

# EEE499 - Real-Time Embedded System Design

## Schedulability Part 2



# RM Least Upper Bound (RM LUB)

$$U = \sum_{i=0}^m (e_i / p_i) \leq m(2^{\frac{1}{m}} - 1)$$

**U:** Utilization of the CPU that is achievable

**e<sub>i</sub>:** Execution time of task i

**m:** Total number of tasks sharing common CPU resources

**p<sub>i</sub>:** Release period of task i

# Response Time Analysis

- a disadvantage of utilization based schedulability testing for RM LUB is that it is sufficient but not necessary
- to supplement this test, we introduce the notion of **Response Time Analysis**

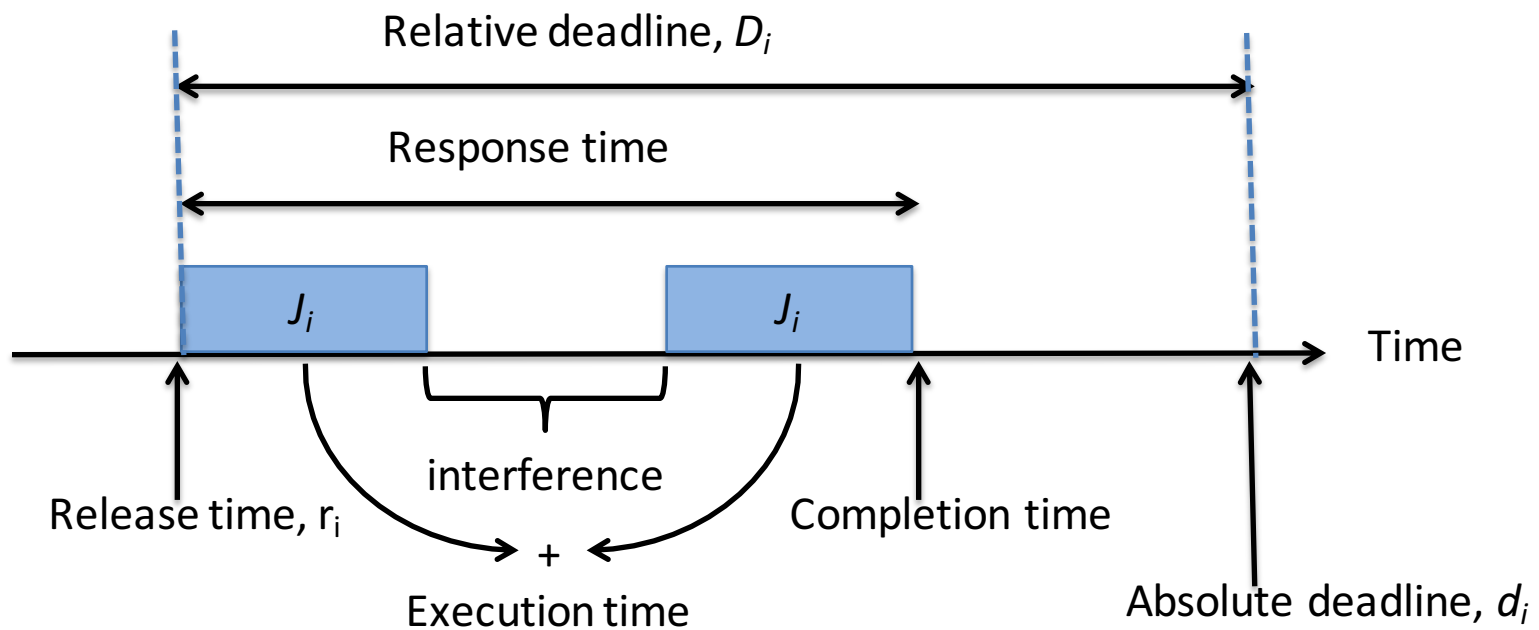
# Response Time Analysis

Response time analysis allows to:

- predicts the worst case response time for each task
- compare each task's response time to its deadline

If all worst case response times are less than their respective deadlines, the system is schedulable

# Response Time



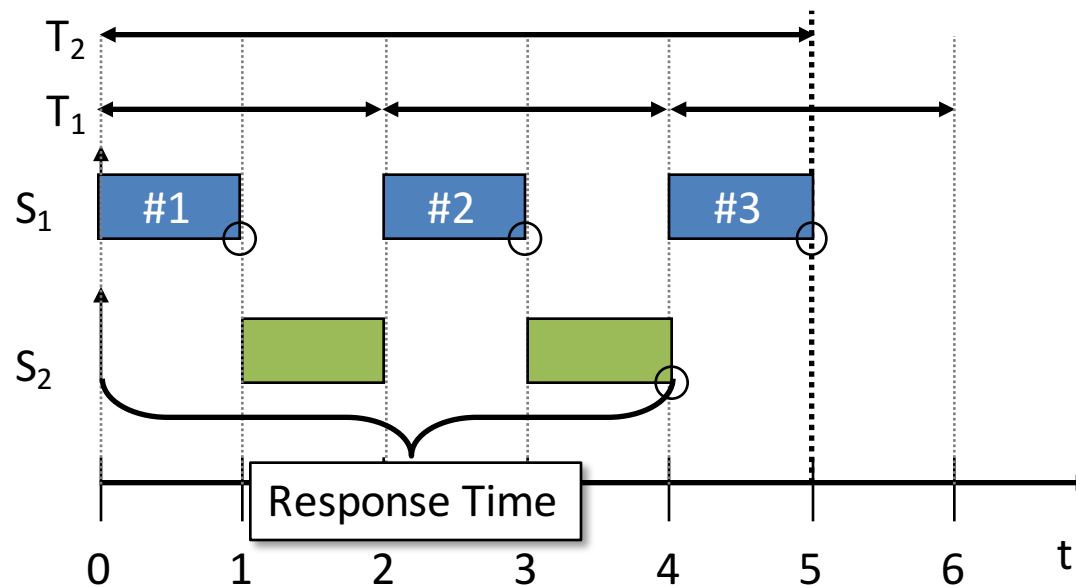
# Response Time

$$S_1 = (2, 1)$$

$$S_2 = (5, 2)$$

$$R_2 = e_2 + \overbrace{e_{1,1} + e_{1,2}}^{I_1}$$

$$R_2 = 2 + 1 + 1 = 4$$



# Response Time

Response time of a task is defined to be the sum of its own worst case execution time and its maximum interference

$$R_i = e_i + I_i \quad (1)$$

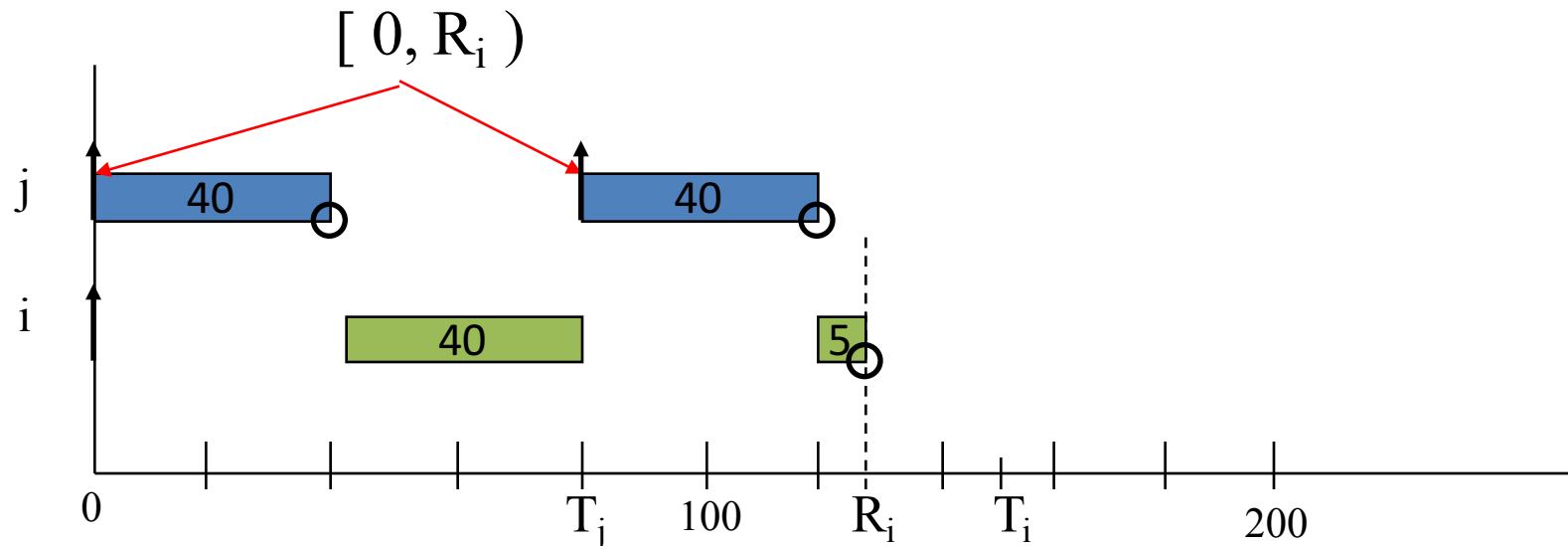
where  $I_i$  is the maximum interference\* that task  $i$  can experience in any time interval  $[t, t + R_i)$

\*The condition for maximum interference occurs when all higher priority tasks are released at the same time as task  $i$ , the *critical instant*.

# Counting Releases

- consider task  $i$  and a higher priority task  $j$
- now, the **number of releases** of task  $j$  in time interval  $0$  to  $R_i$  can be derived as follows:

$$\# \text{ of releases }_j = \lceil R_i / p_j \rceil \quad (2)$$





# Determining Interference

- from this, the maximum interference of task  $j$  on task  $i$  in interval 0 to  $R_i$  is given by:

$$\text{interference}_{\max} = \left\lceil R_i/p_j \right\rceil e_j \quad (3)$$

$[0, R_i)$

- but there may be other higher priority tasks, therefore:

$$I_i = \sum_{j=1}^k \left\lceil R_i/p_j \right\rceil e_j \quad (4)$$

where  $P(j) < P(k)$

# Calculating Response Time

- substituting equation (4) into (1) gives the general expression for response time:

$$R_i = e_i + \sum_{j=1}^k \left\lceil R_i / p_j \right\rceil e_j \quad (5)$$

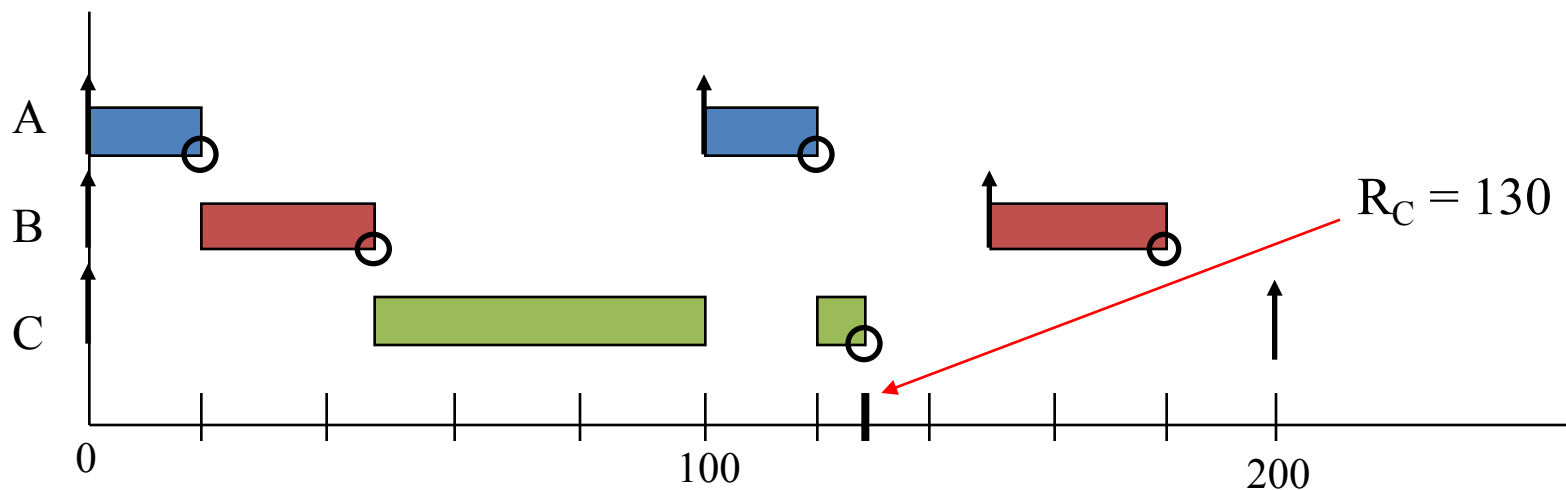
- note the following issue (problem):
  - $R_i$  is on both sides of this equation

# Understanding Response Time

(Example 1 from last class)

$$\begin{aligned} R_C &= e_C + \lceil R_C/p_A \rceil e_A + \lceil R_C/p_B \rceil e_B \\ &= 60 + 2(20) + 1(30) \\ &= 130 \end{aligned}$$

$C_A, T_A$	$C_B, T_B$	$C_C, T_C$
System = {(20, 100), (30, 150), (60, 200)}		



# Calculating Response Time

- essentially, form a recursive relationship with Equation (5) and solve iteratively:

$$w_i^{n+1} = e_i + \sum_{j=1}^k \left\lceil w_i^n / p_j \right\rceil e_j \quad (6)$$

where initial (seed) value  $w_i^0 = e_i$

- the algorithm is then to solve for successive values of  $w_i^{n+1}$  until  $w_i^{n+1} = w_i^n$ , then solution found  $\rightarrow R_i = w_i^n$   
if  $R_i > D_i$ , then task  $i$  can not meet its deadline

# Calculating Response Time

Tâche	$e_i$	$p_i$	$P_i$	$U_i$
A	40	80		
B	10	40		
C	5	20		

Is it schedulable?

# Calculating Response Time

Tâche	$e_i$	$p_i$	$P_i$	$U_i$
A	40	80	3	
B	10	40	2	
C	5	20	1	

Is it schedulable?

# Calculating Response Time

Tâche	$e_i$	$p_i$	$P_i$	$U_i$
A	40	80	3	0.5
B	10	40	2	0.25
C	5	20	1	0.25

Is it schedulable?

$$U = 1 > 0.779$$

# Simple Task Model

- Assumptions:
  1. Tasks are periodic and the period is constant
  2. Completion-time < period
  3. Tasks are independent
  4. Runtime is known and deterministic
  5. all system overheads are negligible or deemed to be included in task computation times
  6. Critical instant - defined as the maximum load condition when all tasks release together
- Constraints
  1. Deadline = period
  2. fixed set of tasks
  3. Preemptive



# Scheduling with Aperiodic Tasks

- the simple task model that we have been able to deal with thus far is restrictive in several ways. Not being able to handle aperiodic tasks is a major restriction.
- one approach is to make aperiodic (or sporadic) tasks resemble periodic tasks
  - consider that an asynchronous task's minimum inter-arrival time can be treated like a period,  $T$
  - with just this assumption one can use response time analysis for both types of tasks

# Scheduling with Aperiodic Tasks

- the simple task model assumption that  $D=T$  is unrealistic for aperiodic tasks
  - typically, an aperiodic task will occur infrequently (large inter-arrival time) but must be serviced quickly ( $D < T$ )
  - therefore priority assignment based upon the period ( $T$ ) will usually not satisfy the requirement to meet the deadline ( $D$ )

# Deadline Monotonic Priority Ordering

- deadline monotonic priority ordering (DMPO) scheme is introduced as follows:
  - the shorter the task deadline, the higher the priority

Tasks	$e_i$	$p_i$	$D_i$	$P_i$	$R_i$
1	3	20	5		
2	3	15	7		
3	4	10	10		
4	3	20	20		

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Tasks	$e_i$	$p_i$	$D_i$	$P_i$	$R_i$
1	3	20	5	1	
2	3	15	7	2	
3	4	10	10	3	
4	3	20	20	4	

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Tasks	$e_i$	$p_i$	$D_i$	$P_i$	$R_i$
1	3	20	5	1	3
2	3	15	7	2	6
3	4	10	10	3	10
4	3	20	20	4	20

# Response Time Analysis

Tasks	$e_i$	$p_i$	$P_i$
1	3	7	
2	3	12	
3	5	20	

- use RM scheduling
- apply the utilization based schedulability test
- use Response Time Analysis to determine whether the system is schedulable

# References

- [1] Siewert, S. Pratt, J. Real-Time Embedded Components and Systems with Linux and RTOS. Mercury Learning and Information, 2016.
- [2] Burns, A. and Wellings, A., “*Real-Time Systems and Programming Languages*”, Chapter 13, Addison Wesley, 1997
- [3] TimeSys Corp, “*The Concise Handbook of Real-Time Systems*”, Version 1.0, 1999