Lab 3 Calculations

a) $x(t) = e^{j2\pi f_0 t}$

$$X(\omega) = 2\pi\,\delta(\omega - \omega_0) \;; \quad \omega_0 = 2\pi f_0$$

b) $x(t) = \cos(2\pi f_0 t) = \cos(\omega_0 t)$

$$x(t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$X(\omega) = \frac{1}{2}\left[2\pi\,\delta(\omega - \omega_0) + 2\pi\,\delta(\omega + \omega_0)\right]$$

$$= \boxed{\pi\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]}$$

c) $x(t) = \sin(2\pi f_0 t) = \sin(\omega_0 t)$

$$x(t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}$$

$$X(\omega) = \frac{1}{2j}\left[2\pi\,\delta(\omega - \omega_0) + 2\pi\,\delta(\omega + \omega_0)\right]$$

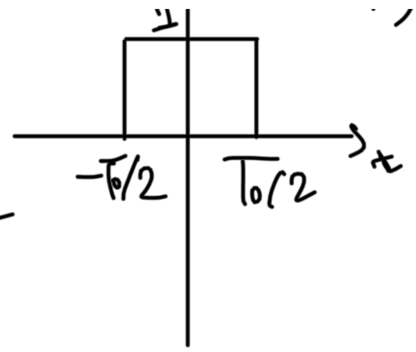
$$= \boxed{\frac{\pi}{j}\left[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)\right]}$$

d) $x(t) = \mathrm{rect}\left(\frac{t}{T}\right)$

x

$\mathrm{Rect}\left(\frac{t}{T_0}\right)$

$$X(w) = \int_{-\infty}^{\infty} \text{rect}\left(\frac{t}{T_0}\right) e^{-jwt}\, dt$$



$$= \int_{-T_0/2}^{T_0/2} 1 \cdot e^{-jwt}\, dt = \frac{-1}{jw}\left[e^{-jwt}\right]_{-T_0/2}^{T_0/2}$$

$$= \frac{-1}{jw}\; e^{\frac{-jwT_0}{2}} - e^{\frac{+jwT_0}{2}}$$

$$= \frac{1}{w}\sin\left(\frac{wT_0}{2}\right) = \boxed{T_0 \,\text{sinc}\left(\frac{wT_0}{2}\right)}$$

e) $e^{-j2\pi f_0 t}\, \text{rect}\left(\frac{t}{T_0}\right)$

Multiply by $e^{-jw_0 t}$ in time domain is shifting by $-w_0$ in Frequency domain

$$X(jw) = T_0 \,\text{sinc}\left(\frac{(w+w_0)T}{2}\right)$$

$$= \boxed{T_0 \operatorname{sinc}\left(\frac{wT_0}{2} + \frac{w_0 T_0}{2}\right)}$$

f) $\cos(2\pi f_0 t)\, \operatorname{rect}\left(\frac{t}{T_0}\right)$

$$\cos(w_0 t)\, \operatorname{rect}\left(\frac{t}{T_0}\right) = \left(\frac{e^{jw_0 t} + e^{-jw_0 t}}{2}\right) \operatorname{rect}\left(\frac{t}{T_0}\right)$$

$$\boxed{X(jw) = \frac{T_0}{2}\left[\operatorname{sinc}\left(\frac{T_0(w-w_0)}{2}\right) + \operatorname{sinc}\left(\frac{T_0(w+w_0)}{2}\right)\right.}$$

g) $x(t) = \operatorname{Rect}\left(\frac{t-t_0}{T_0}\right)$

shifting in time domain is multiplying by $e^{-jt_0 w}$ in freq. domain

$$\boxed{X(jw) = e^{-jt_0 w} \cdot T_0 \operatorname{sinc}\left(\frac{wT_0}{2}\right)}$$

h) $x(t) = e^{jw_0 t} \operatorname{rect}\left(\frac{t-t_0}{T_0}\right)$

shift by w_0 in frequency domain

Shift by w_0 in frequency domain

$$\boxed{X(jw) = e^{-jt_0(w-w_0)}\, T_0 \operatorname{sinc}\left(\frac{(w-w_0)T_0}{2}\right)}$$

i) $x(t) = \cos(2\pi f_0 t)\,\mathrm{rect}\left(\frac{t-t_0}{T_0}\right)$

$$x(t) = \left(\frac{e^{jw_0t} + e^{-jw_0t}}{2}\right)\mathrm{rect}\left(\frac{t-t_0}{T_0}\right)$$

$$\boxed{X(jw) = \frac{T_0}{2}\left[e^{-jt_0(w-w_0)} \operatorname{sinc}\left(\frac{(w-w_0)T_0}{2}\right) + e^{-jt_0(w+w_0)} \operatorname{sinc}\left(\frac{(w+w_0)T_0}{2}\right)\right]}$$

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a) $y(t) = x(t) + \sum_{i=1}^{m} A_i\, x(t-t_1)$

$$\xrightarrow{F.T}$$

$$Y(jw) = X(jw) + \sum_{i=1}^{m} A_i e^{-jwt} X(jw)$$

$$= X(jw)\left[1 + \sum A_i e^{-jwt_i}\right]$$

$$= X(j\omega)\, H(j\omega)$$

Convolution of the signal $x(t)$ with $h(t)$ corresponds to multiplication of the signals $X(j\omega)$ and $H(j\omega)$. Then:

$$H(j\omega) = 1 + \sum_{i=1}^{m} A_i e^{-j\omega t_i}$$

I.F.T

$$h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(1 + \sum_{i=1}^{m} A_i e^{-j\omega t_i}\right) e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi} \left[\int_{-\infty}^{\infty} e^{j\omega t} + \sum_{i=1}^{m} A_i e^{j\omega(t-t_i)}\, d\omega \right]$$

$$\boxed{h(t) = \delta(t) + \sum_{i=1}^{m} \delta(t-t_i)}$$

b) $$\boxed{H(jw) = 1 + \sum_{i=1}^{m} A_i e^{-jwt_i}}$$

c) $$\boxed{Y(jw) = X(jw) \cdot H(jw)}$$

d) $$\boxed{X(jw) = \frac{Y(jw)}{H(jw)}}$$