hw3

Wen Fan(wf85),Zhidong Liu(zl479), Siyu Zhu(sz432)

March 2016

1 Functional Dependencies

- A+ = B+ $\Leftrightarrow$ A $\rightarrow$ B and B $\rightarrow$ A
  Solution:
  Proof from left to right hand :
  Because A $\rightarrow$ A, $A \in A+ = B+$. $A \in B+ \Rightarrow B \rightarrow A$.
  Because B $\rightarrow$ B, $B \in B+ = A+$. $B \in A+ \Rightarrow A \rightarrow B$.
  Proof from right to left hand:
  $\forall K \in B^+, B \rightarrow K, A \rightarrow B \Rightarrow A \rightarrow B \rightarrow K, A \rightarrow K, K \in A^+$. Therefore, $B^+ \subseteq A^+$
  Same reason as above, $A^+ \subseteq B^+$
  $B^+ \subseteq A^+, A^+ \subseteq B^+ \Rightarrow A^+ = B^+$

- X$\rightarrow$Y $\Leftrightarrow$ X is a key of $\pi_{XY}(R)$
  Solution:
  Proof from left to right hand :
  $\forall Tuples\ t_1, t_2 \in R(t_1 = t_2$ allowed), $\pi_X(t_1) = \pi_X(t_2) \Rightarrow \pi_Y(t_1) = \pi_Y(t_2)$.
  We can combine X and Y, then $\pi_X(t_1) = \pi_X(t_2) \Rightarrow \pi_{XY}(t_1) = \pi_{XY}(t_2)$.
  Therefore, according to the definition of key, X is a key of $\pi_{XY}(t_2)$. *Proof from right to left hand:*
  $\forall Tuples\ t_1, t_2 \in R(t_1 = t_2$ allowed), if $\pi_X(t_1) = \pi_X(t_2)$, then $t_1 = t_2$ $\Rightarrow \pi_Y(t_1) = \pi_Y(t_2)$. According to the definition of FD,$X \rightarrow Y$.

- $A \cap B = X(X \neq empty)$, R with $A \cup B$, $X \rightarrow B \Rightarrow R = \pi_A(R) \bowtie \pi_B(R)$
  Solution:
  Assume the attributes in A except from X is A_r, the attributes in B except from X is B_r.
  Prove that $\pi_A(R) \bowtie \pi_B(R) \subseteq R$:
  $\forall$ Tuple $t \in \pi_A(R) \bowtie \pi_B(R)$, $\exists t_1 \in R, t_2 \in R$ and $\pi_X(t_1) = \pi_X(t_2)$

$$t = \pi_{XA_r}(t_1) \bowtie \pi_{XB_r}(t_2) \tag{1}$$

  Because $X \rightarrow B$ and $\pi_X(t_1) = \pi_X(t_2)$, $\pi_B(t_1) = \pi_B(t_2) \Rightarrow \pi_{B_r}(t_1) = \pi_{B_r}(t_2)$. Then (1) becomes:

$$t = \pi_{XA_r}(t_1) \bowtie \pi_{XB_r}(t_2) = \pi_{XA_r}(t_1) + \pi B_r(t_2) = \pi_{XA_r}(t_1) + \pi B_r(t_1)$$

$= \pi_{XA_rB_r}(t_1) = \pi_{A\cup B}t1 = t1$
Therefore $\forall$ Tuple $t \in \pi_A(R) \bowtie \pi_B(R)$, $\exists t_1 \in R, t = t_1 \Rightarrow \pi_A(R) \bowtie \pi_B(R) \subseteq R$
Prove that $R \subseteq \pi_A(R) \bowtie \pi_B(R)$:
$\forall$ Tuple $t \in R$, according to the definition of natural join,

$$t = \pi_A(t) \bowtie \pi_B(t) \in \pi_A(R) \bowtie \pi_B(R) \Rightarrow R \subseteq \pi_A(R) \bowtie \pi_B(R)$$

To sum up, $R = \pi_A(R) \bowtie \pi_B(R)$.

2 ER Diagram

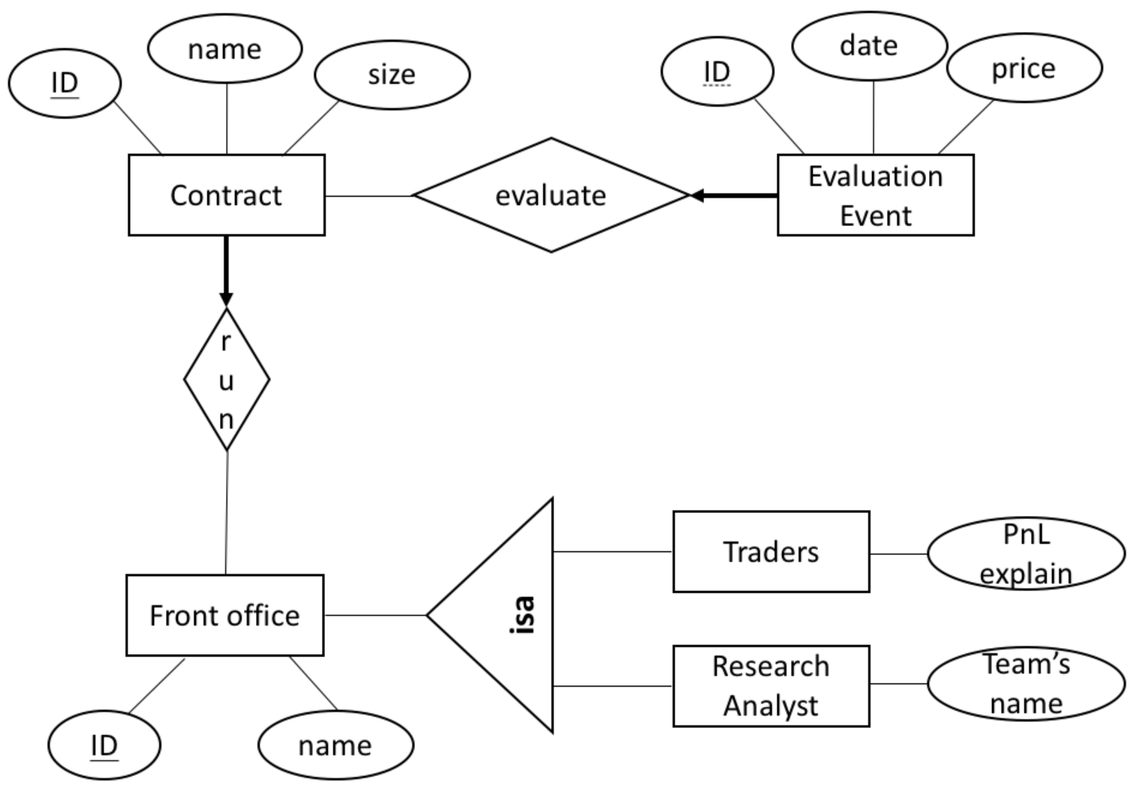

Figure 1: ER Diagram

Note: Evaluation Event is a weak entity depending on Contract, so its ID is underlined with dash lines.

3 Normal Forms

- **Solution**
  No, it is not always beneficial. Decomposition can bring the following problems
  1. Some queries require a join 2. May lose information if not careful 3. Checking FDs may require a join

- **Solution**
  only appears on the LHS of the FDs, so A must be in the key. Since that we can get E and H from A→EH, the key does not contain E or H. From $B \to CDF$ and $CDE \to BGF$, we know that in order to get$(key)^+ = ABCDEFGH$, the key must contain either B or CD (containing both of them will cause a superkey). Computing the attribute closures $(AB)^+$ and $(ACD)^+$, we can verify that both AB and ACD are keys of R. By the property of minimality, they are the only keys of R.

- **Solution**
  BCNF: Functional dependency $B \to CDF$ holds for R. By the Reflexivity of Armstrong's Axiom, $CDF \to F$. Based on the Transitivity, $B \to F$. That is, $B \to F \in F^+$ (Suppose F^+ is the closure of the set of functional dependencies). However, neither "$F \in B$" nor "B contains a key for R" holds here. So by the definition of BCNF, R is not in BCNF.
  3NF: Still use the example above. For functional dependency $B \to F$, F is neither in key AB nor in key ACD. So R is not in 3NF.

- **Solution**
  The minimal cover G of set $F = B \to CDF, A \to EH, CDE \to BGF$ is:

  $$G = B \to C, B \to D, B \to F, A \to E, A \to H, CDE \to B, CDE \to G$$

  According to the definition of 3NF, we first identify that $B \to F$ violates 3NF so we decompose to $ABCDEFGH, BF$. Then $A \to E$ is identified to violate 3NF, so $ABCDEFGH$ is decomposed to $ABCDGH$ and AE. Also, $A \to H$ violates 3NF, $ABCDGH$ is decomposed to $ABCDG$ and AH. At this point, $CDE \to B$ and $CDE \to G$ are not preserved, so we need to add $CBDE$ and $CDEG$ to schema. Finally, the decomposition of R into 3NF is: $ABCDG, AH, AE, BF, CBDE, CDEG$.