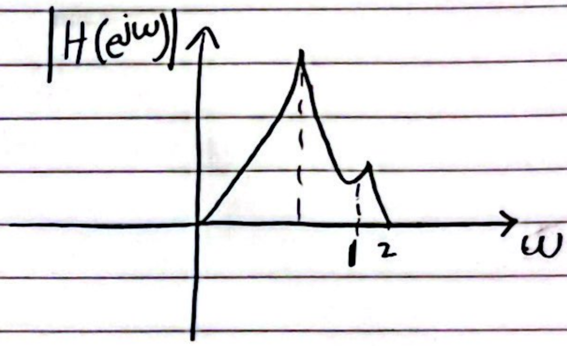
d) $|P_1| = \dfrac{\sqrt{4^2+5^2}}{\sqrt{42}} = 0.988$ $\qquad |P_2| = \dfrac{\sqrt{164}}{\sqrt{165}} = 0.996$

$Re(z_1) = 0.263$ $\qquad Re(P_1) = 0.617$ $\qquad Re(P_2) = 0.778$

$Im(z_1) = 0.964$ $\qquad Im(P_1) = 0.771$ $\qquad Im(P_2) = 0.622$



Convergence:
$|z| > |P_1|$
$1 > |P_1| > |P_2|$



e) System is stable because ROC includes the circle. $|z| > |P_1|$

f) The filter is IIR. Because there are 2 poles, $h[n]$ can't be zero for a finite n

g) $H(e^{jw}) = \dfrac{e^{jw} - z_1}{(e^{jw} - P_1)(e^{jw} - P_2)}$