HW2.

a. 真值表.

1° p q $((p \to q) \to p) \to p$

p	q	$(p \to q)$	$((p \to q) \to p)$	$((p \to q) \to p) \to p$
1	1	1	1	1
1	0	0	1	1
0	1	1	0	1
0	0	1	0	1

2° p q $(p \wedge q) \to (p \vee q)$

p	q	$(p \wedge q)$	$(p \vee q)$	$(p \wedge q) \to (p \vee q)$
1	1	1	1	1
1	0	0	1	1
0	1	0	1	1
0	0	0	1	0

3° p q $(p \to q) \vee (p \to \neg q)$

p	q	$(p \to q)$	$(p \to \neg q)$	$(p \to q) \vee (p \to \neg q)$
1	1	1	0	1
1	0	0	1	1
0	1	1	1	1
0	0	1	1	1

4° p q r $((p \vee q) \to r) \to ((p \to r) \vee (q \to r))$

p	q	r	$(p \vee q)$	$((p \vee q) \to r)$	$(p \to r)$	$(q \to r)$	$((p \to r) \vee (q \to r))$	$((p \vee q) \to r) \to ((p \to r) \vee (q \to r))$
1	1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0	1
1	0	1	1	1	1	1	1	1
1	0	0	1	0	0	1	1	1
0	1	1	1	1	1	1	1	1
0	1	0	1	0	1	0	1	0
0	0	1	0	1	1	1	1	1
0	0	0	0	1	1	1	1	1

1. 谓词转换.

(a) $\forall x\,(P(x) \to A(m,x))$.

(b) $\exists x\, A(P(x), m)$.

(c) $A(m,m)$.

(d) $\forall x\,(S(x) \to \exists y\,(L(y) \to \neg B(x,y)))$.

(e) $\forall x\,(L(x) \to \exists y\,(S(y) \to \neg B(y,x)))$

(f) $\forall x \forall y\,((S(x) \wedge L(y)) \to \neg B(x,y))$.

2. 判断.

(a) 满足 $z < x < y$ 即可，但于自然数集中，$x=0$ 时 z 不存在，不满足.

(b) 满足 $y = 2x = 2z$ 即可，

(c) 满足 $x = z < y+1$ 即可.

3. 证明

1° $(p \wedge q) \wedge r, s \wedge t \vdash q \wedge s$

1	$(p \wedge q) \wedge r$	premise
2	$p \wedge q$	$\wedge e_1, 1$
3	q	$\wedge e_2, 2$
4	$s \wedge t$	premise
5	s	$\wedge e_1, 1$.
6	$q \wedge s$	$\wedge i, 3, 5$

2° $q \to r \vdash (p \to q) \to (p \to r)$

1	$q \to r$	premise
2	$p \to q$	assumption
3	p	assumption
4	q	$\to e, 3, 2$
5	r	$\to e, 4, 1$.
6	$p \to r$	$\to i$, 3-5
7	$(p \to q) \to (p \to r)$	$\to i$, 2-7

3° $\vdash q \to (p \to (p \to (q \to p)))$

1	q	assumption
2	p	assumption
3	$q \to p$	$\to i$, 1-2
4	$p \to (q \to p)$	$\to i$, 2-4
5	$p \to (p \to (q \to p))$	$\to i$, 2-5
6	$q \to (p \to (p \to (q \to p)))$	$\to i$, 1-5

4° $p \to q \wedge r \vdash (p \to q) \wedge (p \to r)$.

1	p	assumption
2	$p \to q \wedge r$	premise
3	$q \wedge r$	$\to e, 1, 2$
4	q	$\wedge e_1, 3$
5	$p \to q$	$\to i, 1-4$
6	p	assumption
7	r	$\wedge e_2, 3$
8	$p \to r$	$\to i, 1-6$
9	$(p \to q) \wedge (p \to r)$	$\wedge i, 5, 7$

5° $p \wedge \neg p \vdash \neg (r \to q) \wedge (r \to q)$

$p \wedge \neg p$ 为矛盾.

根据定理，任公式均可从矛盾推出.

即 $p \wedge \neg p \vdash q$.

将 q 代换则有

$p \wedge \neg p \vdash \neg (r \to q) \wedge (r \to q)$

2. 证明

1° $\exists x (S \to Q(x)) \vdash S \to \exists x Q(x)$

1	$\exists x (S \to Q(x))$	premise
2	x_0 $S \to Q(x_0)$	assumption
3	S	assumption
4	$Q(x_0)$	$\to e, 2, 3$.
5	$\exists x Q(x)$	$\exists x\, i, 4$.
6	$S \to \exists x Q(x)$	$\to i, 3-5$
7	$S \to \exists x Q(x)$	$\exists x\, e\, 1, 2-6$

2° $\forall x P(x) \to S \vdash \exists x (P(x) \to S)$

1	$\neg \exists x (P(x) \to S)$	assumption
2	x_0 $\neg (P(x_0) \to S)$	assumption
3	$\forall x \neg (P(x) \to S)$	$\forall x\, i\ 2$
4	$\neg (P(x) \to S)$	$\forall x\, e\ 3$
5	$P(x) \wedge \neg S$	truth table.
6	$P(x)$	$\wedge e_1\ 5$
7	$\forall x P(x)$	$\forall x\, i\ 6$
8	$\forall x P(x) \to S$	premise
9	S	$\to e, 7, 8$
10	$\neg S$	$\wedge e_2\ 5$
11	$\perp$	$\neg e\ 9\ 10$
12	$\neg\neg \exists x (P(x) \to S)$	$\neg i$, 1-11
13	$\exists x (P(x) \to S)$	$\neg\neg e, 12$

3° $\forall x (P(x) \wedge Q(x)) \vdash \forall x P(x) \wedge \forall x Q(x)$.

1	$\forall x (P(x) \wedge Q(x))$	premise
2	x_0 $P(x_0) \wedge Q(x_0)$	$\forall x\, e\ 1$
3	$P(x_0)$	$\wedge e_1\ 2$
4	$Q(x_0)$	$\wedge e_2\ 2$
5	$\forall x P(x)$	$\forall x\, i\ 2-3$
6	$\forall x Q(x)$	$\forall x\, i\ 2, 4$
7	$\forall x P(x) \wedge \forall x Q(x)$	$\wedge i, 5, 6$

4° $\neg\forall x \neg P(x) \vdash \exists x P(x)$.

1	$\neg\forall x \neg P(x)$	premise
2	x_0 $\neg(\neg P(x_0))$	$\forall x$ e 1.
3	$P(x_0)$	$\neg\neg$e 2
4	$\exists x P(x)$	$\exists x$ i 3.

5° $\forall x \neg P(x) \vdash \neg\exists x P(x)$

1	$\exists x P(x)$	assumption
2	x_0 $P(x_0)$	assumption
3	$\forall x \neg P(x)$	premise
4	x_0 $\neg P(x_0)$	$\forall x$ e 3
5	$\bot$	$\neg$e 2,4
6	$\neg\exists x P(x)$	$\neg$i 1-5