Training a Neural Network with Stochastic Frank-Wolfe

Optimation for Data Science Course Project
Data Science–University of Padua

October 8, 2022

Overview

- Stochastic variants
- Regularization
- Projection free algorithm
- $\min_{x\in\Omega} F(x) = \min_{x\in\Omega} \frac{1}{m}\sum_{i=0}^{m} f_i(x)$
- $\min_{\theta\in\Omega} F(\theta) = -\frac{1}{m}\sum_{i=0}^{m}(y^i log(\hat{y}^i) + (1-y^i)log(1-\hat{y}^i))$

- L_1-ball constraint
- $C(radius) = \{x \in R^n : ||x||_1 \leq radius\}$
- $v_t = \arg\min_{v \in c} \langle \tilde{\nabla}(F_t, v) \rangle$
- Result of the LMO:

$$v_t = \begin{cases} diameter \times sign(-\nabla_{i_k} f(\theta_k)) \cdot, & \text{if } i_k = \arg\max_i |\nabla_i f(\theta_k)|. \\ 0 & \text{otherwise.} \end{cases}$$

- $\theta_{t+1} = \theta_t + \alpha(v_t - \theta_t)$

Algorithm Stochastic Frank-Wolfe method for l_1-ball

Require: Starting from a point inside the region
  for k=1,.... **do**
    Uniformly sample i.i.d. $i_1.i_2, ..., i_b$ from $[1, .., n]$
    $\tilde{\nabla} L(\theta_k) \leftarrow \frac{1}{b} \sum_{j=1}^{b} \nabla f_{i_j}(\theta_k)$
    Set $\hat{\theta}_k = diameter \times sign(-\tilde{\nabla}_{i_k} L(\theta_k))$, with $i_k = \arg\max_i |\tilde{\nabla}_i L(\theta_k)|$
    if $\hat{\theta}_k$ satisfies some specific condition, then STOP
    Set $\theta_{k+1} = \theta_k + \alpha_k(\hat{\theta}_k - \theta_k)$
  end for

Algorithm Stochastic Variance Reduced Frank-Wolfe method for l_1-ball

Require: Starting from a point inside the region
  for t=0,...,S-1 **do**
    take snapshot $\theta_0 = \theta_t$ and compute $\nabla F(\theta_0)$
    for k=1,....m-1 **do**
      Uniformly sample i.i.d. $i_1.i_2, ..., i_b$ from $[1, .., n]$
      $\tilde{\nabla} F(\theta_k) \leftarrow \nabla F(x_0) + \frac{1}{b_k}\sum_{j=1}^{b_k}(\nabla f_{i_j}(x_k) - \nabla f_{i_j}(\theta_0))$
      Set $\hat{\theta}_k = diameter \times sign(-\tilde{\nabla}_{i_k} F(\theta_k))$, with $i_k = \arg\max_i |\tilde{\nabla}_i F(\theta_k)|$
      Set $\theta_{k+1} = \theta_k + \alpha_k(\hat{\theta}_k - \theta_k)$
    end for
  $\theta_{t+1} \leftarrow \theta_{K_t}$
  end for

- Initialization of parameter
- Forward Propagation
- Backward propagation
- Updating Parameters
- Prediction function
- Cost function
- Main function

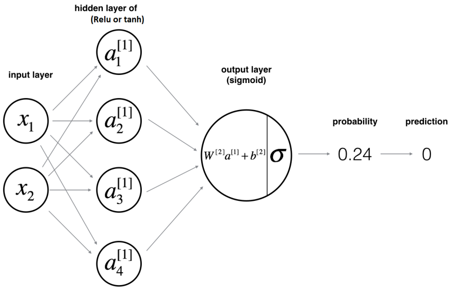


Table: SFW hyperparameters, with ReLu as the activation function

Data set	Learning rate	Batch size	l_1 ball diameter	Epochs	Hidden unit size
fashion mnist	0.001	128	5	20	32
moon	0.0008	32	3	10	16
fruit	0.001	32	20	10	64

Table: SVRF hyperparameters, with ReLu as the activation function

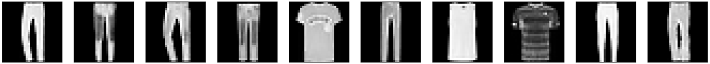
Data set	Learning rate	Batch size	l_1 ball diameter	Epochs	Hidden unit size	inner loop size
fashion mnist	0.005	128	5	20	32	20
moon	0.003	32	3	10	16	10
fruit	0.001	32	20	10	64	10

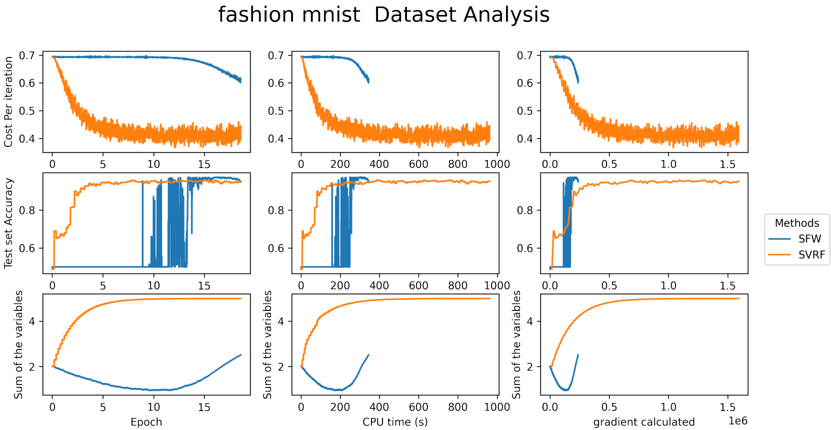
Table: Test Set Accuracy

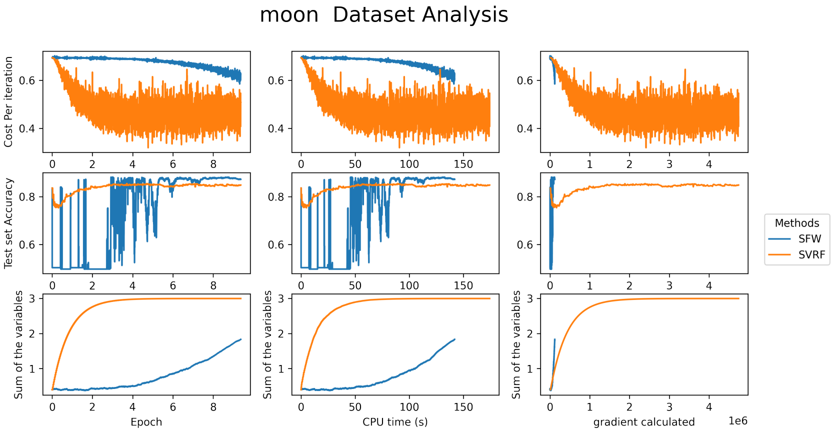
Dataset	SFW	SVRF
fashion mnist	94.9%	**95.2%**
moon	**87.3%**	84.5%
fruit	86.7%	**91.9%**

Table: Training Loss

Dataset	SFW	SVRF
fashion mnist	0.613	**0.4371**
moon	0.625	**0.508**
fruit	0.415	**0.095**




fashion mnist Dataset Analysis
Cost Per iteration
Test set Accuracy
Sum of the variables
Epoch
CPU time (s)
gradient calculated
1e6
Methods
SFW
SVRF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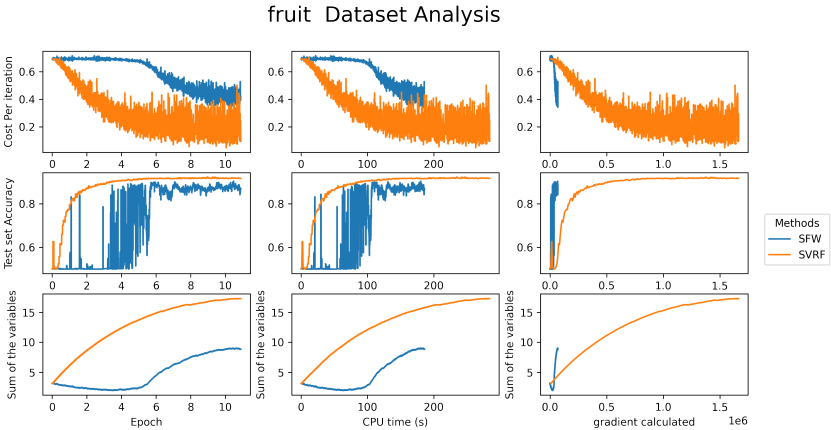
fruit Dataset Analysis
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Methods
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Thank you