

# Data Mining

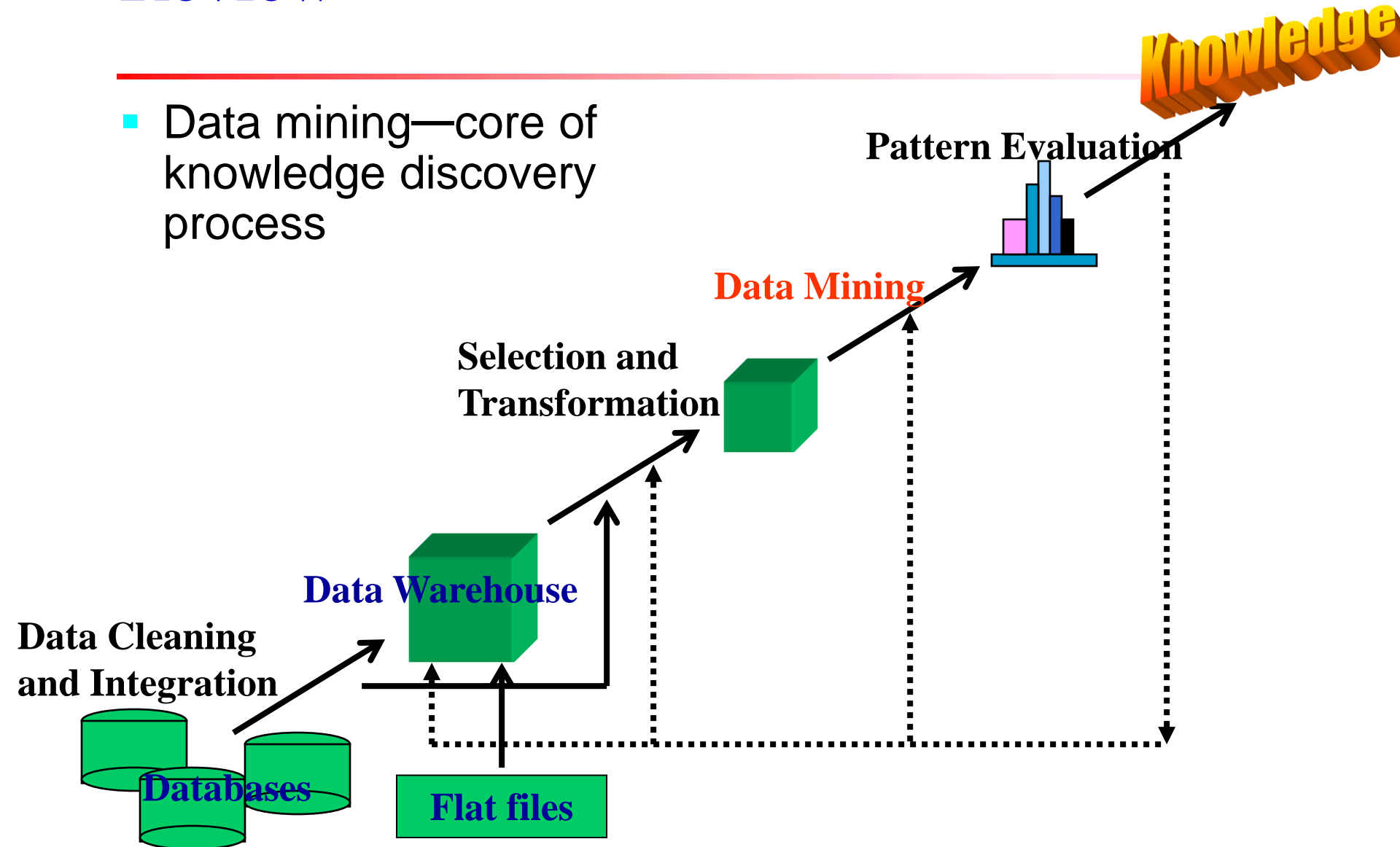
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# Review

- Data mining—core of knowledge discovery process



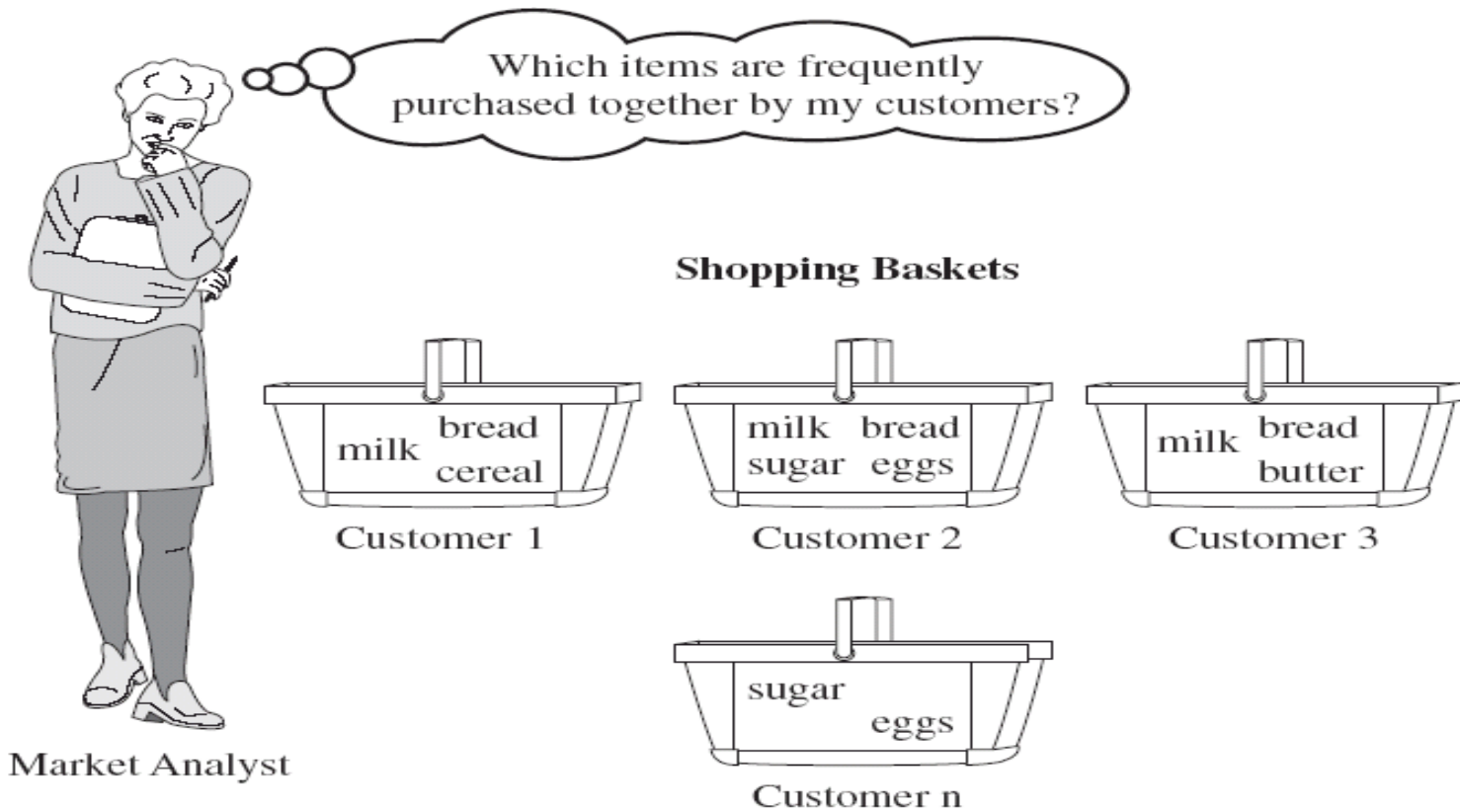
# Mining Association Rules in Large Databases

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- Basic concepts and a road map
- Mining single-dimensional Boolean association rules
- Mining multilevel association rules
- Mining multidimensional association rules
- Summary

# Market Basket Analysis

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# What Is Association Rules Mining?

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## ■ Association rules mining

- Finding frequent patterns, associations among sets of items or objects in transaction databases, relational databases, and other information repositories.

## ■ Examples

- What products were often purchased together? — Beer and diapers?!
- What DNA segments often occur together in DNA sequences?

# What Is Association Rules Mining?

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- Where does the data come from?
  - supermarket transactions, membership cards, discount coupons, customer complaint calls
- Applications
  - Basket data analysis
  - Cross-marketing
  - Catalog design
  - Sale campaign analysis
  - Web log (click stream) analysis
  - DNA sequence analysis

# Basic Concepts

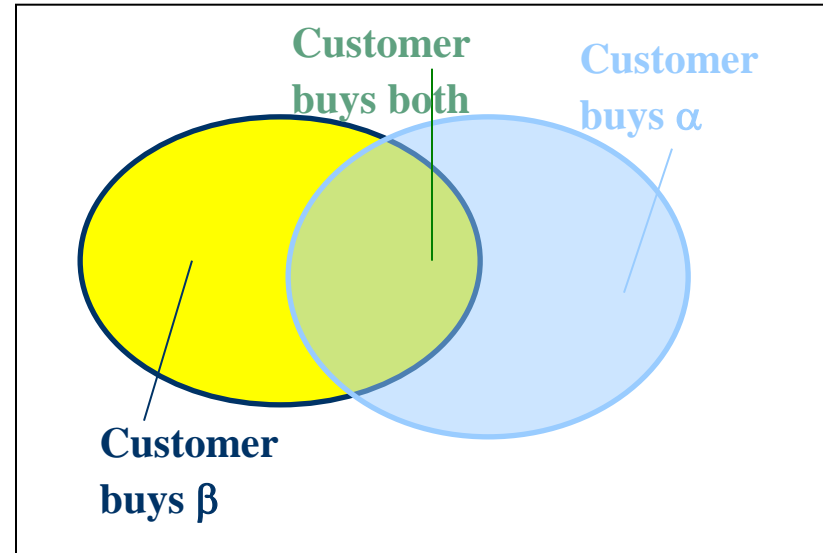
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Transaction-id	Items bought
10	A, B, D
20	A, C, D
30	A, D, E
40	B, E, F
50	B, C, D, E, F

- Item collection  $X = \{x_1, \dots, x_m\}$
- Itemset: a set of items,  $k$ -itemset
- Transaction  $T \subseteq X$ , each  $T$  associates a unique Tid and items bought by a customer
- Rule form  $\alpha \Rightarrow \beta$ ,  $\alpha \subset X$ ,  $\beta \subset X$ ,  $\alpha \cap \beta = \emptyset$

# Basic Concepts

- support,  $s$ , probability that a transaction contains  $\alpha$  and  $\beta$ 
  - support  $(\alpha \Rightarrow \beta) = P(\alpha \cap \beta)$
- Frequent itemset, occurrence greater than a min\_support
- Frequent itemset mining, find all the rules  $\alpha \Rightarrow \beta$  satisfying min\_support
- Let  $\text{sup}_{\min} = 50\%$ ,  
frequent Itemsets  $\{A:3, B:3, D:4, E:3, AD:3\}$   
support  $(A) = 3/5 = 60\%$ , support  $(AD) = 3/5 = 60\%$





# Basic Concepts

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- confidence,  $c$ , conditional probability that a transaction having  $\alpha$  also contains  $\beta$

$$\text{Confidence}(\alpha \Rightarrow \beta) = P(\beta \mid \alpha) = \frac{P(\alpha \cap \beta)}{P(\alpha)} = \frac{\text{count}(\alpha \cap \beta)}{\text{count}(\alpha)}$$

- Measure of rule interestingness
- Rules satisfy min\_support and min\_confidence are strong
- Let  $\text{sup}_{\min} = 50\%$ ,  $\text{conf}_{\min} = 50\%$ ,  
frequent itemsets  $\{A:3, B:3, D:4, E:3, AD:3\}$

Association rules:

$A \Rightarrow D$  (60%, 100%)

$D \Rightarrow A$  (60%, 75%)

# Interestingness Measure: Correlations (Lift)

- *play basketball*  $\Rightarrow$  *eat cereal* [40%, 66.7%] is misleading
  - The overall % of students eating cereal is 75% > 66.7%.
- Measure of dependent/correlated events: **lift**

$$lift = \frac{P(A \cap B)}{P(A)P(B)}$$

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

$$lift(B, C) = \frac{2000 / 5000}{3000 / 5000 * 3750 / 5000} = 0.89 \quad lift(B, \neg C) = \frac{1000 / 5000}{3000 / 5000 * 1250 / 5000} = 1.33$$

# Association Rule Mining: A Road Map

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- **Boolean vs. quantitative** associations (based on the types of values handled)
  - Boolean association rules, only concern presence or absence of items,  $\text{buys}(x, \text{"SQLServer"}) \wedge \text{buys}(x, \text{"DMBook"}) \Rightarrow \text{buys}(x, \text{"DBMiner"})$  [0.2%, 60%]
  - Quantitative association rules, concern quantitative attributes,  $\text{age}(x, \text{"30...39"}) \wedge \text{income}(x, \text{"42...48K"}) \Rightarrow \text{buys}(x, \text{"high resolution TV"})$  [1%, 75%]
- **Single level vs. multiple-level** analysis (based on the levels of abstraction involved)
  - $\text{age}(x, \text{"30...39"}) \Rightarrow \text{buys}(x, \text{"laptop computer"})$
  - $\text{age}(x, \text{"30...39"}) \Rightarrow \text{buys}(x, \text{"computer"})$
- **Single dimension vs. multiple dimensional** associations (based on dimensions involved)

# Mining Association Rules in Large Databases

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- Mining multilevel association rules
- Mining multidimensional association rules
- Summary

# Handling Exponential Complexity

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- Given  $n$  transactions and  $m$  different items:
  - Number of possible association rules:  $O(2^m)$
  - Computation complexity:  $O(nm2^m)$
- **Apriori Principle**
  - Collect single item counts, find large items
  - Find candidate pairs, count them  $\Rightarrow$  large pairs of items
  - Find candidate triplets, count them  $\Rightarrow$  large triplets of items, And so on...
  - **Guiding Principle: Every subset of a frequent itemset has to be frequent**
    - Used for pruning many candidates

# Apriori: A Candidate Generation-and-Test Approach

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- **Apriori** uses prior knowledge of frequent itemsets
- Iterative approach, level-wise search
- The Apriori property (**downward closure property**, **anti-monotone**) of frequent patterns
  - Any subset of a frequent itemset must be frequent
  - If any itemset is infrequent, its superset should not be generated/tested
  - If {beer, diaper, nuts} is frequent, so is {beer, diaper}, every transaction having {beer, diaper, nuts} also contains {beer, diaper}
  - If {beer, diaper} is infrequent, {beer, diaper, nut} cannot be frequent at all

# Apriori: A Candidate Generation-and-Test Approach

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## ■ Method:

- Initially, scan DB once to get frequent 1-itemset
- **Generate** length  $(k+1)$  **candidate** itemsets from length  $k$  **frequent** itemsets
- **Test** the candidates against DB
- Terminate when no frequent or candidate set can be generated

# Apriori Algorithm — An Example

$\text{Sup}_{\min} = 2$

Database D

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

1<sup>st</sup> scan

$C_1$

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

$L_1$

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

$C_2$

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

2<sup>nd</sup> scan

$C_2$

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

$L_2$

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

$C_3$

Itemset	sup
{B, C, E}	2

3<sup>rd</sup> scan

$C_3$

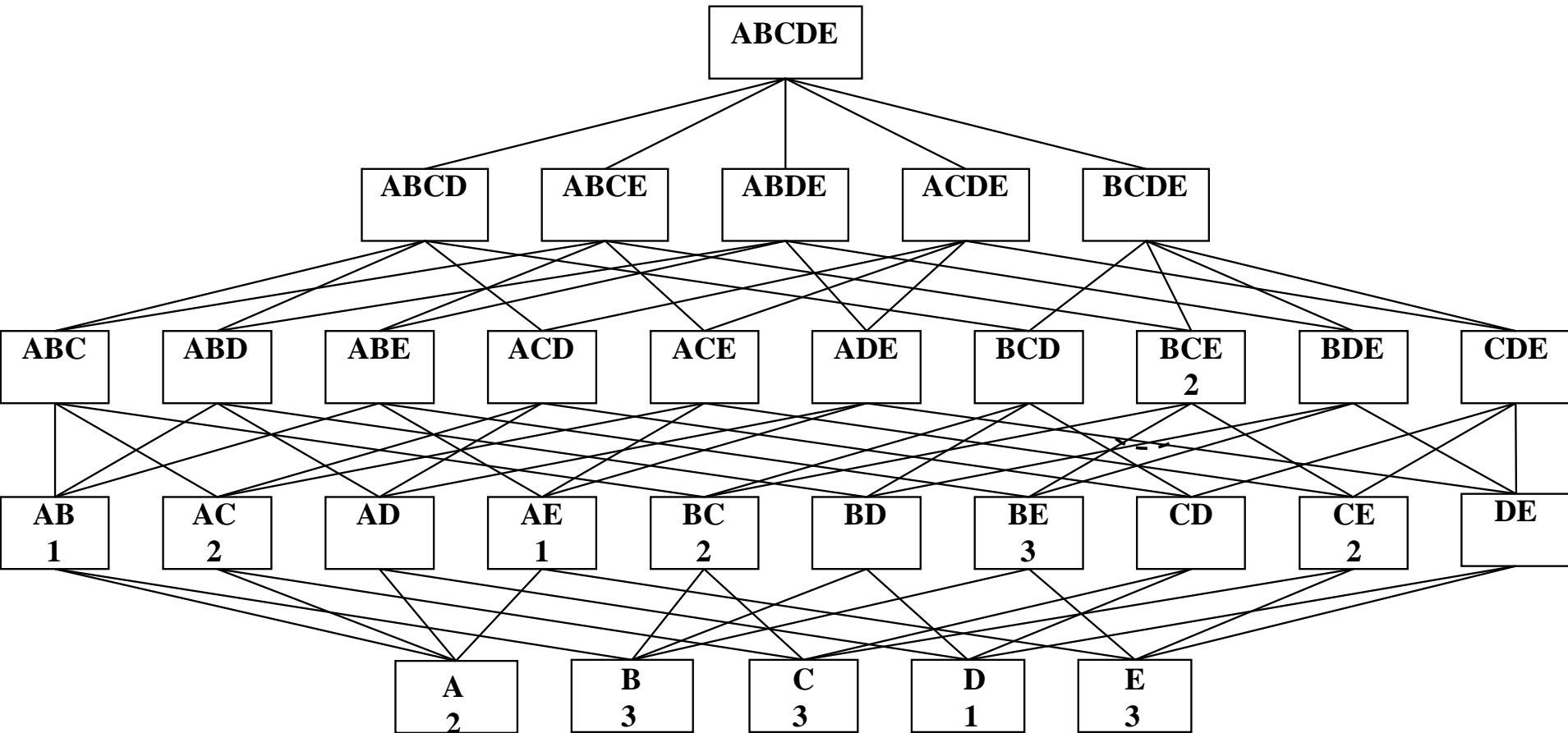
Itemset	sup
{B, C, E}	2

$L_3$

Itemset	sup
{B, C, E}	2

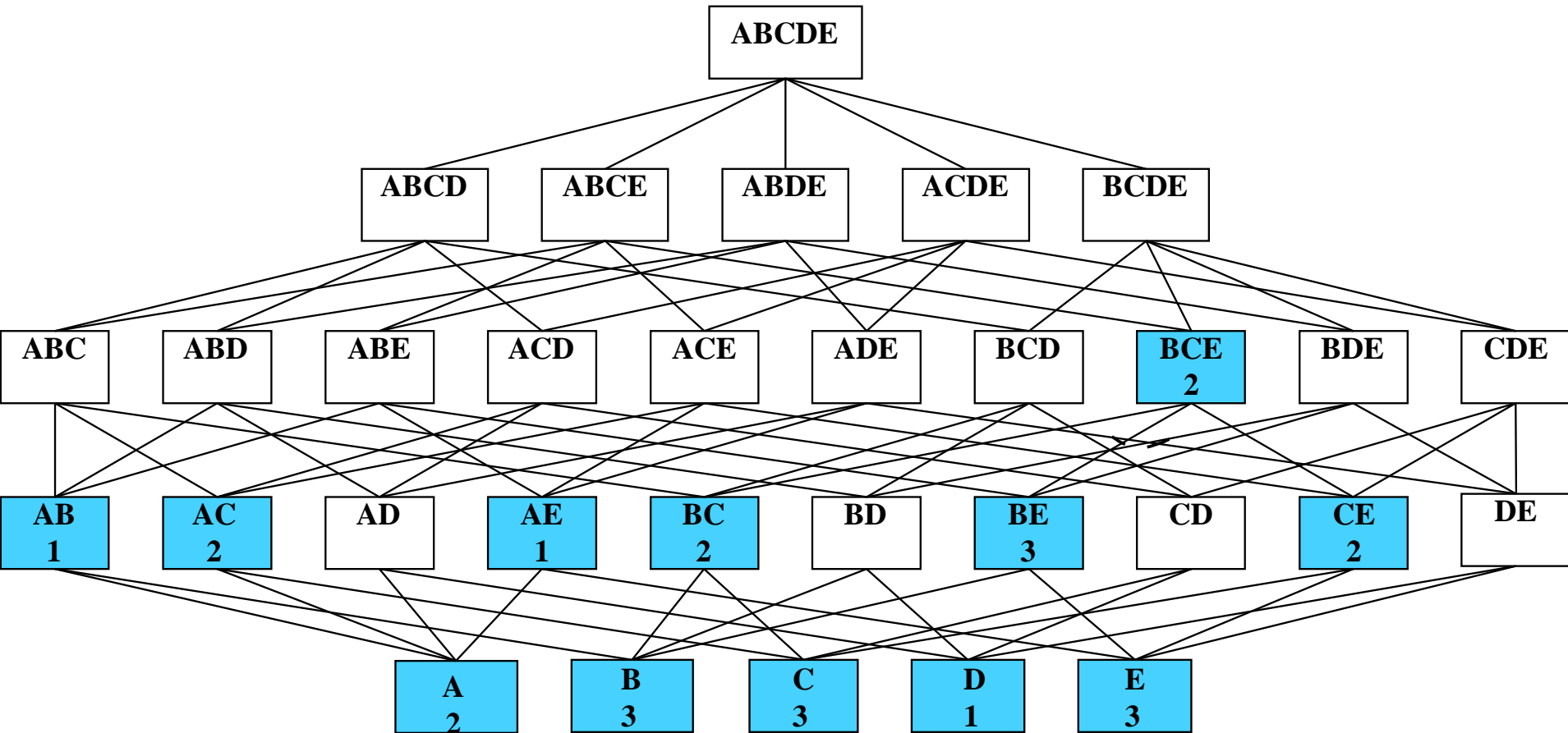


# Apriori Algorithm — An Example



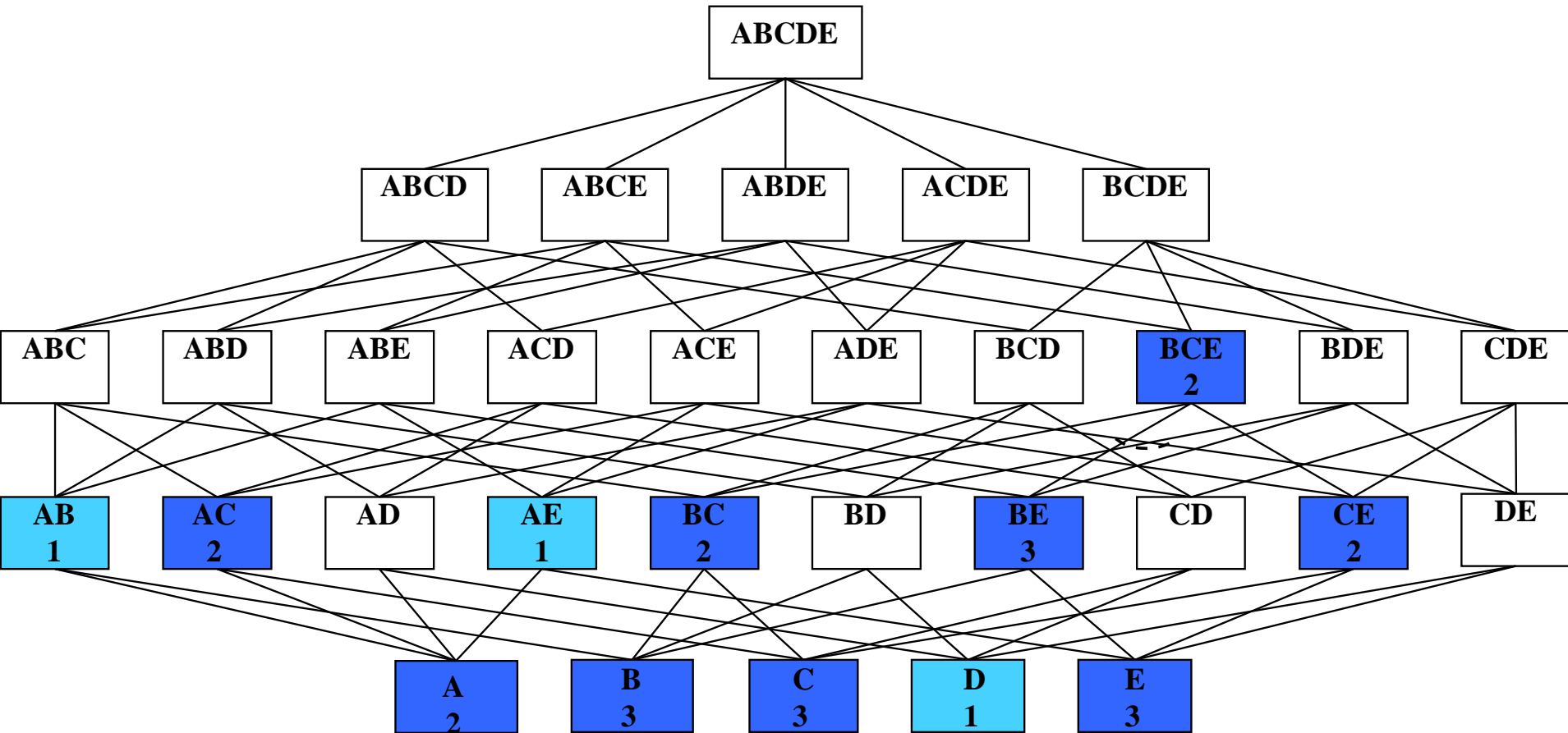
# Apriori Algorithm — An Example

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# Apriori Algorithm — An Example

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# Apriori Algorithm

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## ■ Pseudo-code

$C_k$ : Candidate itemset of size  $k$

$L_k$ : frequent itemset of size  $k$

Input: Database  $D$ ,  $min\_sup$

Output: frequent itemsets  $L$

$L_1 = \{\text{frequent single items from } D\};$

**for** ( $k = 2$ ;  $L_{k-1} \neq \emptyset$ ;  $k++$ ) **do begin**

$C_k =$  candidates generated from  $L_{k-1}$ ;

**for each** transaction  $t \in D$  **do**

increment the count of all candidates in  $C_k$  which are  
contained in  $t$

**end**

$L_k =$  candidates in  $C_k$  with  $min\_support$

**end**

**return**  $L = \cup_k L_k$ ;

# How to Generate Candidates?

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## ■ How to generate candidates?

- Step 1: self-joining  $L_k$
- Step 2: pruning

## ■ Example

- $L_3 = \{abc, abd, acd, ace, bcd\}$
- Self-joining:  $L_3 * L_3$ 
  - $abc$  and  $abd \rightarrow abcd$ ,  $acd$  and  $ace \rightarrow acde$
- Pruning:
  - $acde$  is pruned because  $ade$  is not in  $L_3$
- $C_4 = \{abcd\}$

# How to Generate Candidates?

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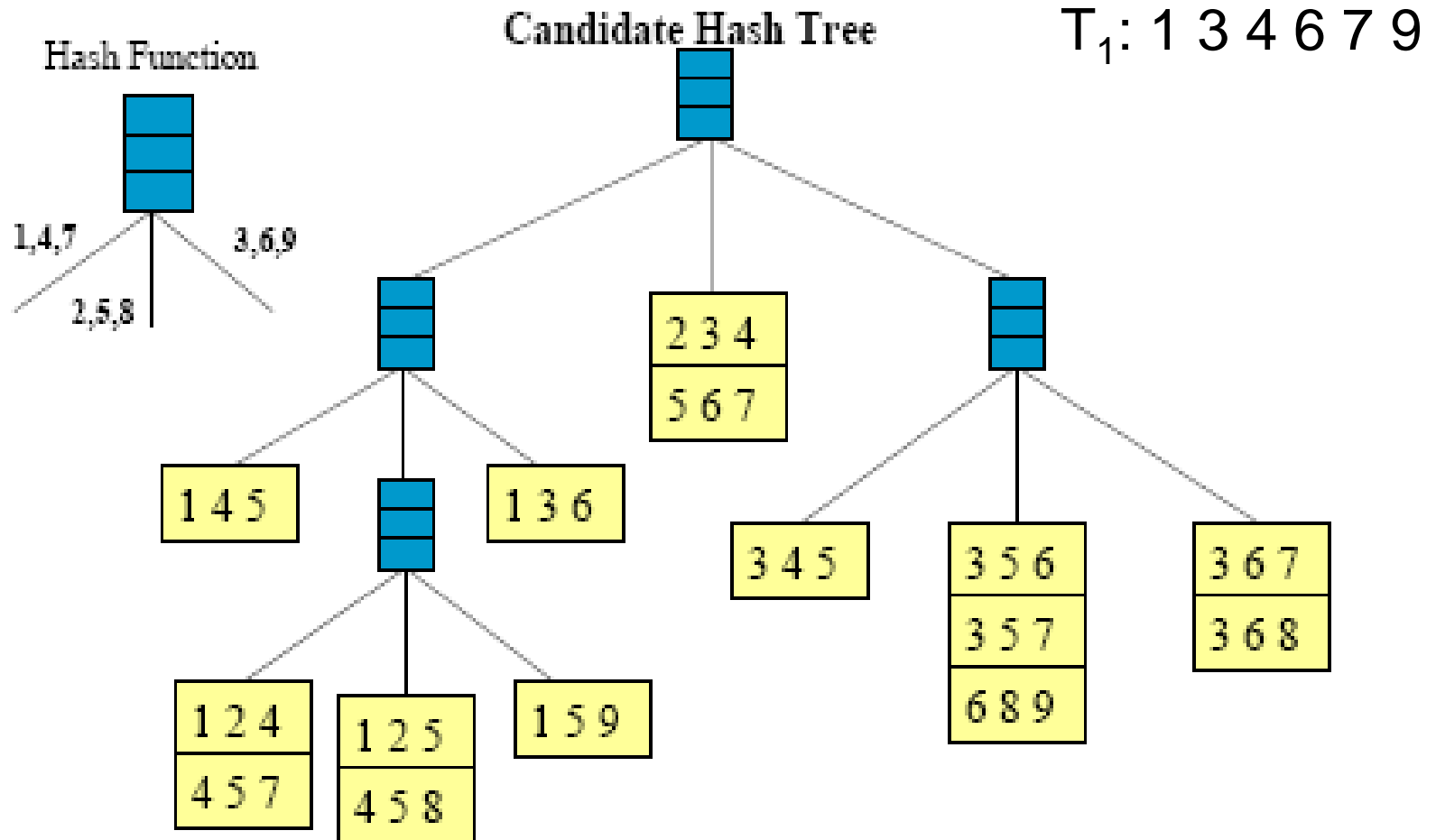
- Suppose the items in  $L_{k-1}$  are listed in order
- Step 1: self-joining  $L_{k-1}$ 
  - for each itemset  $l_1 \in L_{k-1}$
  - for each itemset  $l_2 \in L_{k-1}$
  - if  $(l_1[1]=l_2[1]) \wedge (l_1[2]=l_2[2]) \wedge \dots \wedge (l_1[k-2]=l_2[k-2])$  then
  - $c = l_1 \text{ join } l_2$
  - pruning ( $c$ )
  - end
- end
- Step 2: pruning
  - forall  $(k-1)$ -subsets  $s$  of  $c$  do
  - if ( $s$  is not in  $L_{k-1}$ ) then delete  $c$

# How to Count Supports of Candidates?

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- Why counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf node* of hash-tree contains a list of itemsets and counts
  - *Interior node* contains a hash table

# Example: Counting Supports of Candidates





# Exercise

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1. A database has 9 transactions. Let  $min\_sup = 20\%$ . Please present all the candidates and frequent itemsets at each iteration.

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

# Challenges of Frequent Pattern Mining

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## ■ Challenges

- Multiple scans of transaction database
- Huge number of candidates
- Tedious workload of support counting for candidates

## ■ Improving Apriori

- Reduce passes of transaction database scans
- Shrink number of candidates
- Facilitate support counting of candidates

# Partition: Scan Database Only Twice

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- A. Savasere, E. Omiecinski, and S. Navathe. An efficient algorithm for mining association in large databases. In *VLDB'95*.
- Partitioning technique
  - Partition the data into  $N$  small partitions
  - **Phase 1**: find local frequent itemsets on each data partition. Record all local frequent itemsets.
  - **Phase 2**: Integrate all local frequent itemsets, scan database, find global frequent itemsets.
- **Correctness**: Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions

# Partition: Scan Database Only Twice

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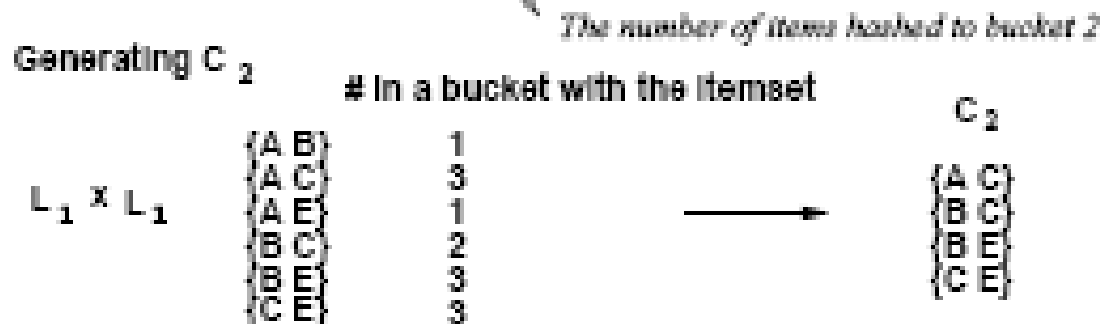
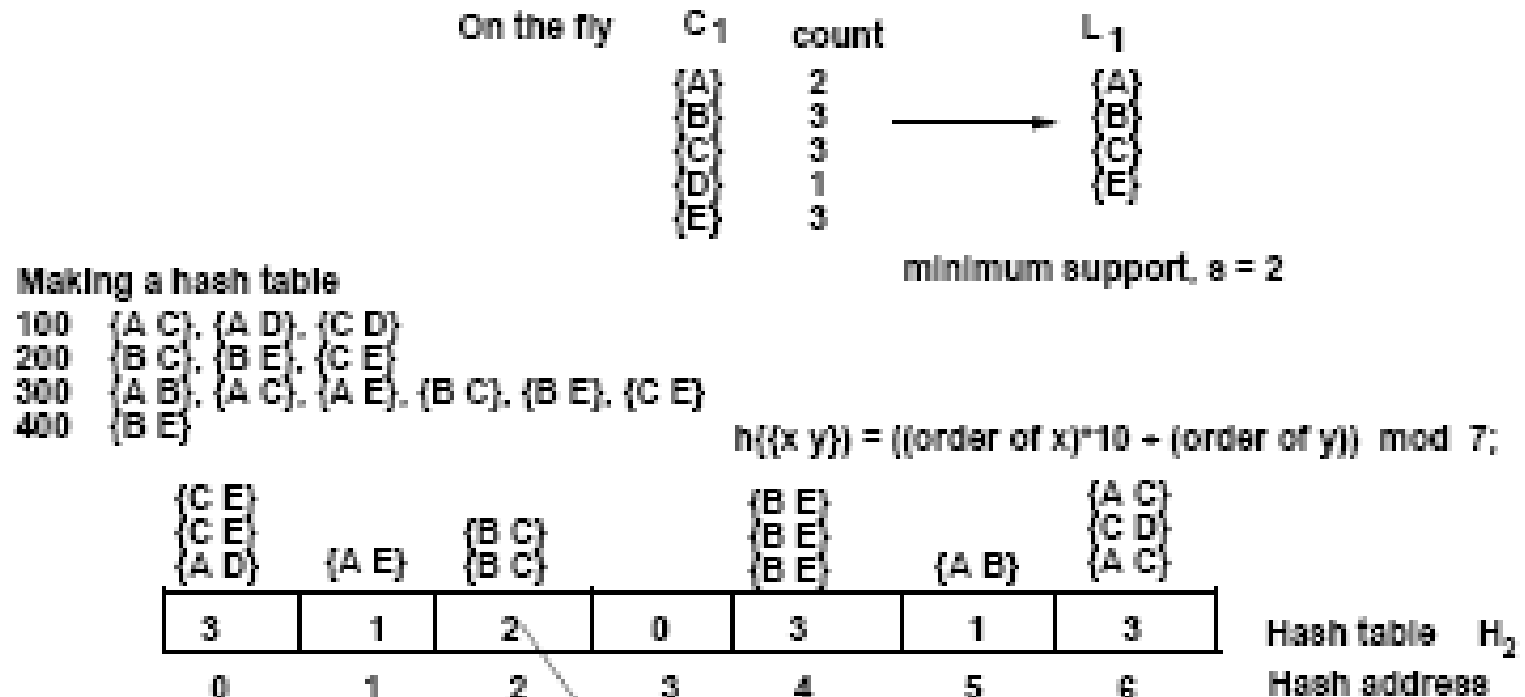
- Each partition can be fit into memory
- Scan database only twice! Reduce I/O cost!
- Execution time scales linearly
- Good for very large-scale database
- Applicable to parallel/distributed computing systems
  - Each processor performs FIM on its local data
  - Central server aggregates local frequent itemsets, broadcast potential global itemsets
  - Each processor scans local data to count the frequency
  - Central server aggregates the counts, find the global itemsets

# DHP: Reduce the Number of Candidates

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- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In *SIGMOD'95*
- Hash-based technique
  - When scanning transactions to generate frequent  $k$ -itemsets,  $L_k$ , generate all  $(k+1)$ -itemsets for each transaction
  - Hash all  $(k+1)$ -itemsets into buckets, increase bucket count
  - If a  $(k+1)$ -itemset bucket count is below  $min\_sup$ , it must be removed from  $(k+1)$  candidate itemsets,  $C_{k+1}$
- **Correctness:** A  $k$ -itemset whose corresponding hashing bucket count is below the threshold cannot be frequent

# DHP: Reduce the Number of Candidates



# DHP: Reduce the Number of Candidates

## ■ Pros

- Reduce the number of candidates,  $C_k$ , especially for  $C_2$ . Size of  $C_2$  is usually huge, reduce  $C_2$  is crucial
- Execution time scales linearly when varying the size of data

Comparison of time (T15.I4.D100)

	Apriori number	DHP number
$L_1$	820	820
$C_2$	335,790	338
$L_2$	207	207
$C_3$	618	618
$L_3$	201	201
$C_4$	184	184
$L_4$	98	98
$C_5$	30	30
$L_5$	23	23
$C_6$	1	1
$L_6$	1	1
total time	39.39	13.91

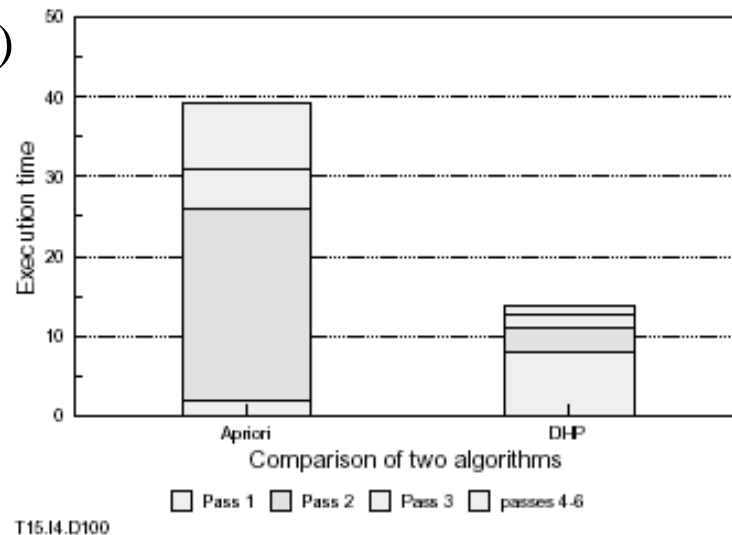
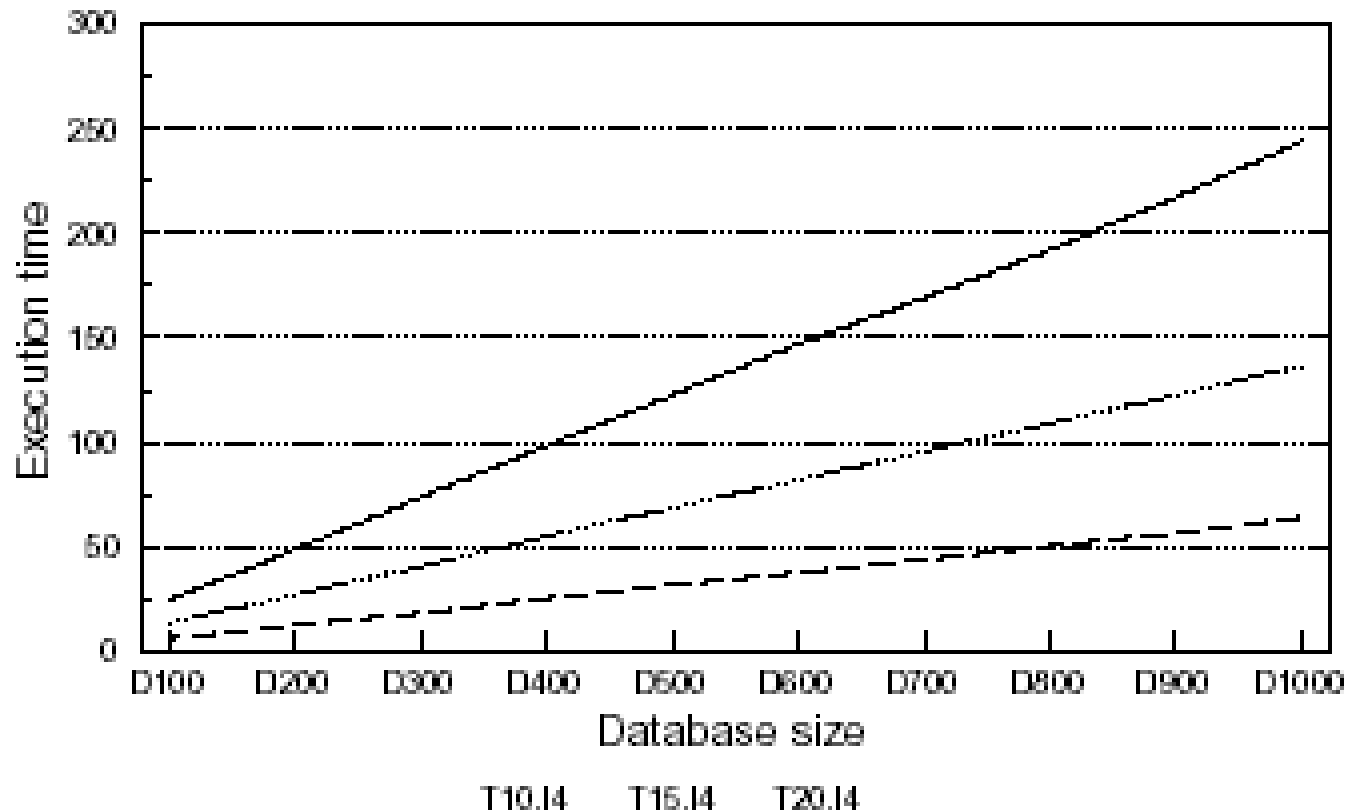


Figure 8: Execution time of Apriori and DHP

Comparison of time (T15.I4.D100)

# DHP: Reduce the Number of Candidates

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Performance of DHP when increasing the size of database



# DHP: Reduce the Number of Candidates

## ■ Cons

- Consume more memory, for hash table
- The larger the hash table, the smaller  $C_k$  and  $L_k$

Results from varying hash table sizes  
(T10.I4.D100)

$ H_2 $	524,288	262,144	131,072	95,536	32,768
$L_1$	559	559	559	559	559
$ \{H_2 \geq s\} $	58	61	75	96	182
$C_2$	81	120	199	394	1355
$L_2$	45	45	45	45	45
$\alpha$	0.0314	0.0320	0.0345	0.0386	0.0545
size of $D_3$	498KB	500KB	507KB	539KB	603KB
$ D_3 $	19,732	19,741	19,755	20,501	21,607
total time	6.44	6.43	6.24	6.77	7.23

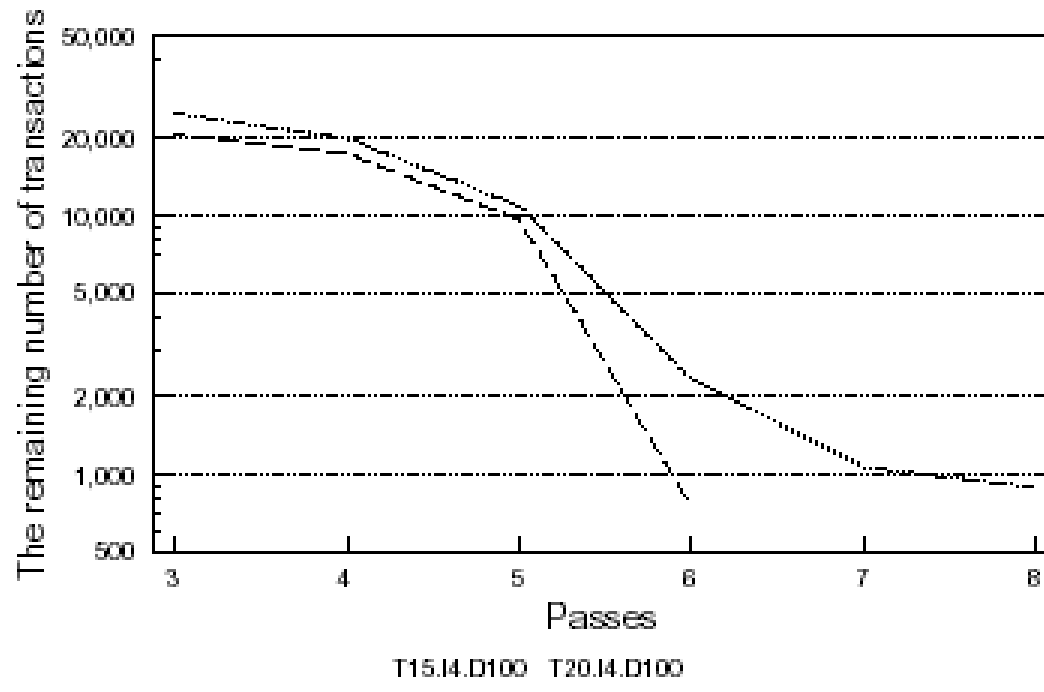
# Transaction Reduction

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- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. In *SIGMOD'95*
- Transaction reduction
  - When scanning transactions to generate frequent  $k$ -itemsets,  $L_k$ , mark the transaction that contains no  $k$ -candidate
  - Remove all the marked transaction
  - The number of transactions drops dramatically

# Transaction Reduction

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The number of original tx's: 100,000  
 $s=0.75\%$

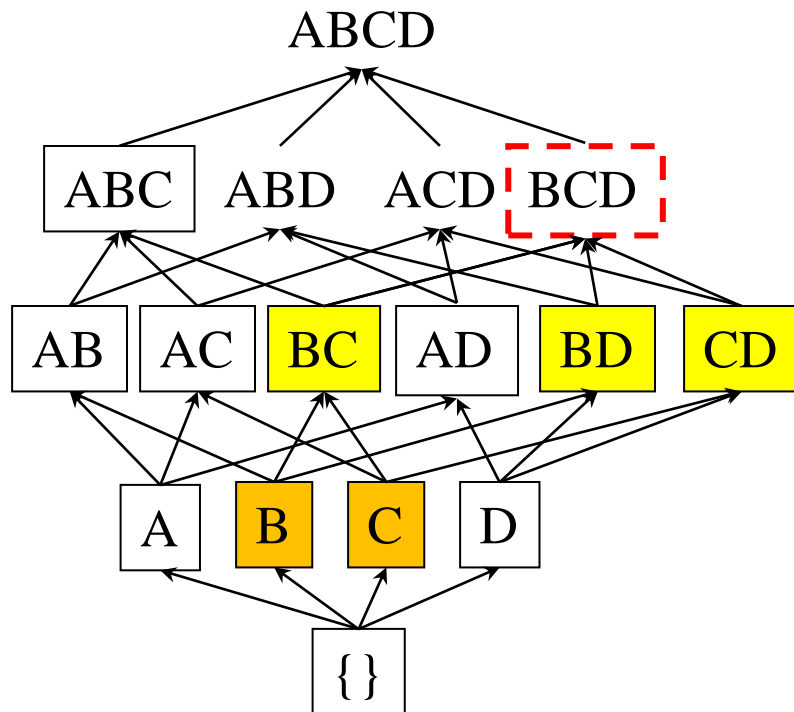
The remaining number of transaction in each pass

# DIC: Reduce Number of Scans

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- S. Brin, R. Motwani, J. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket data. In *SIGMOD'97*
- *Sergey Brin, founder of Google!*
- Partition database into blocks marked by starting points
- New candidate can be added at any starting point once all its subsets are determined frequent
- Reduce the number of database scans

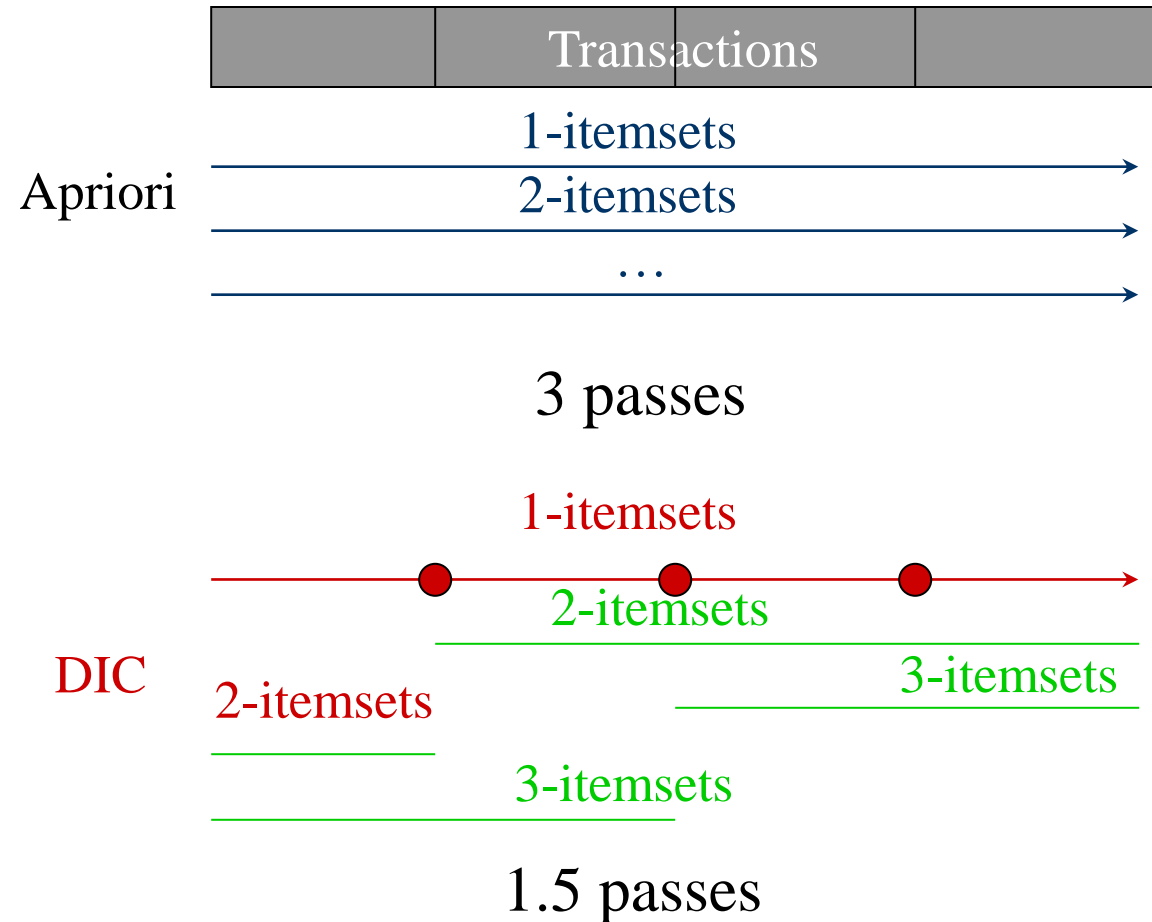
# DIC: Reduce Number of Scans



Itemset lattice

- Once both B and C are determined frequent, new candidate BC is added, the counting of BC begins at the next starting point
- Once all length-2 subsets of BCD are determined frequent, new candidate BCD is added, the counting of BCD begins at the next starting point

# DIC: Reduce Number of Scans



- Assume 40000 transactions, 4 partitions
- Begin counting 2-itemsets after the first 10000 have been read
- Begin counting 3-itemsets after the first 20000 have been read
- Scan database again, count 2 and 3-itemsets
- After 10000 transactions, finish counting 2-itemsets
- After 20000 transactions, finish counting 3-itemsets

# Exercise

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2. A database has 9 transactions. Let  $min\_sup = 20\%$ . Please present all the frequent itemsets generated by DIC in the first iteration. (Note: partition the data into 3 blocks)

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

# Bottleneck of Frequent-pattern Mining

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- Multiple database scans are **costly**
- Mining long patterns needs many passes of scanning and generates lots of candidates
  - To find frequent itemset  $i_1 i_2 \dots i_{100}$ 
    - # of scans: **100**
    - # of Candidates:  $\binom{100}{1} + \binom{100}{2} + \dots + \binom{100}{100} = 2^{100} - 1 =$   
 **$1.27 \times 10^{30}$  !**
- Bottleneck: candidate-generation-and-test
- Can we avoid candidate generation?



# Construct FP-tree from a Transaction Database

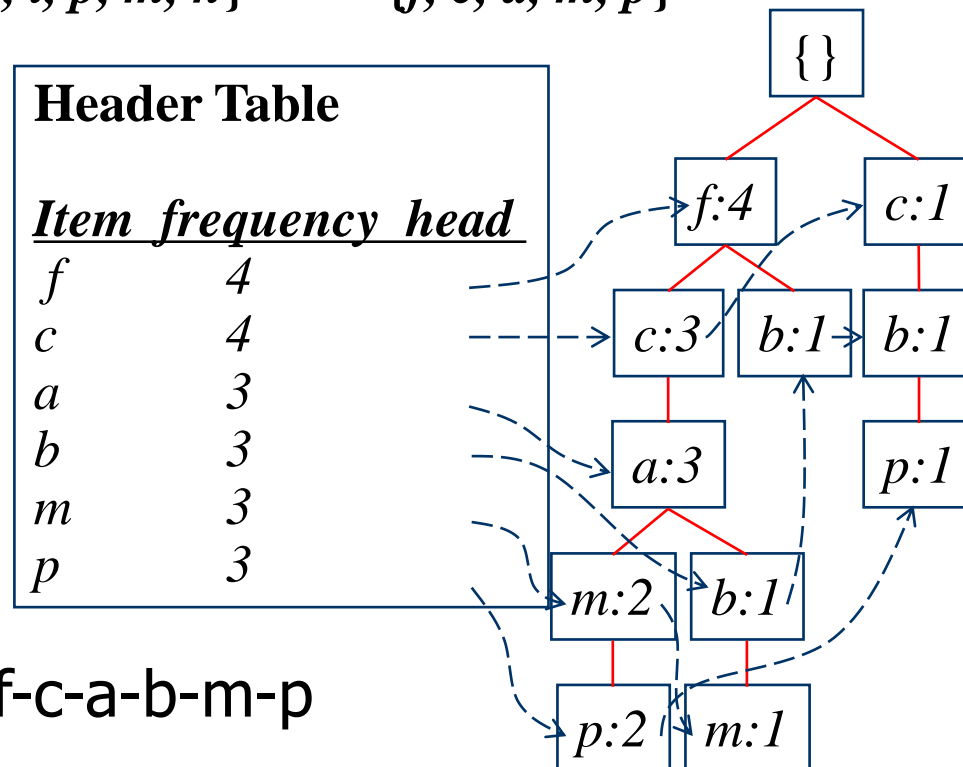
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- Scan DB once, find frequent 1-itemset (single item pattern)
- Sort frequent items in frequency descending order  $L$
- Create the root of the tree, labeled with “null”
- Scan DB again, sort each transaction in  $L$  order, a branch is created for each transaction
  - Increment the count of each node along a common prefix by 1
  - Create nodes for the items following the prefix
- Build a header table, connect each item point in the tree

# Construct FP-tree from a Transaction Database

<i>TID</i>	<i>Items bought</i>	<i>(ordered) frequent items</i>
100	{f, a, c, d, g, i, m, p}	{f, c, a, m, p}
200	{a, b, c, f, l, m, o}	{f, c, a, b, m}
300	{b, f, h, j, o, w}	{f, b}
400	{b, c, k, s, p}	{c, b, p}
500	{a, f, c, e, l, p, m, n}	{f, c, a, m, p}

*min\_support* = 3



**F-list** = f-c-a-b-m-p

# Construct FP-tree from a Transaction Database

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- 1. Scan the transaction database  $D$  once. Collect  $F$ , the set of frequent items, and their support counts. Sort  $F$  in support count descending order as  $L$ , the *list* of frequent items.
- 2. Create the root of an FP-tree, and label it as “null.” For each transaction  $Trans$  in  $D$  do the following:
  - Select and sort the frequent items in  $Trans$  according to the order of  $L$ . Let the sorted frequent item list in the  $Trans$  be  $[p|P]$ , where  $p$  is the first element and  $P$  is the remaining list.
  - Call `insert_tree` ( $[p|P]$ ,  $T$ ), which is performed as follows. If  $T$  has a child  $N$  such that  $N.item-name = p.item-name$ , then increment  $N$ 's count by 1; else create a new node  $N$ , and let its count be 1, its parent link be linked to  $T$ , and its node-link to the nodes with the same *item-name* via the node-link structure.
  - If  $P$  is nonempty, call `insert_tree`( $P$ ,  $N$ ) recursively.

# Benefits of the FP-tree Structure

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## ■ Completeness

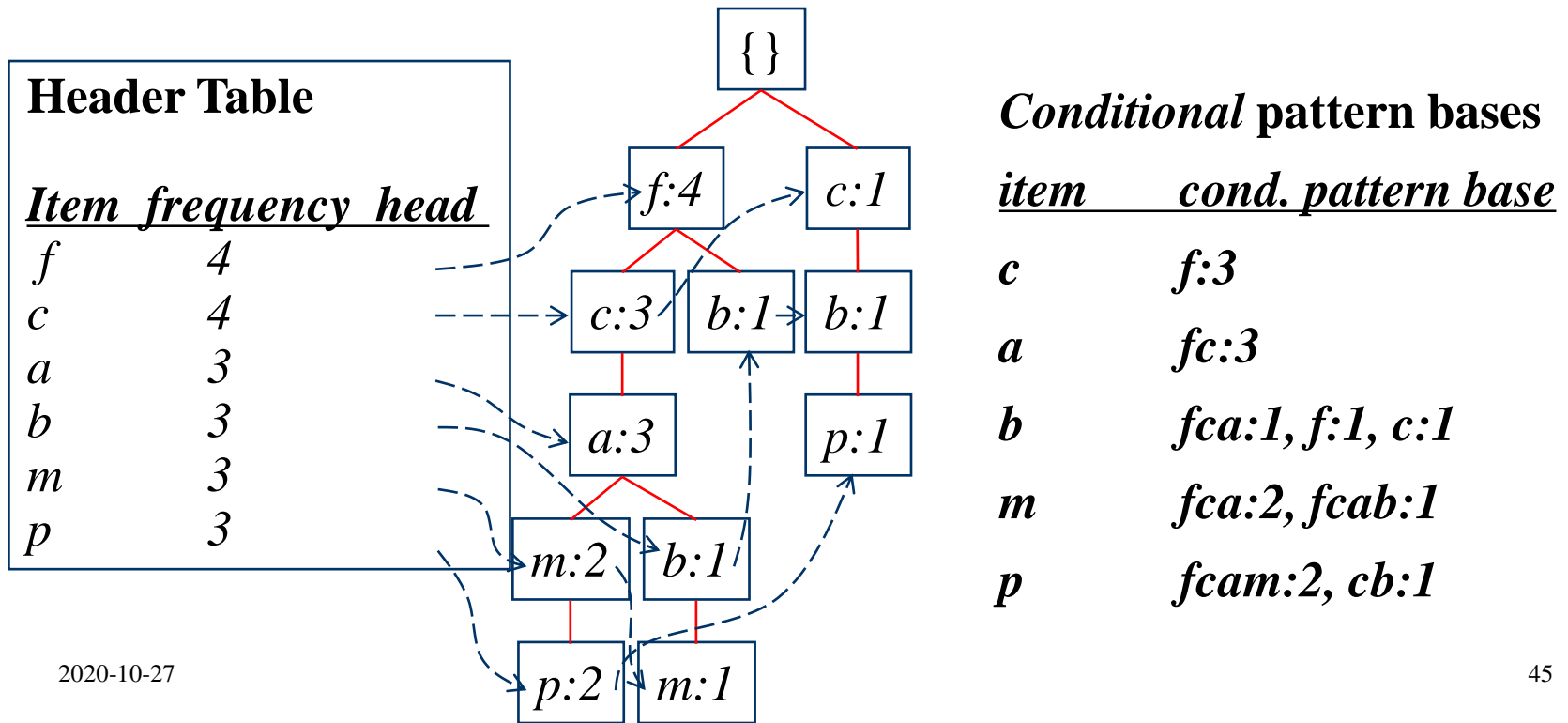
- Preserve complete information for frequent pattern mining
- Never break a long pattern of any transaction

## ■ Compactness

- Reduce irrelevant info—infrequent items are gone
- Items in frequency descending order: the more frequently occurring, the more likely to be shared
- Never be larger than the original database

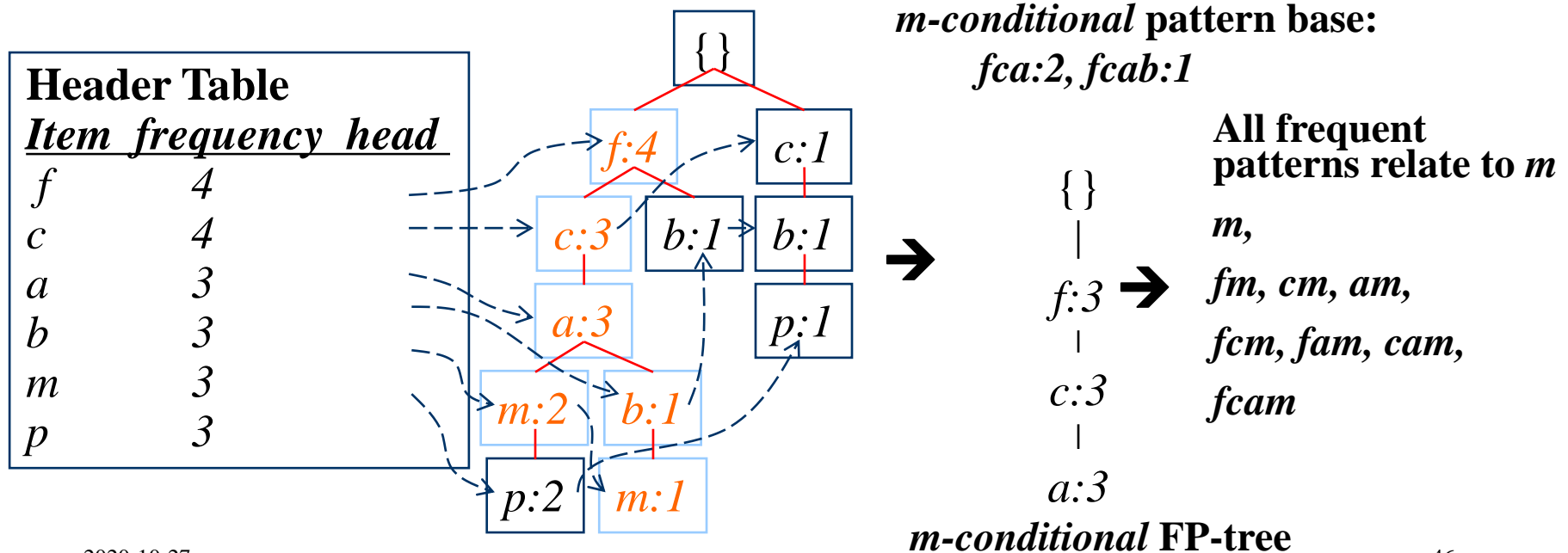
# Construct Conditional Pattern Base

- Start at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item  $x$
- Accumulate all of *transformed prefix paths* of item  $x$  into form  $x$ 's conditional pattern base

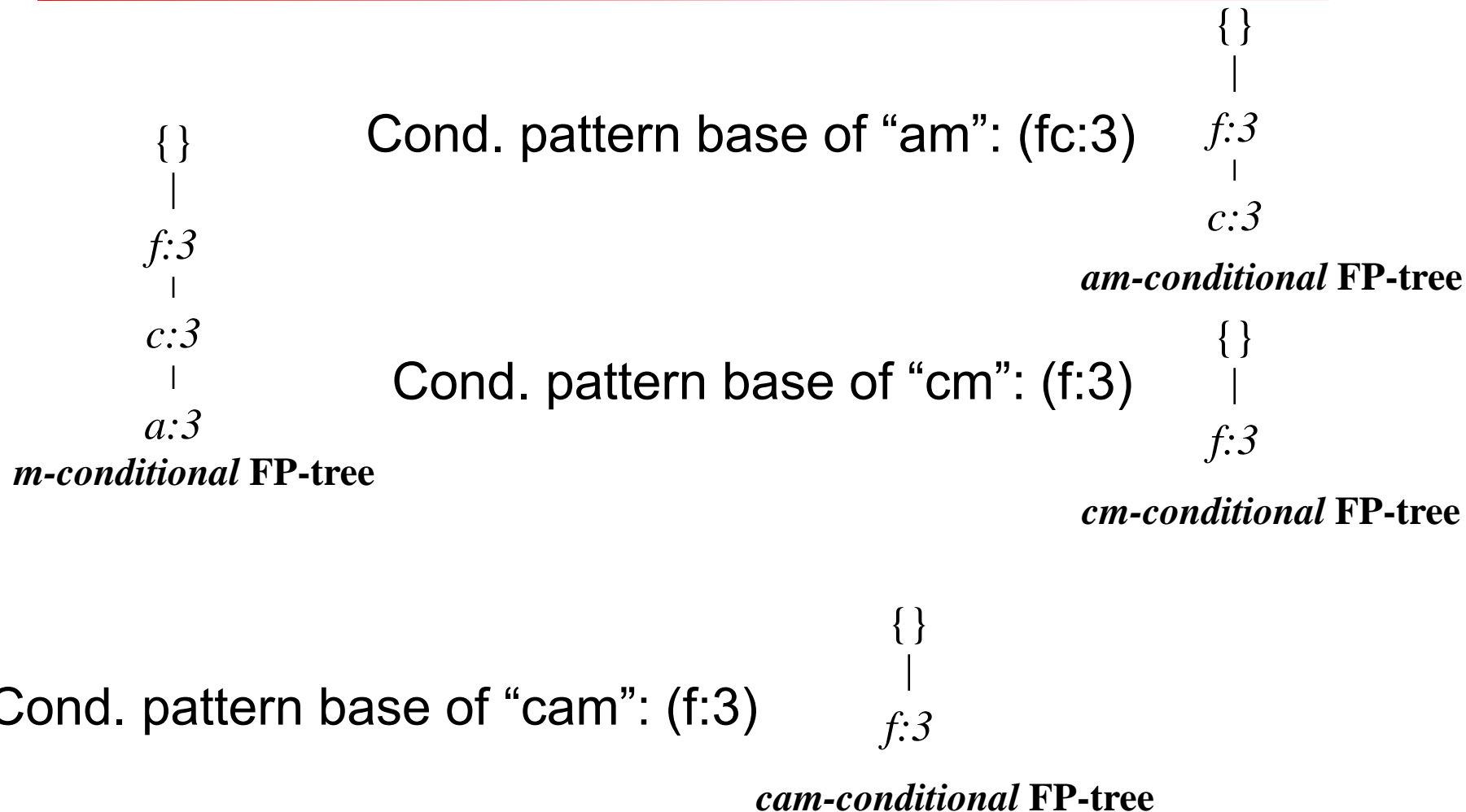


# From Conditional Pattern-bases to Conditional FP-trees

- For each pattern-base
  - Accumulate the count for each item in the base
  - Construct the FP-tree for the frequent items of the pattern base



# Recursion: Conditional FP-tree



# Mining Frequent Patterns With FP-trees

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procedure **FP\_growth**( $Tree, \alpha$ )

- (1) **if**  $Tree$  contains a single path  $P$  then
- (2) **for each** combination (denoted as  $\beta$ ) of the nodes in the path  $P$
- (3)     generate pattern  $\beta \cup \alpha$  with *support\_count* = *minimum support count of nodes in  $\beta$* ;
- (4) **else for each**  $a_i$  in the header of  $Tree$  {
- (5)     generate pattern  $\beta = a_i \cup \alpha$  with *support\_count* =  $a_i.support\_count$ ;
- (6)     construct  $\beta$ 's conditional pattern base and then  $\beta$ 's conditional FP\_tree  $Tree_\beta$  ;
- (7)     **if**  $Tree_\beta$  then
- (8)         call **FP\_growth**( $Tree_\beta, \beta$ ); }



# Exercise

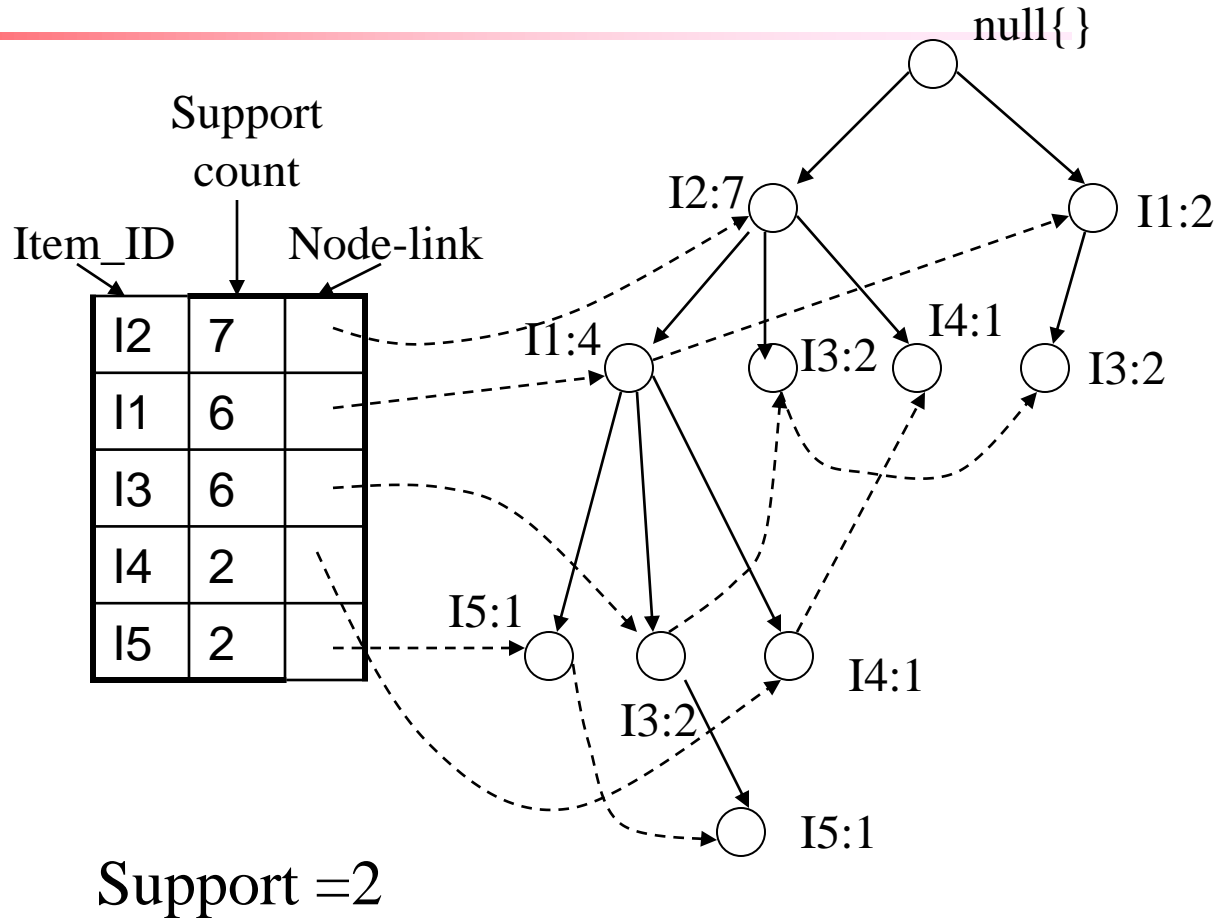
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3. A database has 9 transactions. Let  $min\_sup = 20\%$ . Please construct the FP-tree for the database, the conditional FP-trees, and all the frequent itemsets.

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3

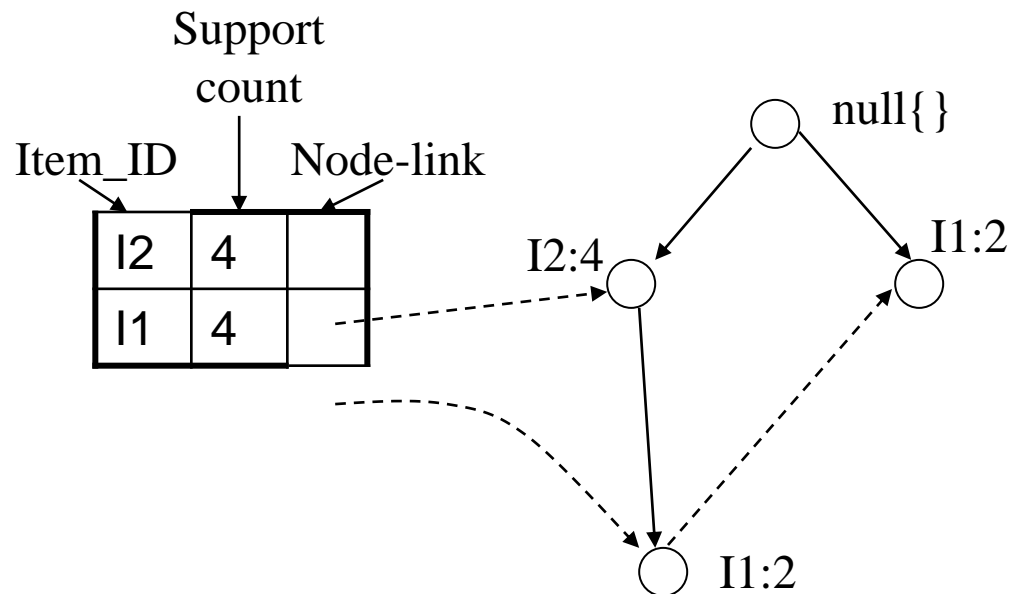
# Solution

TID	List of items_IDs
T100	I1,I2,I5
T200	I2,I4
T300	I2,I3
T400	I1,I2,I4
T500	I1,I3
T600	I2,I3
T700	I1,I3
T800	I1,I2,I3,I5
T900	I1,I2,I3



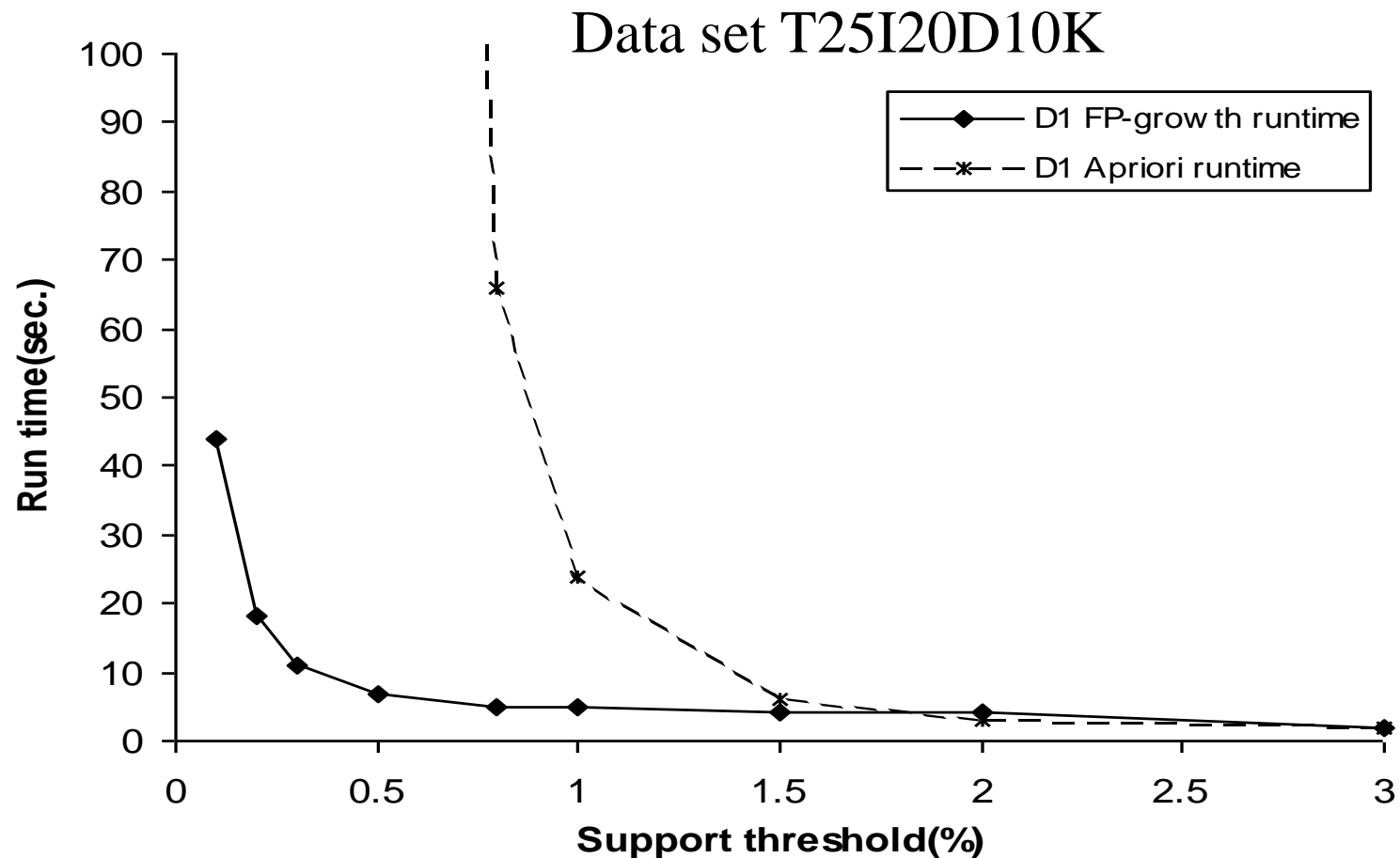
# Solution

<i>item</i>	<i>conditional pattern base</i>	<i>conditional FP-tree</i>	<i>frequent patterns generated</i>
I5	{{I2,I1: 1}, {I2,I1,I3: 1}}	$\langle I2: 2, I1: 2 \rangle$	{I2,I5: 2}, {I1,I5: 2}, {I2,I1,I5: 2}
I4	{{I2,I1: 1}, {I2: 1}}	$\langle I2: 2 \rangle$	{I2,I4: 2}
I3	{{I2,I1: 2}, {I2: 2}, {I1: 2}}	$\langle I2: 4, I1: 2 \rangle, \langle I1: 2 \rangle$	{I2,I3: 4}, {I1,I3: 4}, {I2,I1,I3: 2}
I1	{{I2: 4}}	$\langle I2: 4 \rangle$	{I2,I1: 4}



# FP-Growth vs. Apriori: Scalability With the Support Threshold

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# Why Is FP-Growth the Winner?

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## ■ Divide-and-conquer:

- Decompose both the mining task and DB according to the frequent patterns obtained so far
- Focus searching on smaller databases

## ■ Other factors

- No candidate generation, no candidate test
- Compressed database: FP-tree structure
- Two scans of entire database
- Basic ops—counting local freq items and building sub FP-tree, no pattern search and matching

# Cons

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- Building FP-trees
  - A stack of FP-trees
- Redundant information
  - Transaction abcd appears in a-, ab-, abc-, ac-, c-FP-trees

# Mining Association Rules in Large Databases

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- Basic concepts and a road map
- Mining single-dimensional Boolean association rules
- Mining multilevel association rules
- Mining multidimensional association rules
- Summary

# Mining Multiple-Level Association Rules

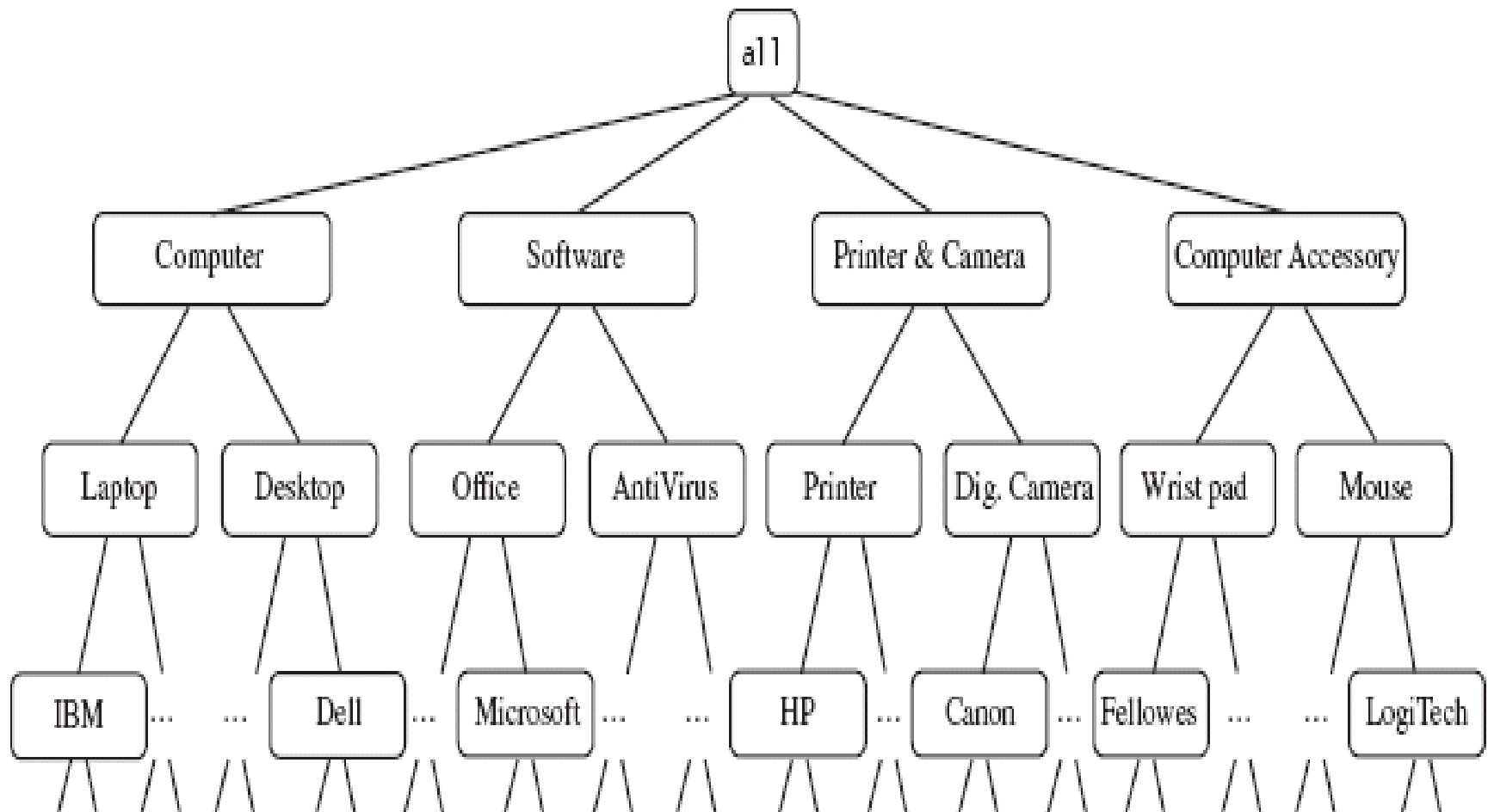
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- Association rules at high concept levels may represent common sense knowledge
- Hard to find association rules at low concept level
  - Items at the lower level usually have lower support, less than min\_support threshold
- Mining association rules at multiple levels of abstraction
- Example: sales in AllElectronics store computer sector



# Example

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# Mining Multiple-Level Association Rules

## ■ Uniform support

- Top-down, level-wise
- Use uniform minimum support for each level
- Perform Apriori at each level
- Optimization: if an ancestor is infrequent, the search on the descendants can be avoided

uniform support

Level 1  
min\_sup = 5%

**Milk**  
[support = 10%]

Level 2  
min\_sup = 5%

**2% Milk**  
[support = 6%]

**Skim Milk**  
[support = 4%]

# Mining Multiple-Level Association Rules

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uniform support

Level 1  
min\_sup = 5%

Milk  
[support = 10%]

Level 2  
min\_sup = 5%

2% Milk  
[support = 6%]

Skim Milk  
[support = 4%]

## ■ Drawbacks

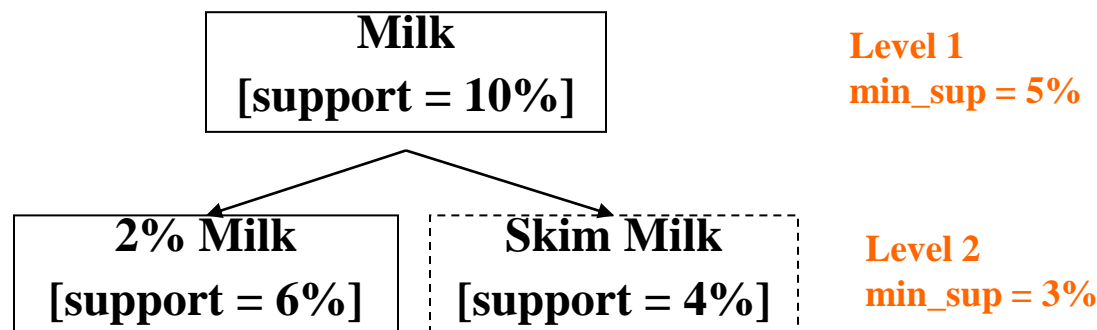
- Miss interesting associations with too high threshold
- Generate too many uninteresting rules with too low threshold

# Mining Multiple-Level Association Rules

## ■ Reduced support

- Top-down, level-wise
- Each concept level has its own minimum support threshold
- The lower level, the smaller threshold
- Perform Apriori at each level

reduced support



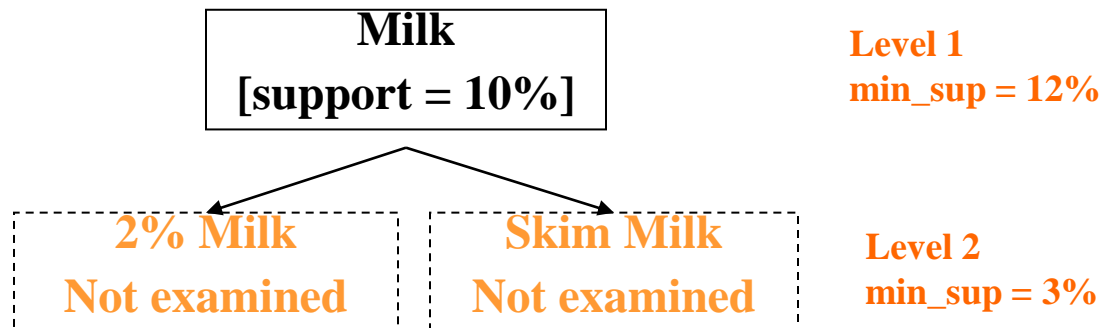
# Mining Multiple-Level Association Rules

## ■ Reduced support

### ■ Optimization -- level-cross filtering by single item

- An item at the  $i$ th concept level is examined *iff* its parent concept at the  $(i-1)$ th level is frequent
- If a concept is infrequent, its descendents are pruned from the database
- Drawbacks
  - Miss associations at low level items which are frequent based on a reduced min\_support, but whose ancestors do not satisfy min\_support

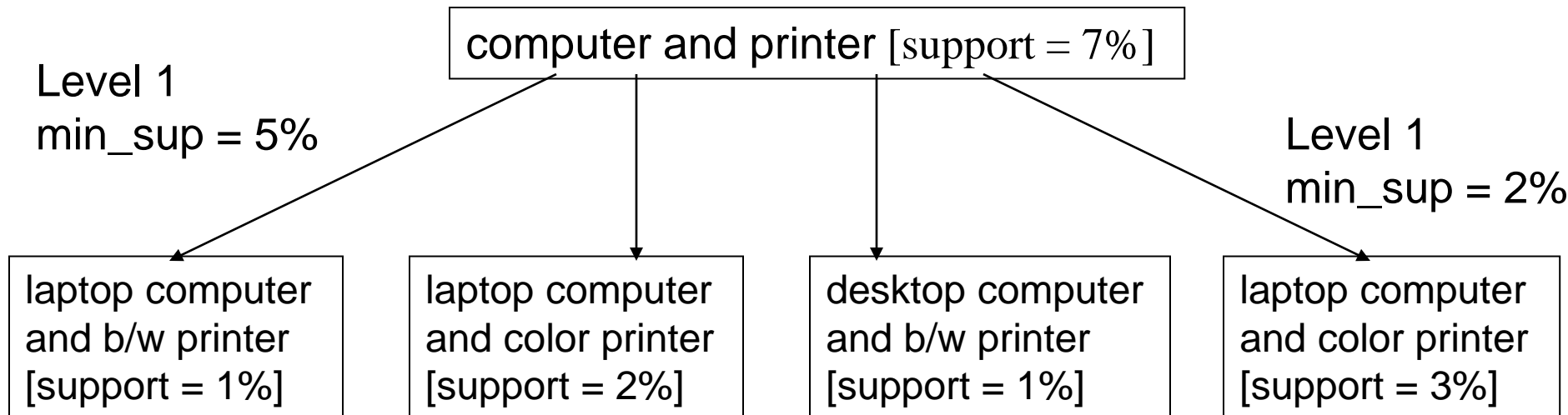
reduced support



# Mining Multiple-Level Association Rules

## ■ Reduced support

- Optimization -- level-cross filtering by  $k$ -itemset
  - Only the children of frequent  $k$ -itemsets are examined
  - Drawback: many valuable patterns may be filtered out

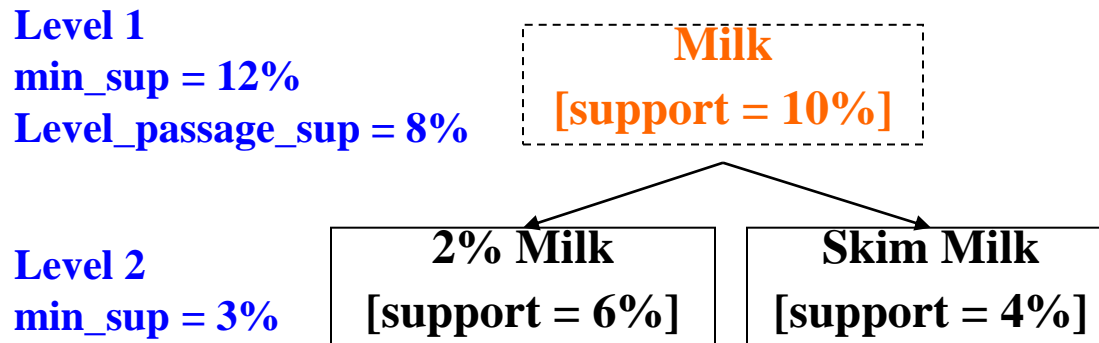


# Mining Multiple-Level Association Rules

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## ■ Reduced support

- Optimization -- Controlled level-cross filtering by single item
  - next level min sup < level passage threshold < min sup
  - Allow the children of items that do not satisfy the min\_sup to be examined if they satisfy the level passage threshold



# Multi-level Association: Redundancy Filtering

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- Some rules may be redundant due to “ancestor” relationships between items
- Example
  - milk  $\Rightarrow$  wheat bread [support = 8%, confidence = 70%]
  - 2% milk  $\Rightarrow$  wheat bread [support = 2%, confidence = 72%]
- We say the first rule is an ancestor of the second rule
- A rule is redundant if its support is close to the “expected” value, based on the rule’s ancestor



# Mining Association Rules in Large Databases

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# Mining Multi-Dimensional Association

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- Single-dimensional rules:

$\text{buys}(X, \text{"milk"}) \Rightarrow \text{buys}(X, \text{"bread"})$

- Multi-dimensional rules:  $\geq 2$  dimensions or predicates

- Inter-dimension assoc. rules (*no repeated predicates*)

$\text{age}(X, \text{"19-25"}) \wedge \text{occupation}(X, \text{"student"}) \Rightarrow \text{buys}(X, \text{"coke"})$

- hybrid-dimension assoc. rules (*repeated predicates*)

$\text{age}(X, \text{"19-25"}) \wedge \text{buys}(X, \text{"popcorn"}) \Rightarrow \text{buys}(X, \text{"coke"})$

- Categorical Attributes: finite number of possible values, no ordering among values

- Quantitative Attributes: numeric, implicit ordering among values — discretization, clustering approaches

# Mining Quantitative Associations

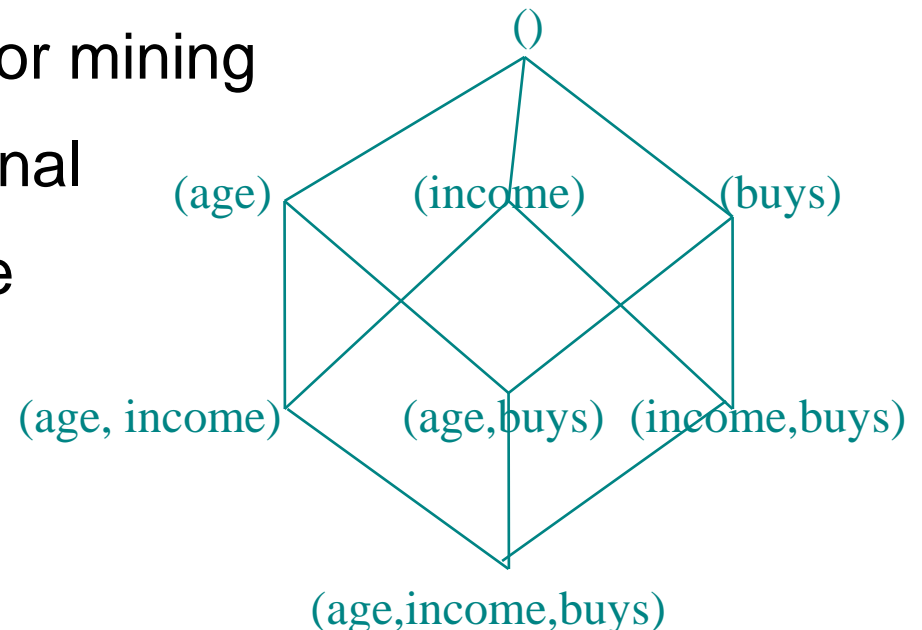
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- Techniques can be used to categorize numerical attributes
  - Static discretization based on predefined concept hierarchies
  - Dynamic discretization based on data distribution
  - Clustering: Distance-based association
    - one dimensional clustering then association

# Static Discretization of Quantitative Attributes

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- Discretized prior to mining using concept hierarchy
- Numeric values are replaced by ranges
- In relational database, finding all frequent  $k$ -predicate sets will require  $k$  or  $k+1$  table scans
- Data cube is well suited for mining
- The cells of a  $n$ -dimensional cuboid correspond to the dimensions
- Mining from data cubes can be much faster



# Quantitative Association Rules

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- Numeric attributes are *dynamically* discretized
  - Such that the confidence or compactness of the rules mined is maximized
- 2-D quantitative association rules:  $A_{\text{quan1}} \wedge A_{\text{quan2}} \Rightarrow A_{\text{cat}}$
- Association rule clustering system (ARCS)
  - Binning: 2-D grid, manageable size
  - Finding frequent predicate sets: scan the database, count the support for each grid cell
  - Clustering the rules: cluster adjacent cells to form a rule

# Quantitative Association Rules

## ■ Example

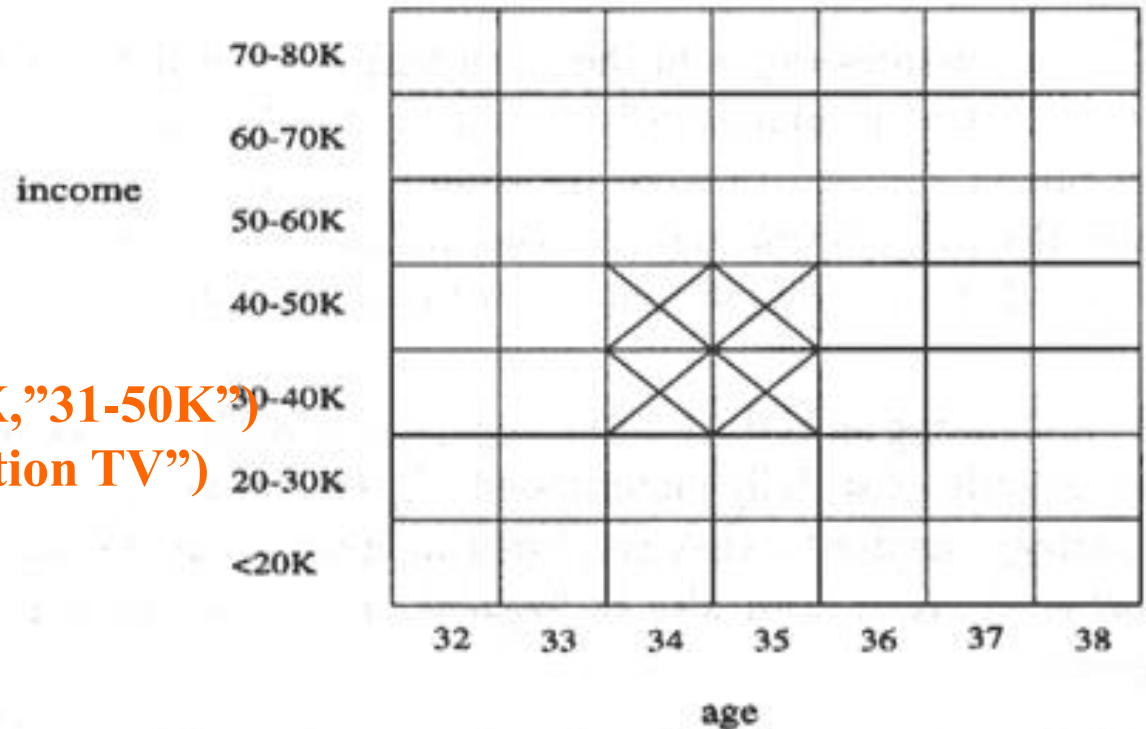
$\text{age}(X, "34") \wedge \text{income}(X, "31-40K") \Rightarrow \text{buys}(X, "high\ resolution\ TV")$

$\text{age}(X, "35") \wedge \text{income}(X, "31-40K") \Rightarrow \text{buys}(X, "high\ resolution\ TV")$

$\text{age}(X, "34") \wedge \text{income}(X, "41-50K") \Rightarrow \text{buys}(X, "high\ resolution\ TV")$

$\text{age}(X, "35") \wedge \text{income}(X, "41-50K") \Rightarrow \text{buys}(X, "high\ resolution\ TV")$

$\text{age}(X, "34-35") \wedge \text{income}(X, "31-50K")$   
 $\Rightarrow \text{buys}(X, "high\ resolution\ TV")$



# Mining Association Rules in Large Databases

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- Basic concepts and a road map
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# Summary

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- Frequent pattern mining—an important task in data mining
- Scalable frequent pattern mining methods
  - Apriori (Candidate generation & test)
  - Partition, DIC, DHP, etc.
  - Projection-based (FP-growth)
- Mining a variety of rules and interesting patterns