University of Texas at Austin 2024 Invitational

Optics B/C Exam

Directions:

- **This is a class set, DO NOT WRITE**
- Write all answers on the answer sheets; any marks elsewhere will not be scored. If additional space is needed on a question, **clearly mark the question number and your team number on the top left of the page** and attach to your test.
- There is no penalty for wrong answers. Answer as many questions as possible, even if you aren't sure if you're correct. For questions with parts that build on one another, points will be awarded for correct answers on parts regardless of previous part completion.
- Units are required whenever giving a numerical answer. Don't worry about significant figures.
- Good luck on the test!

Section 1

Circle True or False

1. True False — In a converging lens, the image of an object located at twice the focal length is real, inverted, and the same size as the object.

2. True False — The critical angle for total internal reflection depends solely on the refractive index of the denser medium.

3. True False — Diffraction occurs prominently when the size of the aperture is comparable or smaller than the wavelength of light passing through it.

4. True False — Polarization of light can only occur through reflection and cannot be achieved by scattering.

5. True False — The resolving power of a microscope depends on the wavelength of the light used and the numerical aperture of the objective lens.

6. True False — Brewster's angle occurs when light reflects at an angle such that the reflected and refracted rays are perpendicular to each other.

7. True False — For a concave mirror, the image of an object placed beyond the center of curvature is virtual.

8. True False — The thin lens equation is valid for lenses of any shape and thickness.

9. True False — Light passing through a prism always undergoes chromatic dispersion.

10. True False — Snell's law can be used to describe the behavior of light as it transitions between two media with the same refractive index.

11. True False — The accuracy of a sextant decreases with increasing altitude of the observed celestial body.

Section 2

Circle the correct answer

1. (3 points) What is the magnification of an object placed 10 cm from a converging lens with a focal length of 5 cm?

   A. -2

   B. $+2$

   C. -0.5

   D. $+0.5$

   E. None of the above

2. (3 points) Which of the following phenomena demonstrates the wave nature of light?

   A. Diffraction

   B. Photoelectric effect

   C. Specular reflection

   D. Refraction

   E. None of the above

3. (3 points) A light ray enters a medium with a refractive index of 1.33 from insulating foam ($n = 1.02$) at an angle of 45°. What is the angle of refraction?

   A. 33.3°

   B. 32.8°

   C. 50.5°

   D. 75.1°

   E. None of the above

4. (3 points) What is the maximum number of bright fringes that can be observed in a double-slit interference experiment with slit separation of 0.1 mm and wavelength of 600 nm?

   A. 331

   B. 333

   C. 335

   D. 337

   E. None of the above

5. (3 points) The focal length of a convex mirror is -10 cm. Where is the image located if the object is placed 15 cm from the mirror?

   A. -4 cm

   B. $+4$ cm

   C. -6 cm

D. +6 cm

E. None of the above

6. (3 points) What happens to light in an optical fiber that results in total internal reflection?

    A. Light travels faster in the core than in the cladding

    B. The core has a higher refractive index than the cladding

    C. Light slows down when it enters the fiber

    D. The core and cladding have the same refractive index

    E. None of the above

7. (3 points) The critical angle for light traveling from glass ($n = 1.5$) to water ($n = 1.33$) is closest to:

    A. 42.1°

    B. 48.8°

    C. 62.5°

    D. 63.3°

    E. None of the above

8. (3 points) Which of the following optical instruments uses two convex lenses to form an image?

    A. Telescope

    B. Periscope

    C. Microscope

    D. Camera

    E. None of the above

9. (3 points) The dispersion of light through a prism causes which of the following phenomena?

    A. Polarization

    B. Chromatic aberration

    C. Total internal reflection

    D. Brewster's angle

    E. None of the above

10. (3 points) What is the correct relationship between the object distance d_o, the image distance d_i, and the focal length f for a convex lens?

    A. $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

    B. $\frac{1}{f} = \frac{d_i}{d_o}$

    C. $\frac{1}{f} = d_o + d_i$

D. $\frac{1}{f} = d_o d_i$

E. None of the above

11. (3 points) A sextant is primarily used to measure the angle between two objects, such as the horizon and a celestial body. What is the primary principle that allows the sextant to function?

A. Polarization of light waves

B. Diffraction of light through a narrow slit

C. Refraction of light through a prism

D. Reflection of light using two mirrors

E. None of the above

Section 3

Write out a short response to the following questions

1. (4 points) How does the vector form of the law of reflection apply to anisotropic materials where the surface properties change with direction?

Answer: The law of reflection can be adjusted to consider anisotropic materials by incorporating the directional dependence of surface properties, which leads to different reflection behaviors based on the material's orientation. In anisotropic materials, surface properties such as the refractive index can vary depending on the direction of light. [2] The vector form of the law of reflection still holds, but it requires modification to account for the variation in reflection depending on the direction. This means that the angle of reflection is no longer uniform across the surface, as it is influenced by the anisotropic nature of the material. [2]

2. (4 points) How does the Abbe number affect the dispersion of light in a prism? Calculate the chromatic aberration for a prism made of crown glass with an Abbe number of 60 when light enters at an angle of 45°.

Answer: The Abbe number is a measure of how much dispersion (the spreading of light into its component colors) a material exhibits. A higher Abbe number indicates lower dispersion, meaning that the material causes less chromatic aberration. [2] For crown glass with an Abbe number of 60, the dispersion is relatively low. Chromatic aberration can be calculated by examining the difference in the refractive index of the material for different wavelengths of light. When light enters the prism at an angle of 45°, the dispersion across wavelengths leads to slight differences in how each color bends, resulting in chromatic aberration. [2]

3. (4 points) Explore how additive color theory is applied in modern display technology (e.g., OLED) and compare it with subtractive color theory in high-quality printing.

Answer: Additive color theory is used in modern display technologies such as OLED screens. In this theory, colors are created by combining different intensities of red, green, and blue (RGB) light. When these colors are mixed together in varying proportions, they can produce almost any color. For example, combining all three in equal intensity results in white light. [2] In contrast, subtractive color theory is used in high-quality printing. This theory involves mixing pigments that absorb certain wavelengths of light and reflect others. The primary colors in subtractive color theory are cyan, magenta, and yellow (CMY), and mixing these pigments results in the absorption of more light, thus producing darker colors. When all three pigments are combined, they ideally absorb all light and create black. [2]

4. (4 points) Calculate the reflectance for light at normal incidence on a metal surface, given its complex refractive index. Discuss how this affects the perception of color for different metals (e.g., gold, silver).

Answer: The reflectance for light at normal incidence on a metal surface can be calculated using the Fresnel equations, which take into account the material's complex refractive index. This complex refractive index includes both the real part, representing how light slows down in the material, and the imaginary part, representing how much light is absorbed by the material. [2] The amount of light reflected at the surface affects how we perceive the color of metals. For instance, gold appears yellow because it reflects red and yellow wavelengths more efficiently while absorbing blue and green. Silver, on the other hand, reflects nearly all wavelengths of visible light, giving it its shiny, white appearance. [2]

5. (4 points) Explain how spherical aberration affects vision and how the eye naturally compensates for this. Discuss the limitations of this compensation in aging populations.

Answer: Spherical aberration occurs when light rays passing through the periphery of a lens (such as the eye's lens) are focused at different points than light rays passing through the center, causing a blurred image. This effect leads to a decrease in image sharpness. [2] The eye compensates for spherical aberration naturally by constricting the pupil, which reduces the amount of light entering through the periphery and allows more focused central rays to dominate. However, as individuals age, the ability to constrict the pupil diminishes, and the eye's lens becomes less flexible, reducing the effectiveness of this compensation and leading to more pronounced visual distortions. [2]

6. (4 points) Model how the arrangement of photoreceptors in the retina can create blind spots and why our brains typically don't notice these spots in normal vision.

Answer: The arrangement of photoreceptors in the retina creates a natural blind spot at the location where the optic nerve exits the eye because there are no photoreceptors (rods or cones) in that region. [2] Despite this blind spot, the brain uses information from the other eye and surrounding visual data to "fill in" the gap, making the blind spot essentially unnoticeable in normal vision. This process is part of how the brain compensates for missing visual information to create a seamless visual experience. [2]

7. (4 points) Explain how the lensmaker's equation can be modified to account for spherical aberration in high-precision optical systems.

Answer: The lensmaker's equation, which is typically used to calculate the focal length of a lens based on its curvature and refractive index, can be modified to account for spherical aberration by including higher-order terms that correct for the varying focal lengths of peripheral rays versus central rays. [2] In high-precision optical systems, spherical aberration is reduced by altering the lens shape to aspheric profiles, which can help focus all incoming rays at a single point, regardless of their position on the lens. [1] Additionally, multi-element lens systems can be designed to counteract aberrations, where different lens surfaces work together to eliminate distortion. [1]

8. (4 points) Explain the Rayleigh scattering polarization effect in the context of the Earth's atmosphere. How does the degree of polarization vary with the angle of observation?

Answer: Rayleigh scattering occurs when sunlight interacts with the small gas molecules in Earth's atmosphere, scattering shorter wavelengths of light (such as blue) more than longer wavelengths. This scattering effect causes the sky to appear blue. [1] The polarization of light increases as a result of Rayleigh scattering because light waves oscillate in a specific direction when scattered, which leads to partial polarization of the scattered light. [1] The degree of polarization depends on the angle of observation: it is maximized at a 90-degree angle from the sun (perpendicular to the light source) and decreases when looking directly toward or away from the sun. [2]

9. (4 points) Derive the threshold condition for laser action in terms of population inversion and discuss the impact of cavity design on laser coherence length.

Answer: For laser action to occur, the population inversion must reach a threshold where the number of atoms in an excited state exceeds the number in the ground state, enabling stimulated emission to dominate over absorption. This threshold condition can be expressed as a gain coefficient exceeding the losses in the laser cavity. [2] The design of the cavity affects the coherence length of the laser, with longer cavities generally leading to longer coherence lengths. This is because the cavity supports fewer modes, allowing for more coherent light emission. [1] Additionally, mirror reflectivity at the ends of the cavity influences the coherence length; highly reflective mirrors help maintain coherence over longer distances. [1]

10. (4 points) Explore how femtosecond lasers can be used in LASIK surgery to correct aberrations in the human eye. Include a discussion of wavefront-guided techniques.

Answer: Femtosecond lasers are used in LASIK surgery to create precise, controlled corneal flaps with minimal damage to surrounding tissues. The extremely short pulse duration of the femtosecond laser allows for highly accurate cuts, reducing the risk of complications and improving the healing process. [2] In addition to reshaping the cornea, femtosecond lasers are employed in wavefront-guided LASIK, which maps the unique imperfections (or higher-order aberrations) of a patient's eye. [1] These aberrations are then corrected with customized laser ablation, providing improved visual outcomes compared to traditional LASIK techniques. [1]

11. (4 points) Explain the role of laser diffraction in reading information stored in CD/DVD media, and discuss how error correction algorithms help in compensating for optical imperfections.

Answer: Laser diffraction is key in reading information stored in CD/DVD media. A laser beam is focused onto the disc surface, and the reflected light is diffracted by the pits and lands encoded on the disc. These differences in reflection are interpreted as binary data, which the reading system converts into digital information. [2] Due to imperfections on the disc's surface, such as scratches or dust, the readout may not be perfect. Error correction algorithms, such as Reed-Solomon coding, are used to detect and correct small errors in the data by adding redundancy and error-checking mechanisms to the encoded information, ensuring accurate data retrieval even in the presence of optical imperfections. [2]

Section 4

Write out your work and answer to the following questions

1. (4 points) A beam of light enters a prism with a refractive index of 1.6 at an angle of 30°. Calculate the angle of refraction, to the nearest hundredth of a degree, inside the prism.

Answer: Using Snell's law, we have:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where $n_1 = 1$ (for air), $\theta_1 = 30^\circ$, and $n_2 = 1.6$ (for the prism). Solving for θ_2:

$$1 \times \sin 30^\circ = 1.6 \times \sin\theta_2$$
$$\sin\theta_2 = \frac{\sin 30^\circ}{1.6} = \frac{0.5}{1.6} = 0.3125$$
$$\theta_2 = \sin^{-1}(0.3125) \approx 18.21^\circ$$

Thus, the angle of refraction inside the prism is approximately $\boxed{18.21^\circ}$.

2. (4 points) A plano-convex lens has a radius of curvature of 20 cm. Calculate the focal length, to the nearest tenth of a centimeter, of the lens if it is made of glass with a refractive index of 1.5.

Answer: The focal length of a plano-convex lens is given by the lensmaker's equation:

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For a plano-convex lens, $R_1 = 20\,\text{cm}$ and $R_2 = \infty$ (because one surface is flat). The refractive index $n = 1.5$. Substituting into the equation:

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{20} - \frac{1}{\infty}\right)$$
$$\frac{1}{f} = 0.5 \times \frac{1}{20} = \frac{0.5}{20} = 0.025$$
$$f = \frac{1}{0.025} = 40\,\text{cm}$$

Thus, the focal length of the lens is $\boxed{40.0\,\text{cm}}$.

3. (4 points) Calculate the total internal reflection angle, to the nearest hundredth of a degree, for light moving from water ($n = 1.33$) to air.

Answer: The critical angle for total internal reflection can be calculated using the formula:

$$\sin\theta_c = \frac{n_2}{n_1}$$

where $n_1 = 1.33$ (for water) and $n_2 = 1$ (for air). Solving for θ_c:

$$\sin\theta_c = \frac{1}{1.33} = 0.7519$$

$$\theta_c = \sin^{-1}(0.7519) \approx 48.75^\circ$$

Thus, the total internal reflection angle is approximately $\boxed{48.75^\circ}$.

4. (4 points) A converging lens has a focal length of 20 cm. An object is placed 30 cm from the lens. Calculate the position relative to the lens, to the nearest tenth of a centimeter, and the nature (real or virtual, magnified or diminished) of the image.

Answer: Using the lens formula:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

where $f = 20$ cm (focal length) and $d_o = 30$ cm (object distance). Solving for d_i (image distance):

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{20} - \frac{1}{30} = \frac{3-2}{60} = \frac{1}{60}$$

$$d_i = 60\,\text{cm}$$

Since the image distance is positive, the image is real. To determine the magnification (M):

$$M = -\frac{d_i}{d_o} = -\frac{60}{30} = -2$$

The negative magnification indicates that the image is inverted. Therefore, the image is real, inverted, and magnified by a factor of 2. Thus, the image is located $\boxed{60.0\,\text{cm}}$ from the lens, and it is real and magnified.

5. (4 points) A beam of light passes through a double slit, with slit separation of 0.5 mm, and produces an interference pattern on a screen placed 1.5 meters away. If the wavelength of light is 600 nm, calculate the distance, to the nearest tenth of a millimeter, between the central bright fringe and the first-order bright fringe.

Answer: The distance between the central bright fringe and the first-order bright fringe can be calculated using the formula for double-slit interference:

$$y = \frac{m\lambda L}{d}$$

where: - $m = 1$ (for the first-order fringe), - $\lambda = 600\,\text{nm} = 600 \times 10^{-9}\,\text{m}$ (wavelength of light), - $L = 1.5\,\text{m}$ (distance to the screen), - $d = 0.5\,\text{mm} = 0.5 \times 10^{-3}\,\text{m}$ (slit separation).

Substituting the values:

$$y = \frac{1 \times 600 \times 10^{-9} \times 1.5}{0.5 \times 10^{-3}} = \frac{900 \times 10^{-9}}{0.5 \times 10^{-3}} = 1.8 \times 10^{-3}\,\text{m} = 1.8\,\text{mm}$$

Thus, the distance between the central bright fringe and the first-order bright fringe is approximately $\boxed{1.8\,\text{mm}}$.

6. (4 points) Light of wavelength 500 nm passes through a diffraction grating with 10^4 lines per centimeter. Calculate the angle, to the nearest hundredth of a degree, where the second-order maximum occurs.

Answer: The angle of the second-order maximum can be found using the diffraction grating formula:

$$d \sin\theta = m\lambda$$

where: - $m = 2$ (second-order maximum), - $\lambda = 500\,\text{nm} = 500 \times 10^{-9}\,\text{m}$ (wavelength of light), - $d = \frac{1}{10^4\,\text{lines/cm}} = \frac{1}{10^6\,\text{lines/m}} = 10^{-6}\,\text{m}$ (grating spacing).

Substituting the values:

$$10^{-6} \sin\theta = 2 \times 500 \times 10^{-9}$$

$$\sin\theta = \frac{1000 \times 10^{-9}}{10^{-6}} = 1$$

Since $\sin\theta = 1$, $\theta = 90°$.

Thus, the second-order maximum occurs at $\boxed{90.00°}$.

7. (4 points) A telescope's objective lens has a focal length of 2 meters, and its eyepiece has a focal length of 10 cm. Calculate the magnification produced by this telescope, assuming normal adjustment.

Answer: The magnification M of a telescope is given by the formula:

$$M = \frac{f_{\text{objective}}}{f_{\text{eyepiece}}}$$

where $f_{\text{objective}} = 2$ meters $= 200$ cm and $f_{\text{eyepiece}} = 10$ cm.

Substituting the values:

$$M = \frac{200}{10} = 20$$

Thus, the magnification produced by the telescope is $\boxed{20}$.

8. (4 points) A monochromatic light source has a wavelength of 400 nm in air. If the light enters water, which has a refractive index of 1.33, calculate the new wavelength of the light in water, to the nearest nanometer.

Answer: The wavelength of light in a medium is given by:

$$\lambda_{\text{medium}} = \frac{\lambda_{\text{air}}}{n}$$

where $\lambda_{\text{air}} = 400\,\text{nm}$ and $n = 1.33$ (refractive index of water).

Substituting the values:

$$\lambda_{\text{medium}} = \frac{400}{1.33} \approx 300.75\,\text{nm}$$

Rounding to the nearest nanometer, the new wavelength of the light in water is $\boxed{301\,\text{nm}}$.

9. (4 points) A concave lens has a focal length of −20 cm. If an object is placed 15 cm from the lens, calculate the image distance, to the nearest hundredth of a centimeter, and describe the nature (real or virtual, magnified or diminished) of the image.

Answer: Using the lens formula:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

where $f = -20\,\text{cm}$ and $d_o = 15\,\text{cm}$. Solving for d_i (image distance):

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-20} - \frac{1}{15} = \frac{-3-4}{60} = \frac{-7}{60}$$

$$d_i = \frac{60}{-7} \approx -8.57\,\text{cm}$$

The negative sign indicates that the image is virtual. Since the absolute value of d_i is less than d_o, the image is also diminished. Thus, the image is located at $\boxed{-8.57\,\text{cm}}$, and it is virtual and diminished.

10. (4 points) A light ray traveling through glass (refractive index 1.5) strikes a glass-to-air boundary at an angle of incidence of 40°. Calculate the angle of refraction, to the nearest hundredth of a degree, of the light as it passes into the air.

Answer: Using Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where $n_1 = 1.5$ (refractive index of glass), $\theta_1 = 40°$ (angle of incidence), and $n_2 = 1$ (refractive index of air). Solving for θ_2 (angle of refraction):

$$1.5 \times \sin 40° = 1 \times \sin\theta_2$$

$$\sin\theta_2 = 1.5 \times 0.6428 = 0.9642$$

$$\theta_2 = \sin^{-1}(0.9642) \approx 74.62°$$

Thus, the angle of refraction is approximately $\boxed{74.62°}$.

11. (4 points) In a double-slit experiment, the distance between the slits is 0.13 mm, and the distance to the screen is 1 meter. If the second-order bright fringe is observed at a position 12 mm away from the central maximum, calculate the wavelength of the light, to the nearest nanometer.

Answer: The position of the m-th order bright fringe in a double-slit experiment is given by:

$$y = \frac{m\lambda L}{d}$$

where:

- $y = 12\,\text{mm} = 12 \times 10^{-3}\,\text{m}$ (position of the second-order bright fringe)
- $m = 2$ (second-order)
- $L = 1\,\text{m}$ (distance to the screen)
- $d = 0.13\,\text{mm} = 0.13 \times 10^{-3}\,\text{m}$ (slit separation)

Substituting the values:

$$12 \times 10^{-3} = \frac{2 \times \lambda \times 1}{0.13 \times 10^{-3}}$$

$$\lambda = \frac{12 \times 10^{-3} \times 0.13 \times 10^{-3}}{2 \times 1} = 780 \times 10^{-9}\,\text{m} = 780\,\text{nm}$$

Thus, the wavelength of the light is $\boxed{780\,\text{nm}}$.

12. (4 points) A certain telescope has an angular resolution of $1.22 \times \frac{\lambda}{D}$, where λ is the wavelength of light and D is the diameter of the objective lens. If the telescope's objective lens has a diameter of 1 meter and the wavelength of light is 500 nm, calculate the angular resolution to the nearest thousandth of an arcsecond.

Answer: The formula for angular resolution is:

$$\theta = 1.22 \times \frac{\lambda}{D}$$

where $\lambda = 500\,\text{nm} = 500 \times 10^{-9}\,\text{m}$ and $D = 1\,\text{m}$.

Substituting the values:

$$\theta = 1.22 \times \frac{500 \times 10^{-9}}{1} = 6.1 \times 10^{-7}\,\text{radians}$$

To convert this to arcseconds, recall that $1\,\text{radian} = 206,265\,\text{arcseconds}$:

$$\theta = 6.1 \times 10^{-7} \times 206,265 \approx 0.126\,\text{arcseconds}$$

Thus, the angular resolution is approximately $\boxed{0.126\,\text{arcseconds}}$.

13. (4 points) A laser emits light with a power of 2 mW and a wavelength of 650 nm. Calculate the energy, in joules, of a single photon emitted by the laser.

Answer: The energy of a single photon is given by the formula:

$$E = \frac{hc}{\lambda}$$

where: - $h = 6.626 \times 10^{-34}\,\text{J} \cdot \text{s}$ (Planck's constant), - $c = 3.0 \times 10^{8}\,\text{m/s}$ (speed of light), - $\lambda = 650\,\text{nm} = 650 \times 10^{-9}\,\text{m}$ (wavelength of the laser light).

Substituting the values:

$$E = \frac{6.626 \times 10^{-34} \times 3.0 \times 10^{8}}{650 \times 10^{-9}} = \frac{1.9878 \times 10^{-25}}{650 \times 10^{-9}} \approx 3.058 \times 10^{-19}\,\text{J}$$

Thus, the energy of a single photon is approximately $\boxed{3.06 \times 10^{-19}\,\text{J}}$.

14. (4 points) A concave mirror has a focal length of 12 cm. Calculate the magnification of the image if the object is placed 8 cm from the mirror.

Answer: First, we use the mirror equation to find the image distance:

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

where:

- $f = 12$ cm (focal length of the concave mirror)
- $d_o = 8$ cm (object distance)

Solving for d_i (image distance):

$$\frac{1}{12} = \frac{1}{8} + \frac{1}{d_i}$$

$$\frac{1}{d_i} = \frac{1}{12} - \frac{1}{8} = \frac{2-3}{24} = -\frac{1}{24}$$

$$d_i = -24\,\text{cm}$$

Now, the magnification M is given by:

$$M = -\frac{d_i}{d_o} = -\frac{-24}{8} = 3$$

Thus, the magnification of the image is $\boxed{3}$, meaning the image is magnified and virtual (since the image distance is negative).

15. (8 points) Prove that when light is incident at Brewster's angle, the reflected light is completely polarized perpendicular to the plane of incidence. *Hint: Start by using Fresnel's equations and prove that the reflection coefficient for light polarized parallel to the plane of incidence goes to zero at Brewster's angle.*

Answer: Brewster's angle, denoted as θ_B, is the angle of incidence at which light, when reflected from the boundary between two media, becomes fully polarized perpendicular to the plane of incidence. The condition for Brewster's angle is given by:

$$\theta_B = \tan^{-1}\left(\frac{n_2}{n_1}\right)$$

where:

- n_1 is the refractive index of the first medium,
- n_2 is the refractive index of the second medium.

At Brewster's angle, the reflected and refracted rays are perpendicular, meaning:

$$\theta_r + \theta_t = 90^\circ$$

where θ_r is the reflection angle and θ_t is the refraction angle.

To describe the reflection and transmission of light at the boundary, we use Fresnel's equations for the reflection coefficients of parallel and perpendicular polarizations.

- For parallel-polarized light ($r_\parallel$):

$$r_\parallel = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t}$$

- For perpendicular-polarized light ($r_\perp$):

$$r_\perp = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t}$$

where:

- θ_i is the angle of incidence
- θ_t is the angle of refraction, related by Snell's law: $n_1 \sin\theta_i = n_2 \sin\theta_t$

At Brewster's angle, the reflected light is completely polarized perpendicular to the plane of incidence. This implies that the reflection coefficient for the parallel component $r_\parallel$ must be zero. Let's prove that this is true.

We know that at Brewster's angle:

$$\theta_r + \theta_t = 90^\circ \quad \Rightarrow \quad \theta_t = 90^\circ - \theta_B$$

Using Snell's law:

$$n_1 \sin\theta_B = n_2 \sin\theta_t = n_2 \cos\theta_B$$

Substituting this into Fresnel's equation for $r_\parallel$:

$$r_\parallel = \frac{n_2 \cos\theta_B - n_1 \cos(90^\circ - \theta_B)}{n_2 \cos\theta_B + n_1 \cos(90^\circ - \theta_B)}$$

Since $\cos(90^\circ - \theta_B) = \sin\theta_B$, we get:

$$r_\parallel = \frac{n_2 \cos\theta_B - n_1 \sin\theta_B}{n_2 \cos\theta_B + n_1 \sin\theta_B}$$

At Brewster's angle, $\tan\theta_B = \frac{n_2}{n_1}$, or equivalently, $n_1 \tan\theta_B = n_2$, which implies:

$$n_1 \sin\theta_B = n_2 \cos\theta_B$$

Substituting this into the equation for $r_\parallel$:

$$r_\parallel = \frac{n_2 \cos\theta_B - n_2 \cos\theta_B}{n_2 \cos\theta_B + n_2 \cos\theta_B} = 0$$

Thus, at Brewster's angle, the reflection coefficient for parallel-polarized light is zero, meaning no parallel-polarized light is reflected.

Since $r_\parallel = 0$ at Brewster's angle, no light polarized parallel to the plane of incidence is reflected. Therefore, the reflected light is fully polarized perpendicular to the plane of incidence (s-polarized). ■