Part(2) Calculations:

$x(t)$ is periodic with T:

$$x(t+T) = x(t)$$

Fourier Series Expansion of $x(t)$:

$$x(t) = \sum_{k=-\infty}^{\infty} X_k \, e^{\frac{j2\pi kt}{T}}$$

$\uparrow$ Expansion Coefficients

Approximation of $x(t)$ is $\tilde{x}(t)$

$$\tilde{x}(t) = \sum_{k=-K}^{+K} X_k \, e^{\frac{+j2\pi kt}{T}}$$

Finite

$$X_K = \frac{1}{T} \int_{-T/2}^{T/2} x(t) \, e^{-j\frac{2\pi kt}{T}}$$

for $t \in [-\frac{T}{2}, \frac{T}{2})$ $\leftarrow$ one period

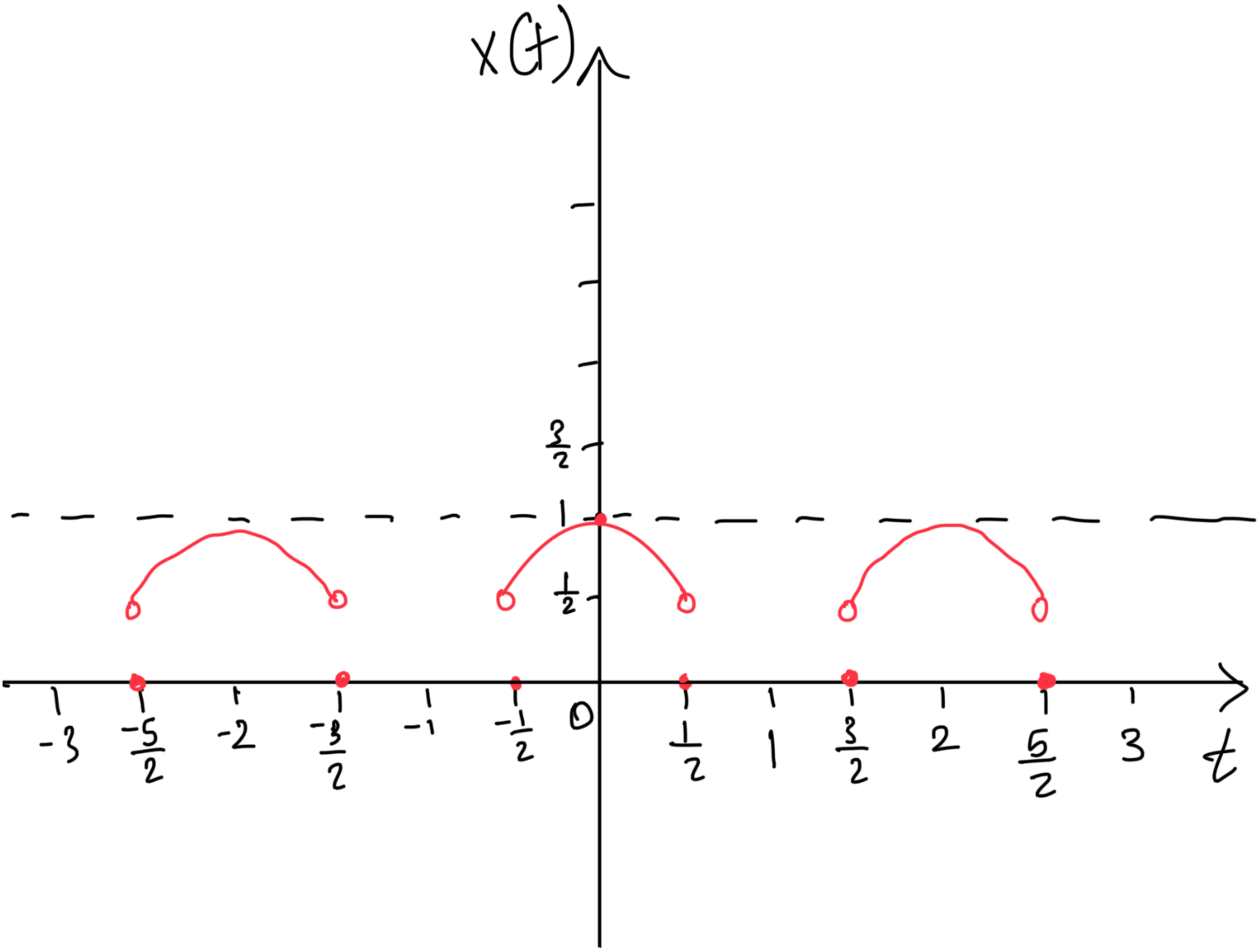
$$x(t)=\begin{cases} 1-2t^2, & -\frac{W}{2}<t<\frac{W}{2} \\ 0, & \text{otherwise} \end{cases}$$

For one Period

$$\boxed{x(t)=\begin{cases} 1-2t^2, & -\frac{1}{2}<t<\frac{1}{2} \\ 0, & \text{otherwise} \end{cases} \quad \text{Given } W=1}$$

Domain

$$-1.5T<t<1.5T \xrightarrow{T=2} \boxed{-3<t<3}$$



Integral :

$$X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{\frac{-j2\pi k t}{T}}\, dt$$

$$x(t) = \begin{cases} 1-2t^2, & -\frac{w}{2} < t < \frac{w}{2} \\ 0, & \text{otherwise} \end{cases}$$

for $-\frac{w}{2} < t < \frac{w}{2}$

$$X_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t) \quad e^{\frac{-j2\pi k t}{T}}\, dt$$

$$X_k = \frac{1}{T}\left[\int_{-T/2}^{-w/2} \cancel{0 \cdot e^{\frac{-j2\pi k t}{T}} dt}^{\,0} + \int_{-w/2}^{w/2} (1-2t^2)\cdot e^{\frac{-j2\pi k t}{T}} dt\right.$$

$$\int^{T/2} \quad e^{\frac{-j\cancel{2}\pi k t}{}} \quad \Big)$$

$$+\int_{W/2} 0 \cdot \cancel{e^{\frac{\;}{T}}}\, dt \Big) \quad 0$$

$$X_k = \frac{1}{T}\left[\underbrace{\int_{-W/2}^{W/2} e^{\frac{-j2\pi k t}{T}}\, dt}_{A=} + \underbrace{\int_{-W/2}^{W/2} -2t^2 e^{\frac{-j2\pi k t}{T}}\, dt}_{=B}\right]$$

direct Integration

Integration by Parts 2 times

$$X_k = \frac{1}{T}\left[A + B\right]$$

$$A = \int_{-W/2}^{W/2} e^{\frac{-j2\pi k t}{T}}\, dt = \frac{-T}{j2\pi k}\left[e^{\frac{-j2\pi k t}{T}}\right]_{-W/2}^{W/2}$$

$$= \frac{T}{\pi k}\, \frac{-e^{\frac{-j\cancel{2}\pi k W}{\cancel{2}T}} + e^{\frac{+j\cancel{2}\pi k W}{\cancel{2}T}}}{2j}$$

$$= \boxed{\frac{T}{\pi k}\sin\left(\frac{\pi k W}{T}\right)}$$

$$B = -2\int^{W/2} t^2 e^{\frac{-j2\pi k t}{T}}\, dt$$

$-W/2$

$u = t^2 \qquad dV = e^{\frac{-j2\pi kt}{T}}$

$du = 2t\,dt \qquad V = \frac{-T}{j2\pi k} e^{\frac{-j2\pi kt}{T}}$

$$= \underbrace{\frac{-t^2 T}{j2\pi k} e^{\frac{-j2\pi kt}{T}} \Bigg]_{-W/2}^{W/2}}_{\text{direct Integration}} - \int_{-W/2}^{W/2} \frac{-tT}{j\pi k} e^{\frac{-j2\pi kt}{T}} dt$$

$u = \frac{-tT}{j\pi k} \quad dv = e^{\frac{-j2\pi kt}{T}} dt$

$du = \frac{-T}{j\pi k} \quad V = \frac{-T}{j2\pi k} e^{\frac{-j2\pi kt}{T}}$

$$= \frac{TW^2}{2\pi k} \cdot \sin\left(\frac{\pi k W}{T}\right) - \left[\frac{-tT^2}{2\pi^2 k^2} e^{\frac{-j2\pi kt}{T}} \Bigg|_{-W/2}^{W/2} + \frac{T^2}{2\pi^2 k^2} \int_{-W/2}^{W/2} e^{\frac{-j2\pi kt}{T}} dt \right]$$

$$= \frac{TW^2}{2\pi k} \cdot \sin\left(\frac{\pi k W}{T}\right) - \left[\frac{-T^2 W}{2\pi^2 k^2} \cos\left(\frac{\pi k W}{T}\right) \right.$$

^{2}

$$+\frac{1}{\pi^2 k^2}\cos\left(\frac{\pi k w}{T}\right)\Bigg]$$

$$X_k = \frac{1}{T}\left[A+B\right]$$

$$= \frac{1}{T}\Bigg[\frac{T}{wk}\sin\left(\frac{\pi k w}{T}\right) + \frac{Tw^2}{2\pi k}\sin\left(\frac{\pi k w}{T}\right)$$

$$+\frac{T^2 w}{2\pi^2 k^2}\cos\left(\frac{\pi k w}{T}\right) - \frac{T^2}{\pi^2 k^2}\cos\left(\frac{\pi k w}{T}\right)\Bigg]$$

$$= \frac{1}{T}\Bigg[\frac{2T\pi k - w^3 k T}{2\pi w k^2}\sin\left(\frac{\pi k w}{T}\right) -$$

$$\frac{T^2\pi^2 k^2 w - 2T^2\pi^2 k^2}{2\pi^4 k^4}\cos\left(\frac{\pi k w}{T}\right)\Bigg]$$

$$\boxed{X_k = \frac{2\pi k - w^3 k}{2\pi w k^2}\sin\left(\frac{\pi k w}{T}\right) - \frac{T(w-2)}{2\pi^2 k^2}\cos\left(\frac{\pi k w}{T}\right)}$$