

1.a P: c<sub>0</sub>

N: c<sub>1</sub>

$$I(P, N) = I(10, 10) = 1$$

## 1) Gain (Gender)

Gender p<sub>i</sub> n<sub>i</sub> I(p<sub>i</sub>, n<sub>i</sub>)

M 6 4 0.971

F 4 6 0.871

$$E(\text{Gender}) = \frac{10}{20} I(6, 4) + \frac{10}{20} I(4, 6)$$
$$= 0.971$$

$$\text{Gain}(\text{Gender}) = I(p, n) - E(\text{Gender})$$

$$= 0.029$$

## 2) Gain (CarType)

CarType p<sub>i</sub> n<sub>i</sub> I(p<sub>i</sub>, n<sub>i</sub>)

Family 1 3 0.811

Sport 8 0 0

Luxury 1 7 0.544

$$E(\text{CarType}) = \frac{4}{20} \cdot 0.811 + \frac{8}{20} \cdot 0 + \frac{8}{20} \cdot 0.544$$
$$= 0.380$$

$$G(\text{CarType}) = I(p, n) - E(\text{CarType})$$

$$= 1 - 0.380$$

$$= 0.620$$

### 3) Gain( ShirtSize)

ShirtSize pi ni I(pi, ni)

S	3	2	0.971
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M	3	4	0.985
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L	2	2	1
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XL	2	2	1
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$$E(\text{ShirtSize}) = \frac{5}{20} \cdot 0.971 + \frac{7}{20} \cdot 0.985 + \frac{2}{20} \cdot 2 \\ = 0.988$$

$$G(\text{ShirtSize}) = I(P, N) - E(\text{ShirtSize})$$

$$= 0.212$$

b)

CarType?

~~Family~~      Sports      ~~Luxury~~

Shirt Size?

~~Small~~    ~~Medium~~    Large    Extra Large

1

C0

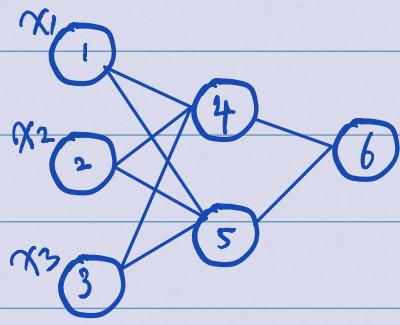
Shirt Size?

~~Large~~      C0      C1

2. (a)

将 Gender 中 M 设为 0, F 设为 1; CarType 中 Family 设为 0, Sports 设为 1, Luxury 设为 2; 将 Shirt Size 中 small 设为 0, Medium 设为 1, Large 设为 2, Extra Large 设为 3。

可设计出如下神经网络：



输入  $x_1: 0(M), 1(F)$

$x_2: 0(\text{Family}), 1(\text{Sports}), 2(\text{Luxury})$

$x_3: 0(\text{small}), 1(\text{Medium}), 2(\text{Large}), 3(\text{Extra Large})$

输出: 0(C0) 1(C1)

(b)

初始参数: learning rate: 0.9

$x_1 \ x_2 \ x_3 \ w_{14} \ w_{15} \ w_{24} \ w_{25} \ w_{34} \ w_{35} \ w_{46} \ w_{56} \ \theta_4 \ \theta_5 \ \theta_6$

0 0 0 0.2 -0.3 0.4 0.1 -0.5 0.2 -0.3 -0.2 -0.4 0.2 0.1

正向传播

Unit j	New input, $I_j$	Output, $O_j$
4	-0.4	$\frac{1}{1+e^{-0.4}} = 0.401$
5	0.2	0.550
6	-0.130	0.468

反向传播

$$Err_j = O_j(1-O_j) \sum_k Err_k w_{jk}$$

unit j	Err j
6	$(0.468)(1-0.468)(1-0.468) = 0.132$
5	$(0.55)(1-0.55)(0.132)(-0.2) = -0.0065$
4	$(0.468)(1-0.468)(0.132)(-0.3) = -0.0099$

更新参数

weight or bias	New value
$w_{46}$	$-0.3 + (0.9)(0.132)(0.401) = -0.25$
$w_{56}$	$-0.2 + (0.9)(0.132)(0.550) = -0.13$
$w_{14}$	$0.2 + (0.9)(-0.0099)(0) = 0.2$
$w_{15}$	$-0.3 + (0.9)(-0.0065)(0) = -0.3$

W24

$$0.4 + (0.9)(-0.0099)(0) = 0.4$$

W25

$$0.1 + (0.9)(-0.0065)(0) = 0.1$$

W34

$$-0.5 + (0.9)(-0.99)(0) = -0.5$$

W35

$$0.2 + (0.9)(-0.0065)(0) = 0.2$$

θ6

$$0.1 + (0.9)(0.132) = 0.22$$

θ5

$$0.2 + (0.9)(-0.0065) = 0.19$$

θ4

$$-0.4 + (0.9)(-0.0099) = -0.41$$

3.  $P(\text{class} = 'C_0') = \frac{1}{2}$

$$P(\text{class} = 'C_1') = \frac{1}{2}$$

$$P(\text{Gender} = 'F' | \text{class} = 'C_0') = \frac{2}{5}$$

$$P(\text{Gender} = 'F' | \text{class} = 'C_1') = \frac{3}{5}$$

$$P(\text{Car Type} = 'Family' | \text{class} = 'C_0') = \frac{1}{10}$$

$$P(\text{Car Type} = 'Family' | \text{class} = 'C_1') = \frac{3}{10}$$

$$P(\text{Shirt Size} = 'Large' | \text{class} = 'C_0') = \frac{1}{5}$$

$$P(\text{Shirt Size} = 'Large' | \text{class} = 'C_1') = \frac{1}{5}$$

$$z = (\text{Gender} = F, \text{Car Type} = \text{Family}, \text{Shirt Size} = \text{Large})$$

$$P(z | \text{class} = 'C_0') = \frac{2}{5} \times \frac{1}{10} \times \frac{1}{5} = \frac{1}{125}$$

$$P(z | \text{class} = 'C_1') = \frac{3}{5} \times \frac{3}{10} \times \frac{1}{5} = \frac{9}{250}$$

$$P(z | \text{class} = 'C_0') \cdot P(\text{class} = 'C_0') = \frac{1}{2} \cdot \frac{1}{125} = \frac{1}{250}$$

$$P(z | \text{class} = 'C_1') \cdot P(\text{class} = 'C_1') = \frac{1}{2} \cdot \frac{9}{250} = \frac{9}{500}$$

A<sup>1</sup>, B<sup>1</sup>, C<sup>1</sup>

因为  $\frac{9}{500} > \frac{1}{250}$ , 所以 z 将会分配到 C<sub>1</sub> 类

#### 4. (a) 第一輪聚类:

$$A_1 A_2 = 7.34$$

$$A_1 A_3 = 6.403$$

$$A_1 B_2 = 3.741$$

$$A_1 B_3 = 5.744$$

$$A_1 C_2 = 3.741$$

$$A_1 C_3 = 5.477$$

$$A_1 C_4 = 4.582$$

$$A_2 B_1 = 9.889$$

$$A_2 B_3 = 10.049$$

$$B_1 B_2 = 2.449$$

$$B_1 B_3 = 9.643$$

$$B_1 C_2 = 5.83$$

$$B_1 C_3 = 10$$

$$B_1 C_4 = 8.774$$

$$A_2 C_1 = 4.123$$

$$A_2 C_2 = 7.874$$

$$B_2 C_1 = 10.816$$

$$B_2 C_2 = 11.045$$

$$C_1 C_2 = 11.874$$

$$C_1 C_3 = 9.643$$

$$C_1 C_4 = 8.366$$

cluster1:  $A_1 \quad A_3 \quad B_3 \quad C_2 \quad C_3 \quad C_4$

cluster2:  $B_1 \quad B_2$

cluster3:  $C_1 \quad A_2$

新加入的点:

$$D = \text{centroid}_1 = \frac{A_1 + A_3 + B_3 + C_2 + C_3 + C_4}{6} = (4.5, 4.5, 6.83)$$

$$E = \text{centroid}_2 = \frac{B_1 + B_2}{2} = (1.5, 2, 1.5)$$

$$F = \text{centroid}_3 = \frac{A_2 + C_1}{2} = (10.5, 7, 2)$$

(b)

#### 第二輪聚类:

$$A_1 D = 3.13 \quad A_1 E = 4.3 \quad A_1 F = 8.73$$

$$A_2 D = 7.33 \quad A_2 E = 9.02 \quad A_2 F = 2.06$$

$$A_3 D = 4.22 \quad A_3 E = 8.86 \quad A_3 F = 7.5$$

$$B_1 D = 7.64 \quad B_1 E = 1.22 \quad B_1 F = 11.2$$

$$B_2 D = 5.64 \quad B_2 E = 1.2^2 \quad B_2 F = 8.39$$

$$B_3D = 3.03 \quad B_3E = 8.63 \quad B_3F = 10.30$$

$$C_1D = 9.26 \quad C_1E = 11.81 \quad C_1F = 2.66$$

$$C_2D = 3.63 \quad C_2E = 4.94 \quad C_2F = 10.73$$

$$C_3D = 5.7 \quad C_3E = 9.35 \quad C_3F = 7.95$$

$$C_4D = 1.59 \quad C_4E = 7.64 \quad C_4F = 7.5$$

cluster1: A<sub>1</sub> A<sub>3</sub> B<sub>3</sub> C<sub>2</sub> C<sub>3</sub> C<sub>4</sub>

cluster2: B<sub>1</sub> B<sub>2</sub>

cluster3: A<sub>2</sub> C<sub>1</sub>

第二轮算法后聚类与第一轮算法后相同，表明聚类已完成。

极最后的三个 cluster 如上所示。