# Linear Regression

## Guided Practice

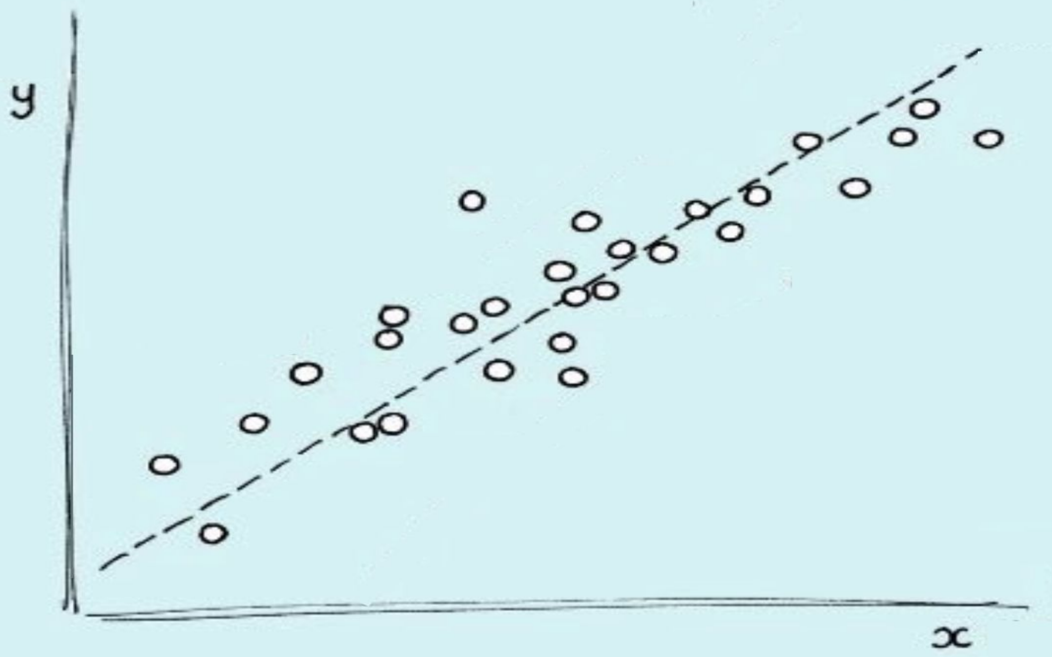


Image by Dan White

# Poll

A. Project Score

B. Hours of Sleep

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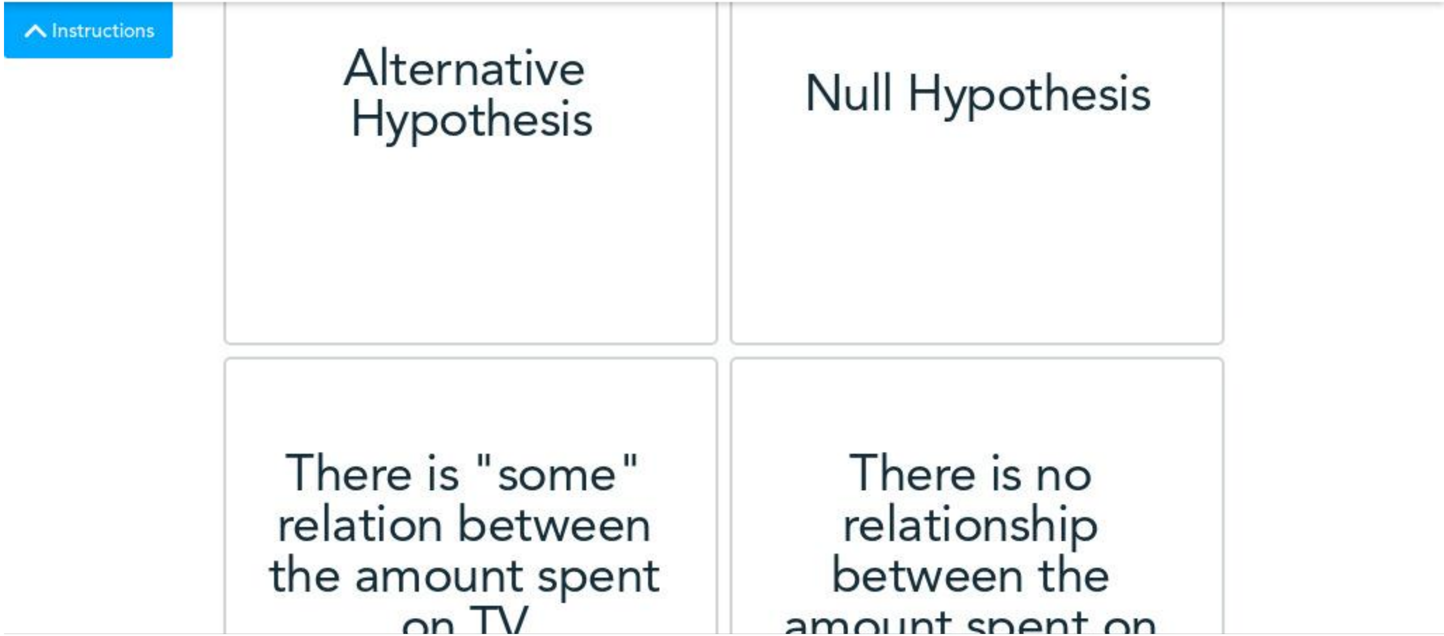
# Matching Pairs

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# Fill in the Blanks

Describe each of the variables of the equation:

$\hat{y}$ predicted value of our [redacted] variable

$b0$ [redacted] when x=0

$b1$ [redacted] of the regression line

$x$ an [redacted] variable

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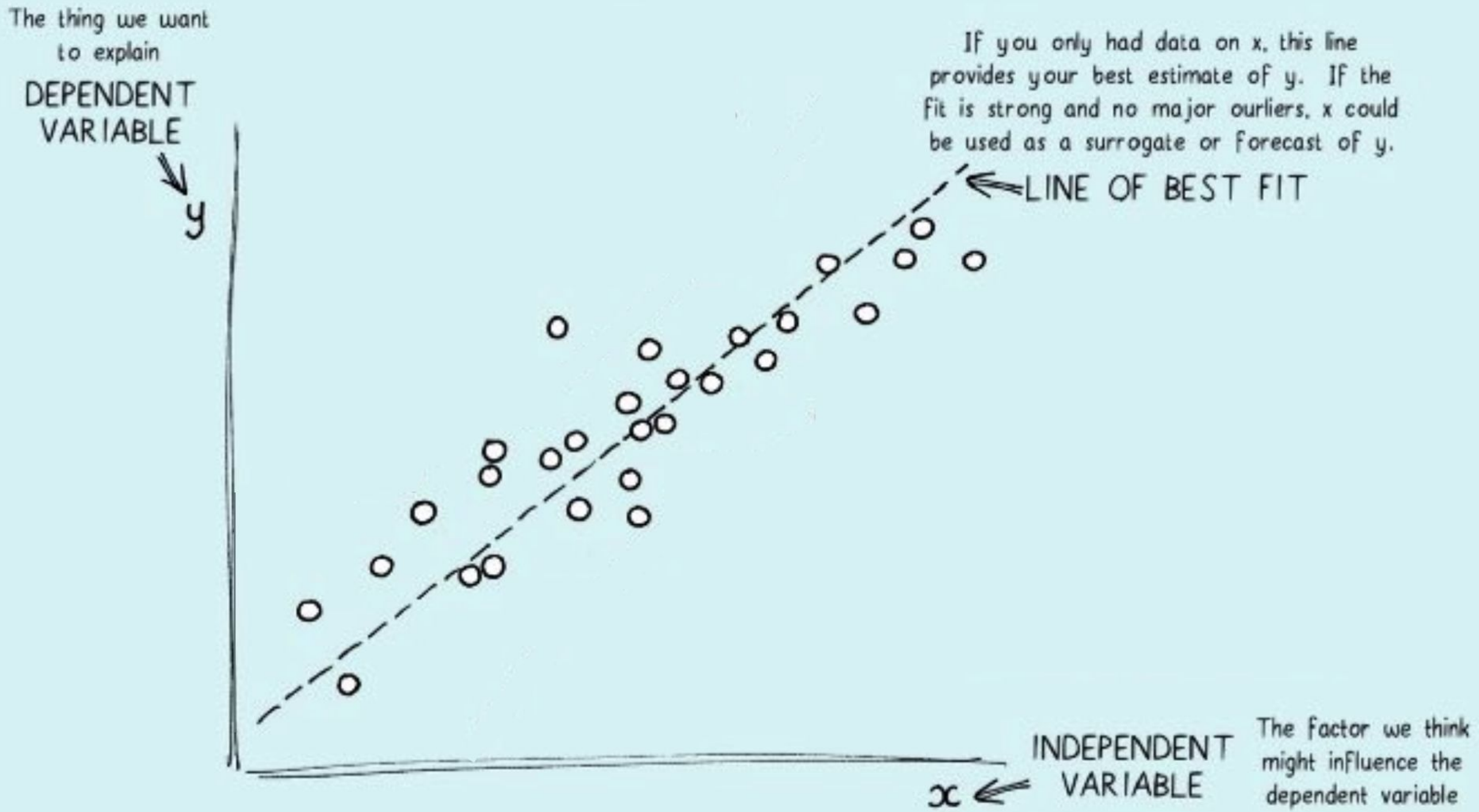
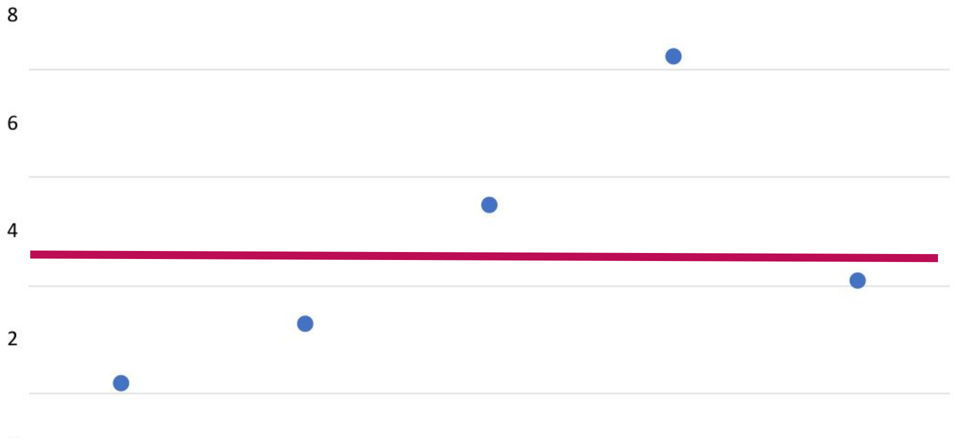
# Linear Regression Goals



# How do we find the line of best fit?

For a dataset with only one variable, the best fit line is the mean value of the data points.

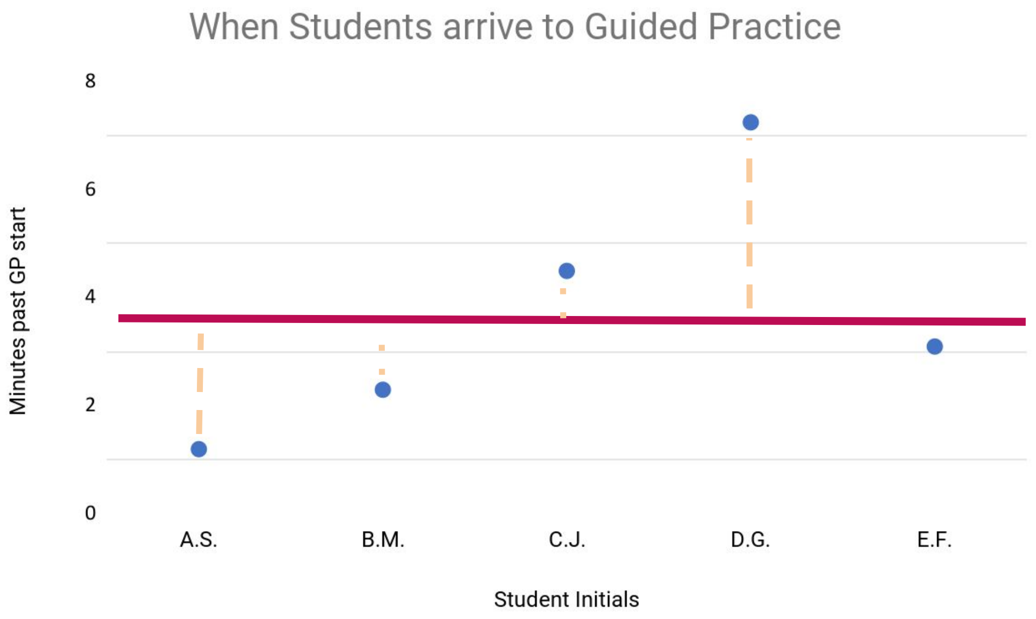



$$\text{Mean} = \frac{\text{Sum of All Data Points}}{\text{Number of Data Points}}$$

# Line of Best fit

For a dataset with only one variable, the best fit line is the mean value of the data points.

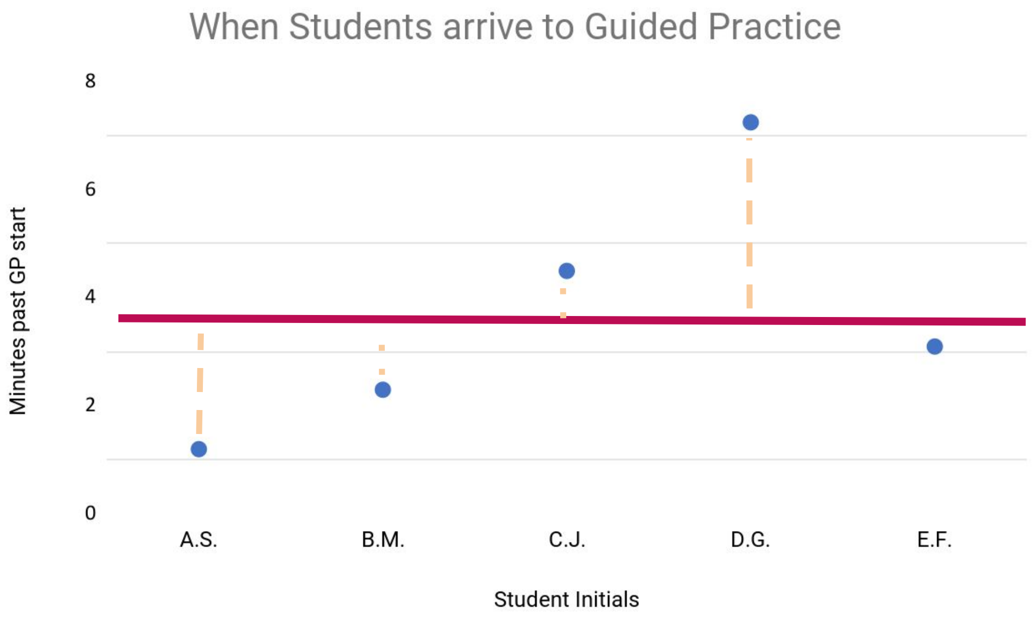
| Student Initial | Minutes past start of GP |
|---|---|
| A.S. | 1.2 |
| B.M. | 2.3 |
| C.J. | 4.5 |
| D.G. | 7.25 |
| E.F. | 3.1 |
| **Mean:** | **3.67** |



# Line of Best fit

The sum of the distances (error) above the mean has the same absolute value as the distances (error) below the mean.

| Student Initial | Minutes past start of GP | $(y - \bar{y})$ |
|---|---|---|
| A.S. | 1.2 | -2.47 |
| B.M. | 2.3 | -1.37 |
| C.J. | 4.5 | 0.83 |
| D.G. | 7.25 | 3.58 |
| E.F. | 3.1 | -0.57 |
| Mean: | 3.67 | |

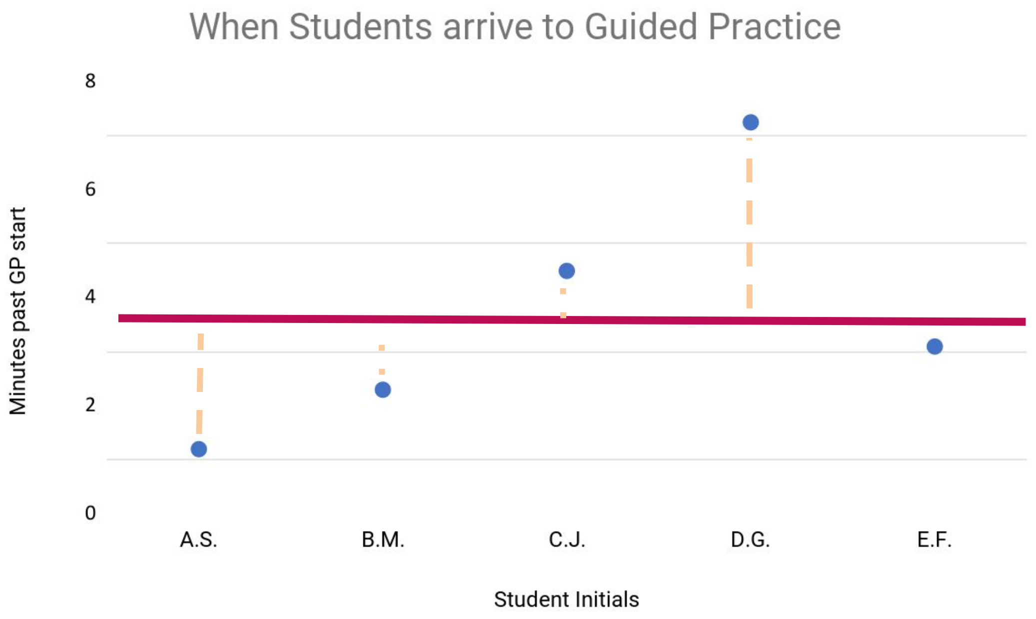



Distance = Residual = Error

# Line of Best fit

The goal of the line of best fit is to minimize the Sum of Squares Error.

| Error (E) | Square Error (SE) |
|---|---|
| -2.47 | 6.1009 |
| -1.37 | 1.8769 |
| 0.83 | 0.6889 |
| 3.58 | 12.8164 |
| -0.57 | 0.3249 |
| -3.67 | 13.4689 |
| **Sum:** | **35.2769** |



# Line of Best fit

Ordinary Least Square (OLS) regression utilizes the principle of least squares, i.e. minimize the sum of the squares of the differences.

| Error (E) | Square Error (SE) |
|---|---|
| -2.47 | 6.1009 |
| -1.37 | 1.8769 |
| 0.83 | 0.6889 |
| 3.58 | 12.8164 |
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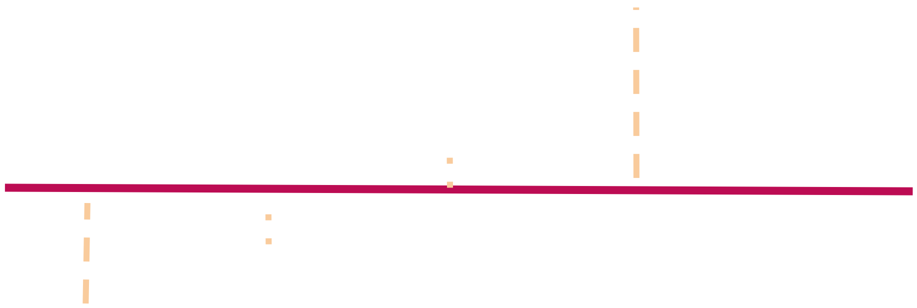

# Open Ended Question

Ready? Enter your answer here.

## How to Edit

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**Equation of a Line**

$$y = mx + b$$

**Simple Linear Regression Equation**

$$\hat{y} = b_0 + b_1 x$$

**Multiple Linear Regression Equation**

$$\hat{y} = b_0 + b_1 x_1 + b_{\square} x_{\square}$$

# Multiple Linear Regression Equation

$$\hat{y} = b_0 + b_1x_1 + b_{\square}x_{\square}$$

- $\hat{y}$ *the predicted value of our dependent variable y*
- $b_0$ *the y-intercept when x is equal to* 0
- $b_1$ *coefficient for variable 1*
- $x_1$ *first independent variable*
- $b_{\square}$ *coefficient for variable n*
- $x_{\square}$ *the nth independent variable*

# Multiple Linear Regression Equation

$$\hat{y} = b_0 + b_1x_1 + b_{\square}x_{\square}$$

$$b_1 = \frac{\sum(x_2-\overline{x_2})^2 \sum(x_1-\overline{x_1})(y-\bar{y}) - \sum(x_1-\overline{x_1})(x_2-\overline{x_2}) \sum(x_2-\overline{x_2})(y-\bar{y})}{\sum(x_1-\overline{x_1})^2 \sum(x_2-\overline{x_2})^2 - (\sum(x_1-\overline{x_1})(x_2-\overline{x_2}))^2}$$

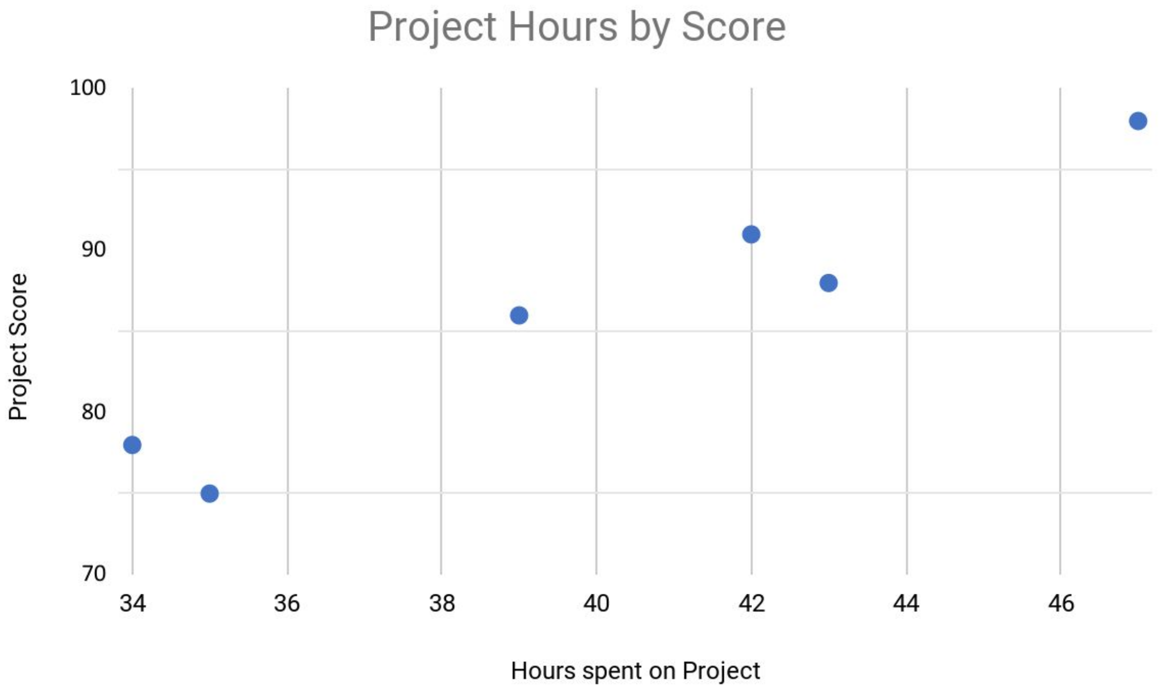
$$b_2 = \frac{\sum(x_1-\overline{x_1})^2 \sum(x_2-\overline{x_2})(y-\bar{y}) - \sum(x_1-\overline{x_1})(x_2-\overline{x_2}) \sum(x_1-\overline{x_1})(y-\bar{y})}{\sum(x_1-\overline{x_1})^2 \sum(x_2-\overline{x_2})^2 - (\sum(x_1-\overline{x_1})(x_2-\overline{x_2}))^2}$$

$$b_0 = \bar{y} - b_1\overline{x_1} - b_2\overline{x_2}$$

# Linear Regression Example

Let's explore the relationship between number of hours spent on a project and the score a project was graded.

| Project Hours (x) | Project Score (y) |
|---|---|
| 34 | 78 |
| 35 | 75 |
| 39 | 86 |
| 42 | 91 |
| 43 | 88 |
| 47 | 98 |



# Linear Regression Example

We need to run our calculations to determine our coefficients, i.e. **fit our model.**

$$\hat{y} = b_0 + b_1 x$$

$$b_1 = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

$$b_0 = \bar{y} - b_1 \bar{x}$$

| Project Hours (x) | Project Score (y) | $(x-\bar{x})$ | $(y-\bar{y})$ | $(x-\bar{x})(y-\bar{y})$ | $(x-\bar{x})^2$ |
|---|---|---|---|---|---|
| 34 | 78 | -6 | -8 | 48 | 36 |
| 35 | 75 | -5 | -11 | 55 | 25 |
| 39 | 86 | -1 | 0 | 0 | 1 |
| 42 | 91 | 2 | 5 | 10 | 4 |
| 43 | 88 | 3 | 2 | 6 | 9 |
| 47 | 98 | 7 | 12 | 84 | 49 |
| **40** | **86** | | **Sum:** | **203** | **124** |

# Simple Linear Regression Equation

- $b_1 = \dfrac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$ = 1.63

- $b_0 = \bar{y} - b_1\bar{x}$ = 86 - (1.63 x 40) = 20.8

$\hat{y} = b_0 + b_1x$ $\hat{y} = 20.8 + 1.63x$

# Simple Linear Regression Equation

$$\hat{y} = 20.8 + 1.63x$$

With our linear model we could predict the number of hours spent for a score of 75:

$$80 = 20.8 + 1.63x$$

$$x = 36.31 \textit{ hours}$$

# Simple Linear Regression Equation

$$\hat{y} = 20.8 + 1.63x$$

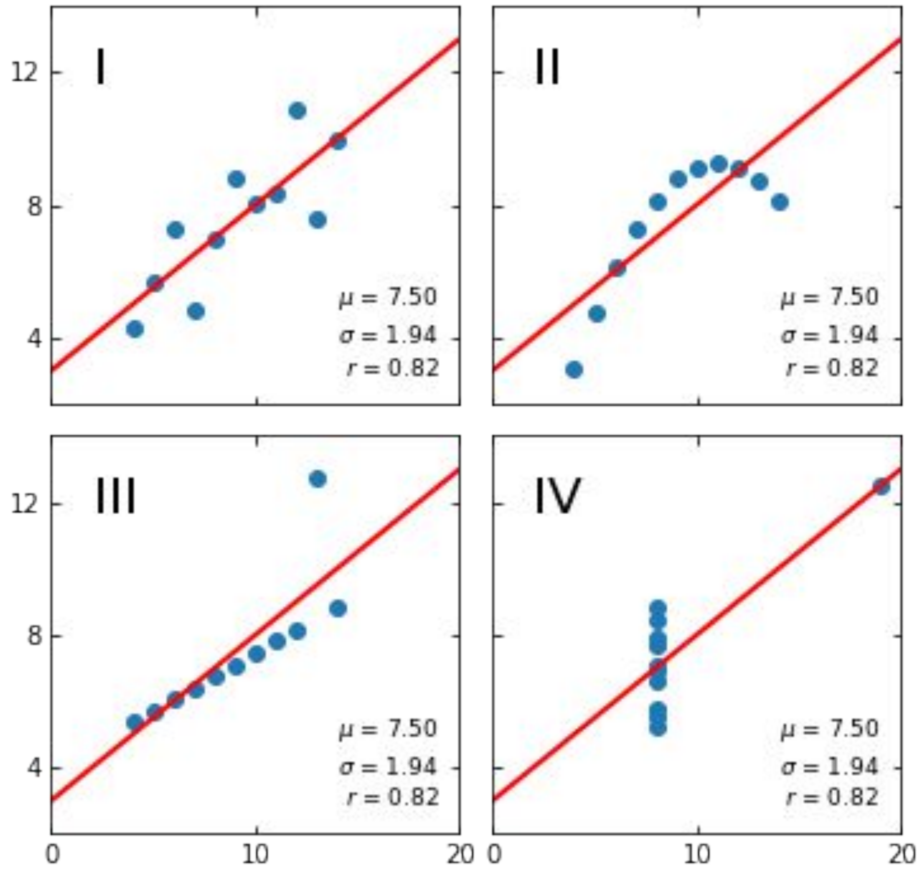
With our linear model we could predict the number of hours spent for a score of 80:

$$80 = 20.8 + 1.63x$$

$$x = 36.31 \; \textit{hours}$$

# Anscombe's Quartet: The Importance of Visualization

```python
import matplotlib.pyplot as plt
import numpy as np

x = [10, 8, 13, 9, 11, 14, 6, 4, 12, 7, 5]
y1 = [8.04, 6.95, 7.58, 8.81, 8.33, 9.96, 7.24, 4.26, 10.84, 4.82, 5.68]
y2 = [9.14, 8.14, 8.74, 8.77, 9.26, 8.10, 6.13, 3.10, 9.13, 7.26, 4.74]
y3 = [7.46, 6.77, 12.74, 7.11, 7.81, 8.84, 6.08, 5.39, 8.15, 6.42, 5.73]
x4 = [8, 8, 8, 8, 8, 8, 8, 19, 8, 8, 8]
y4 = [6.58, 5.76, 7.71, 8.84, 8.47, 7.04, 5.25, 12.50, 5.56, 7.91, 6.89]

datasets = {
    'I': (x, y1),
    'II': (x, y2),
    'III': (x, y3),
    'IV': (x4, y4)
}

fig, axs = plt.subplots(2, 2, sharex=True, sharey=True, figsize=(6, 6),
                        gridspec_kw={'wspace': 0.08, 'hspace': 0.08})
axs[0, 0].set(xlim=(0, 20), ylim=(2, 14))
axs[0, 0].set(xticks=(0, 10, 20), yticks=(4, 8, 12))

for ax, (label, (x, y)) in zip(axs.flat, datasets.items()):
    ax.text(0.1, 0.9, label, fontsize=20, transform=ax.transAxes, va='top')
    ax.tick_params(direction='in', top=True, right=True)
    ax.plot(x, y, 'o')

    # linear regression
    p1, p0 = np.polyfit(x, y, deg=1)  # slope, intercept
    ax.axline(xy1=(0, p0), slope=p1, color='r', lw=2)

    # add text box for the statistics
    stats = (f'$\\mu$ = {np.mean(y):.2f}\n'
             f'$\\sigma$ = {np.std(y):.2f}\n'
             f'$r$ = {np.corrcoef(x, y)[0][1]:.2f}')
    #bbox = dict(boxstyle='round', fc='blanchedalmond', ec='orange', alpha=0.5)
    ax.text(0.95, 0.07, stats, fontsize=9,
            transform=ax.transAxes, horizontalalignment='right')

plt.show()
```



# Assumptions of Linearity(after)

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The very first step **after building a linear regression model** is to check whether your model meets the assumptions of linear regression. These assumptions are a vital part of assessing whether the model is correctly specified.

https://www.godatadrive.com › blog › basic-guide-to-test-...

What Are the Assumptions of Linear Regression? - DataDrive

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Why do we need to check assumptions before we run regression?

If the assumptions of regression analysis are met, then **the errors associated with one variable are not correlated with the errors of any other variables** . Independence of residuals can be examined via the Durban – Watson statistic which tests for correlations between errors.

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Why is it important to examine the assumptions of linearity when using ...

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# Let's open our notebook

https://github.com/mojo-flat/Linear-Regression-GP-2

# LINEAR REGRESSION

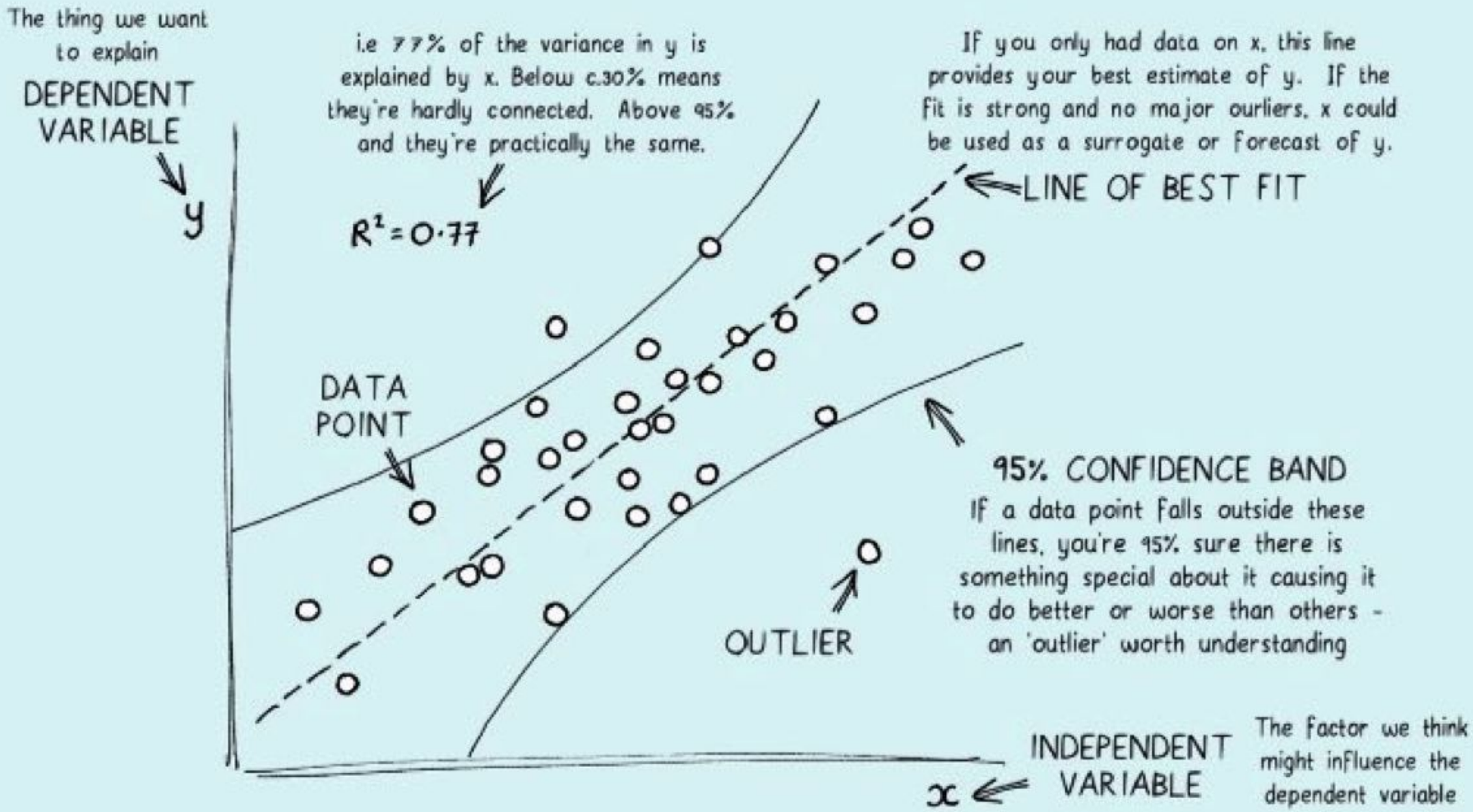


Image by Dan White

# Interpreting R Squared

- Percentage of variation in the dependent variable explained by the independent variable. Values between 0 & 1

$$SS_{residual} = \sum (y - \hat{y})^2$$

$$R^2 = 1 - \frac{SS_{residual}}{SS_{total}}$$

$$SS_{total} = \sum (y - \bar{y})^2$$

An R-squared value of 0.928 can be described conceptually as:

> ***92.8% of the variations in dependent variable** $score$ **are explained by the independent variables** $hours$ **in our model.***

# Interpreting Model Coefficients

Understand the **Marginal Effect** of the independent variable on the dependent variable.

Given a one-unit change in the independent variable, how much is the mean of the dependent variable changed.

$b_1$ x + 1 = Increase in mean score of 1.637 points

$b_0$ x =0, score is equal to 20.8 points

# Hypothesis Testing

To determine if our independent variable has a statistically significant relationship with the dependent variable, we conduct a hypothesis test.

The null hypothesis should contain an equality (=, ≤ ,≥):
Average NBA Player's Height = 2.0m (6ft 7in)

The alternate hypothesis should not have an equality (≠,<,>):
Average NBA Player's Height ≠ 2.0m (6ft 7in)

- $H0 : \mu = 3.5$
- $H1 : \mu \neq 3.5$

The null hypothesis should contain an equality (=, ≤ ,≥):
old scores ≥ new scores

The alternate hypothesis should not have an equality (≠,<,>):
old scores < new scores

$H0 : \mu \geq \mu$
$H1 : \mu < \mu$

- $H_0$ - The average NBA player's height is 2.0m tall.
- $H_1$ - The average NBA player's height is not 2.0m tall.

- $H_0$ - The old average of the scores is equal to or greater than the new average of the scores.
- $H_1$ - The old average of the scores is less than the new average of the scores.

# Hypothesis Testing

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## Aim is to reject the null hypothesis

# p-values & Hypothesis Tests

*From:*
## *Interpreting Significance and p-values*

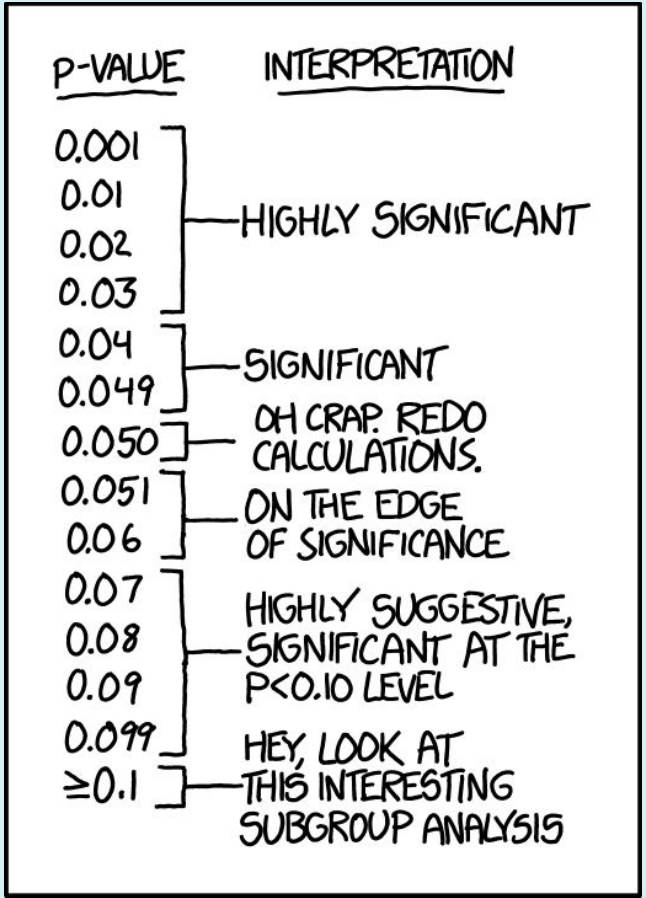
We reject or fail to reject a null hypothesis based on an associated significance level or p-value.

> The p-value represents a probability of observing your results (or something more extreme) given that the null hypothesis is true

Applied to a regression model, p-values associated with coefficients indicate the probability of observing the associated coefficient given that the null-hypothesis is true. As a result, very small p-values indicate that coefficients are statistically significant. A very commonly used cut-off value for the p-value is 0.05.

Just like for statistical significance, rejecting the null hypothesis at an alpha level of 0.05 is the equivalent for having a 95% confidence interval around the coefficient that does not include zero. In short

> The p-value represents the probability that the coefficient is actually zero.



# Interpreting p-values

OLS Regression Results

| | | | |
|---|---|---|---|
| **Dep. Variable:** | score | **R-squared:** | 0.928 |
| **Model:** | OLS | **Adj. R-squared:** | 0.910 |
| **Method:** | Least Squares | **F-statistic:** | 51.79 |
| **Date:** | Sun, 05 Jun 2022 | **Prob (F-statistic):** | 0.00198 |
| **Time:** | 20:10:26 | **Log-Likelihood:** | -12.874 |
| **No. Observations:** | 6 | **AIC:** | 29.75 |
| **Df Residuals:** | 4 | **BIC:** | 29.33 |
| **Df Model:** | 1 | | |
| **Covariance Type:** | nonrobust | | |

| | coef | std err | t | P>\|t\| | [0.025 | 0.975] |
|---|---|---|---|---|---|---|
| **const** | 20.5161 | 9.158 | 2.240 | 0.089 | -4.911 | 45.944 |
| **hours** | 1.6371 | 0.227 | 7.196 | 0.002 | 1.005 | 2.269 |

| | | | |
|---|---|---|---|
| **Omnibus:** | nan | **Durbin-Watson:** | 2.912 |
| **Prob(Omnibus):** | nan | **Jarque-Bera (JB):** | 0.929 |
| **Skew:** | -0.588 | **Prob(JB):** | 0.628 |
| **Kurtosis:** | 1.473 | **Cond. No.** | 357. |

- Prob(F-statistic) p-value: likelihood that we observe our score values by random chance if **linear model** had no statistically significant relationship.

- P>|t| p-value: likelihood that we observe our score values by random chance if **hours** spent had no statistically significant relationship.

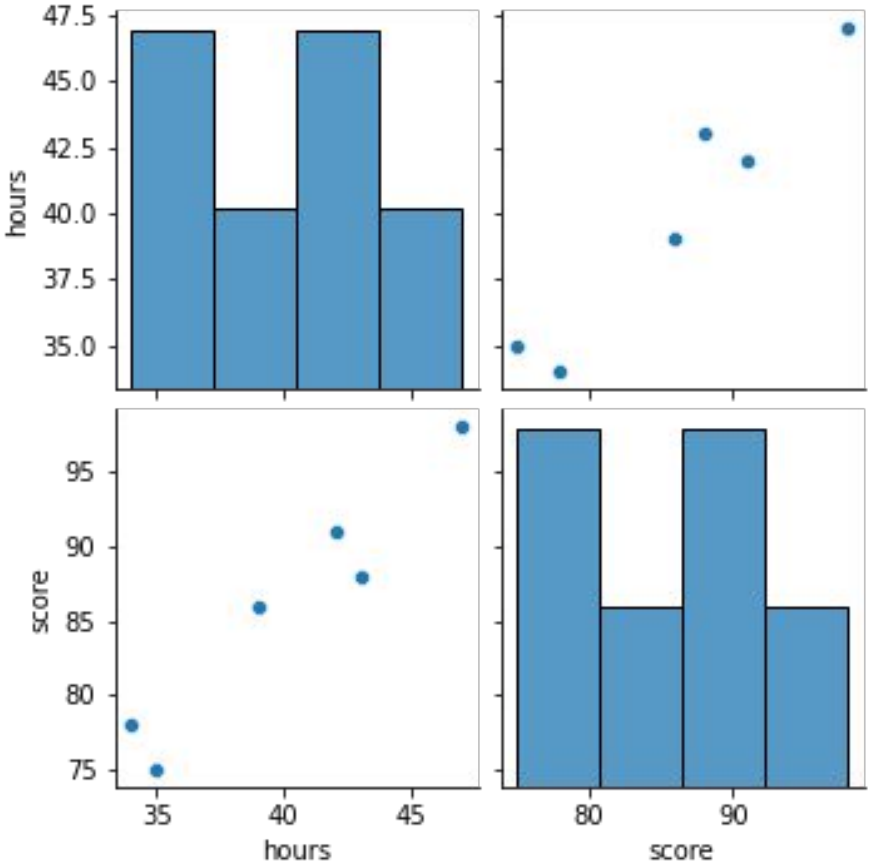
alpha 0.05 > p

# Assumptions of Linear Regression

- Linearity: there is a linear relationship between the independent and dependent variables
- Normality: residuals are normally distributed
- Homoscedasticity: the variance for the residual is the same for any value of x
- Independence: observations are independent of one another

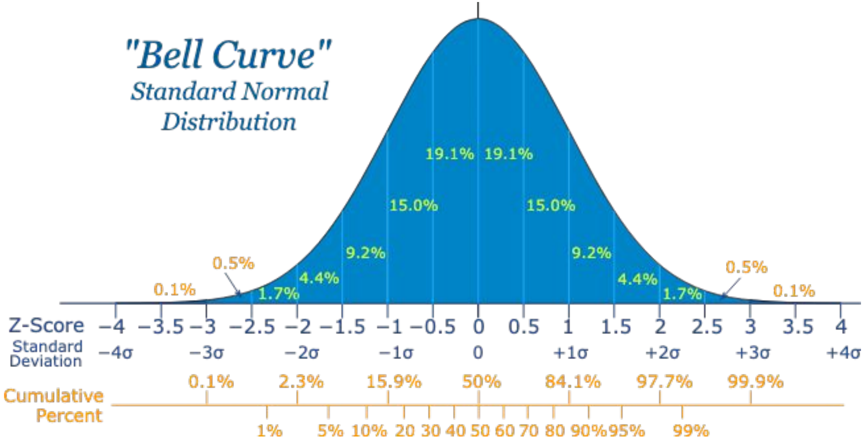
# Assumption of Linearity

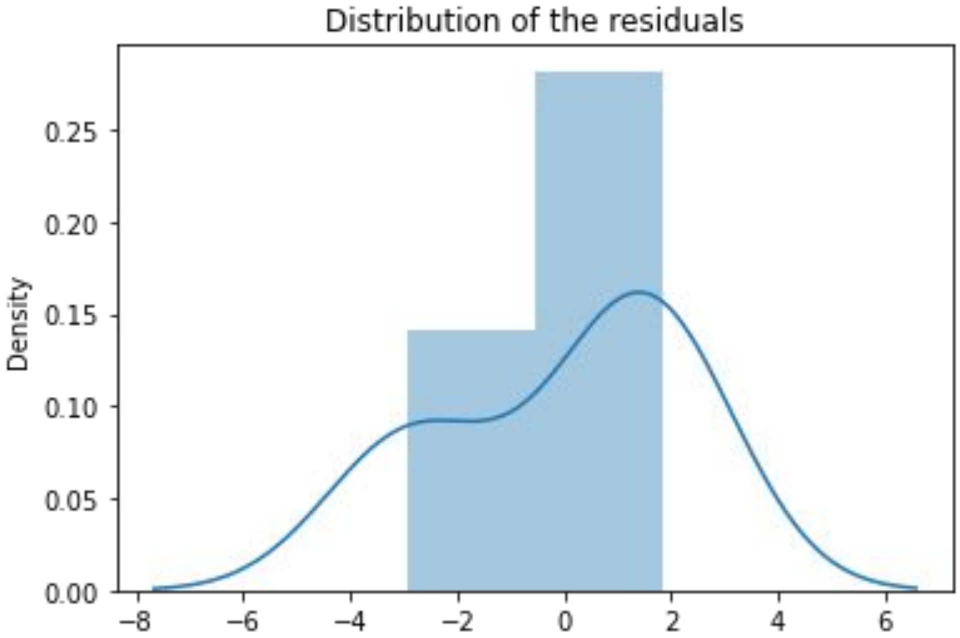
- **Linearity**: there is a linear relationship between the independent and dependent variables



# Assumption of Normality

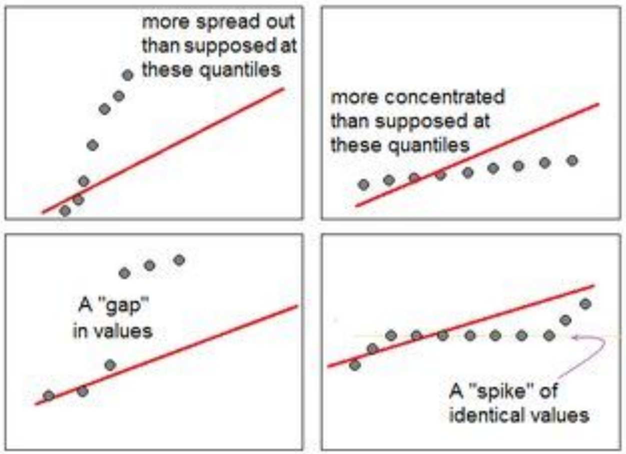
- **Normality**: residuals are normally distributed (symmetric about the mean)

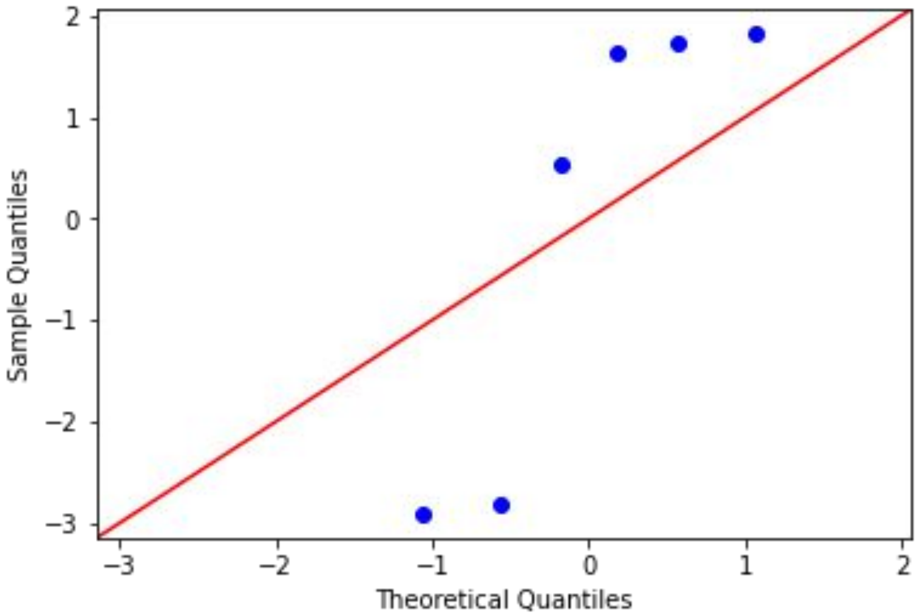


# Normality(quantile-quantile plot)

Plot the quantiles of the residuals to compare distributions, if points lie close to or along 45° line from *x-axis*, samples have similar distributions.

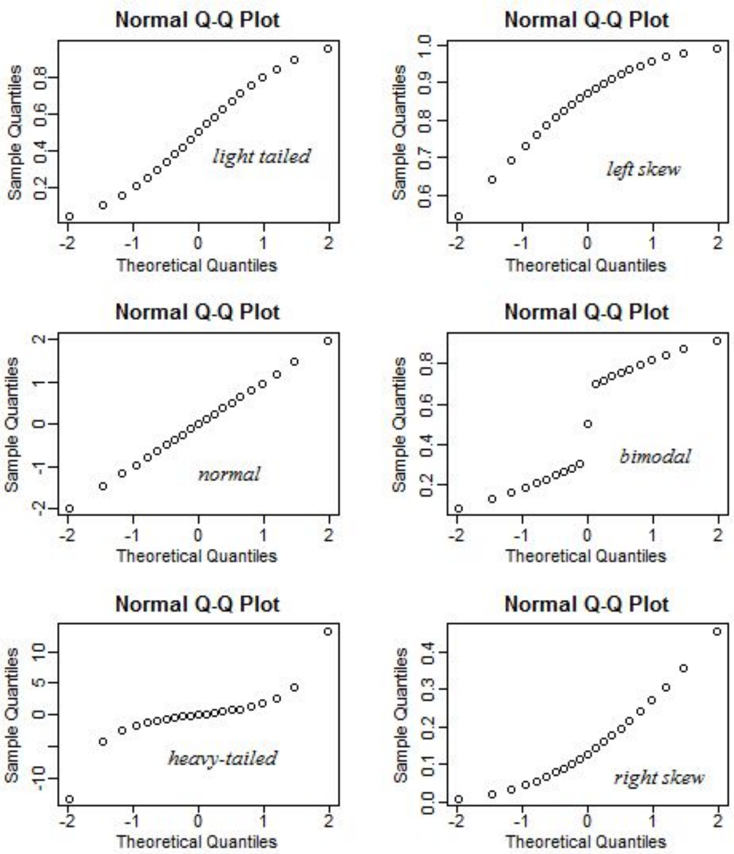


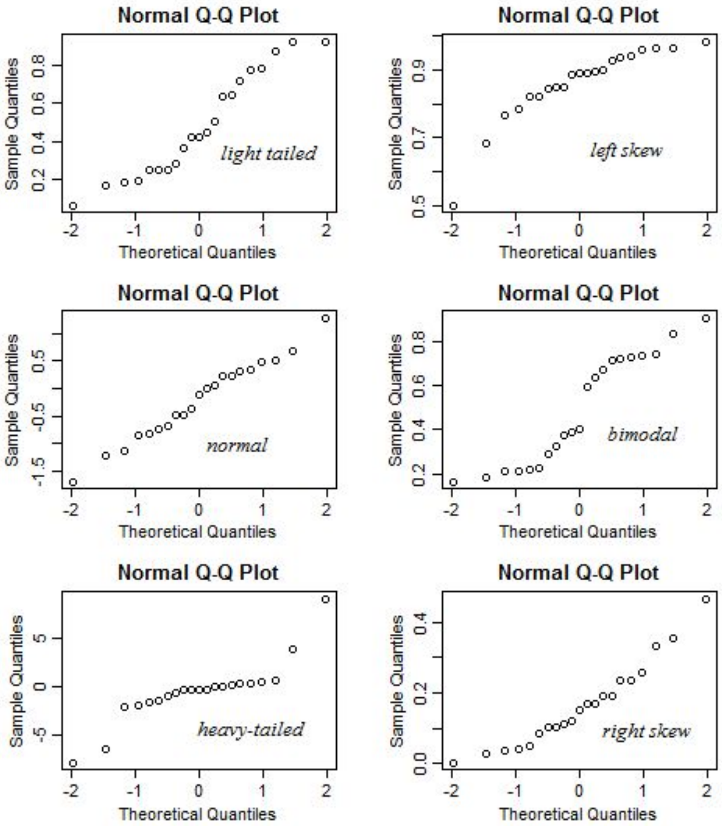

Our Q-Q plot indicates a gap where before it *y* quantiles are lower than the *x* quantiles and after which the *y* quantiles are higher.

# Q-Q Plot

## Without randomness

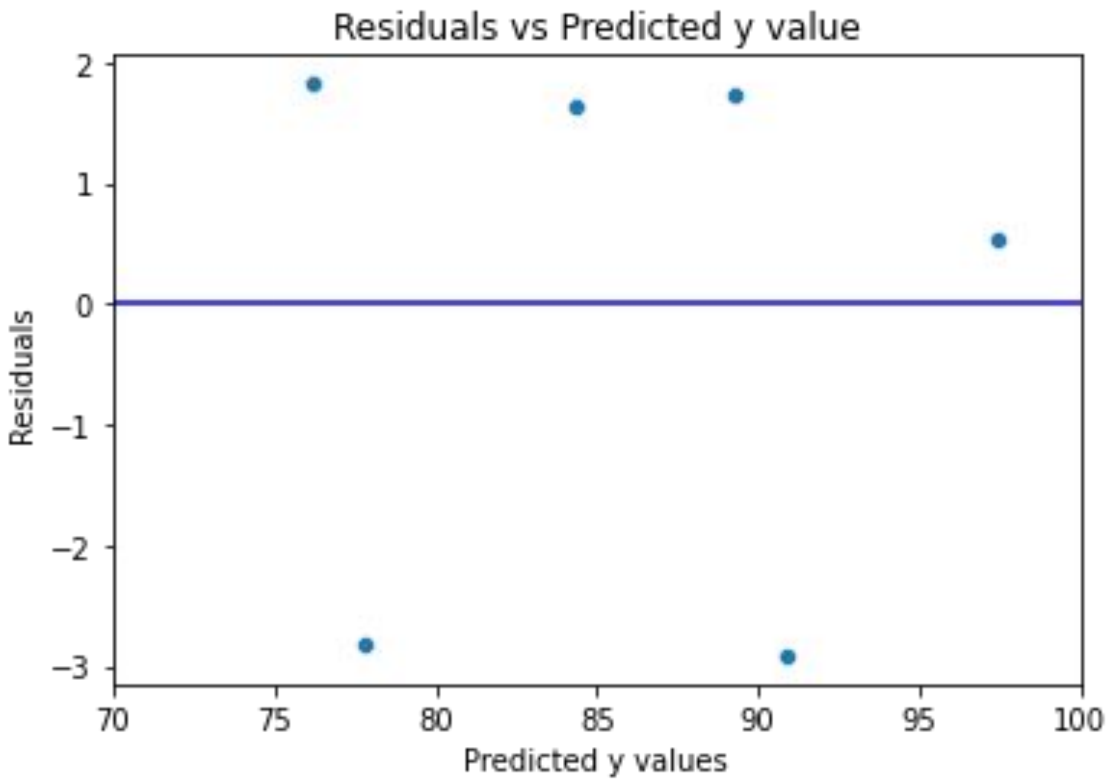



## With randomness



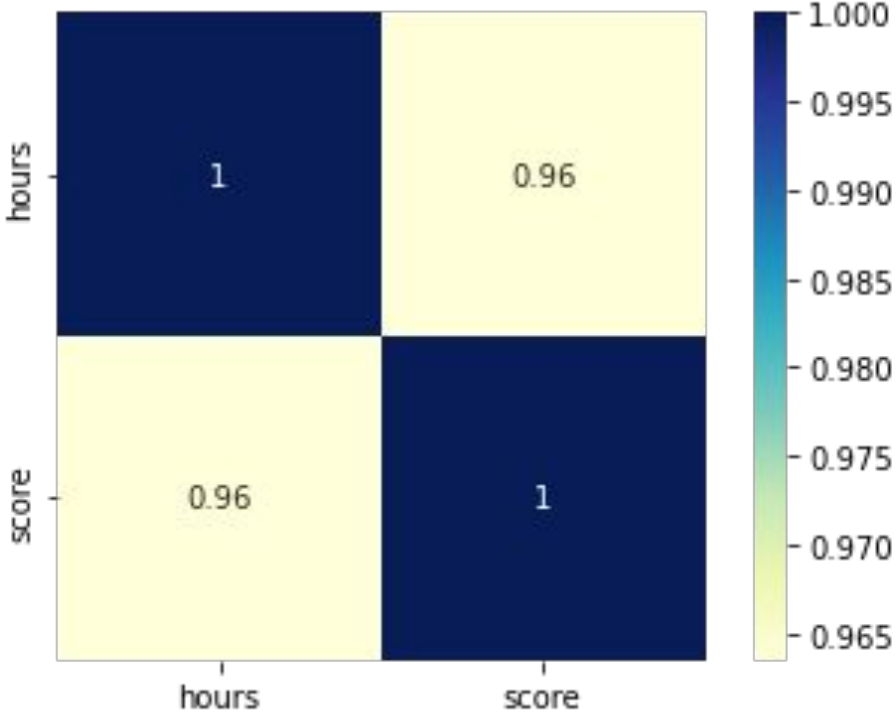
# Assumption of Homoscedasticity

- Homoscedasticity: the variance for the residual is the same for any value of x



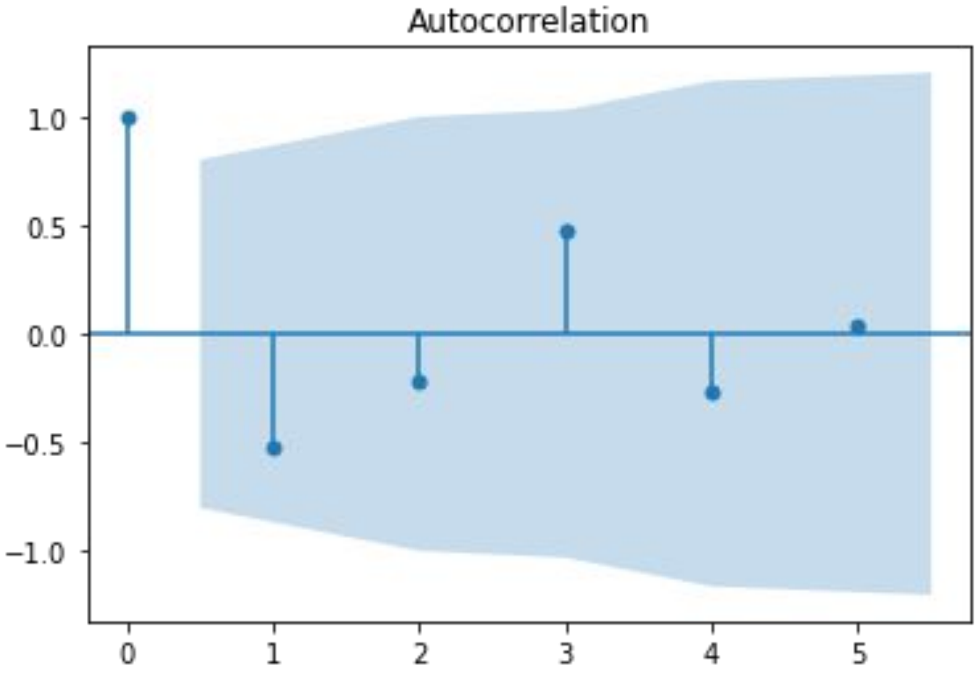
# Assumption of Independence

- No perfect multicollinearity

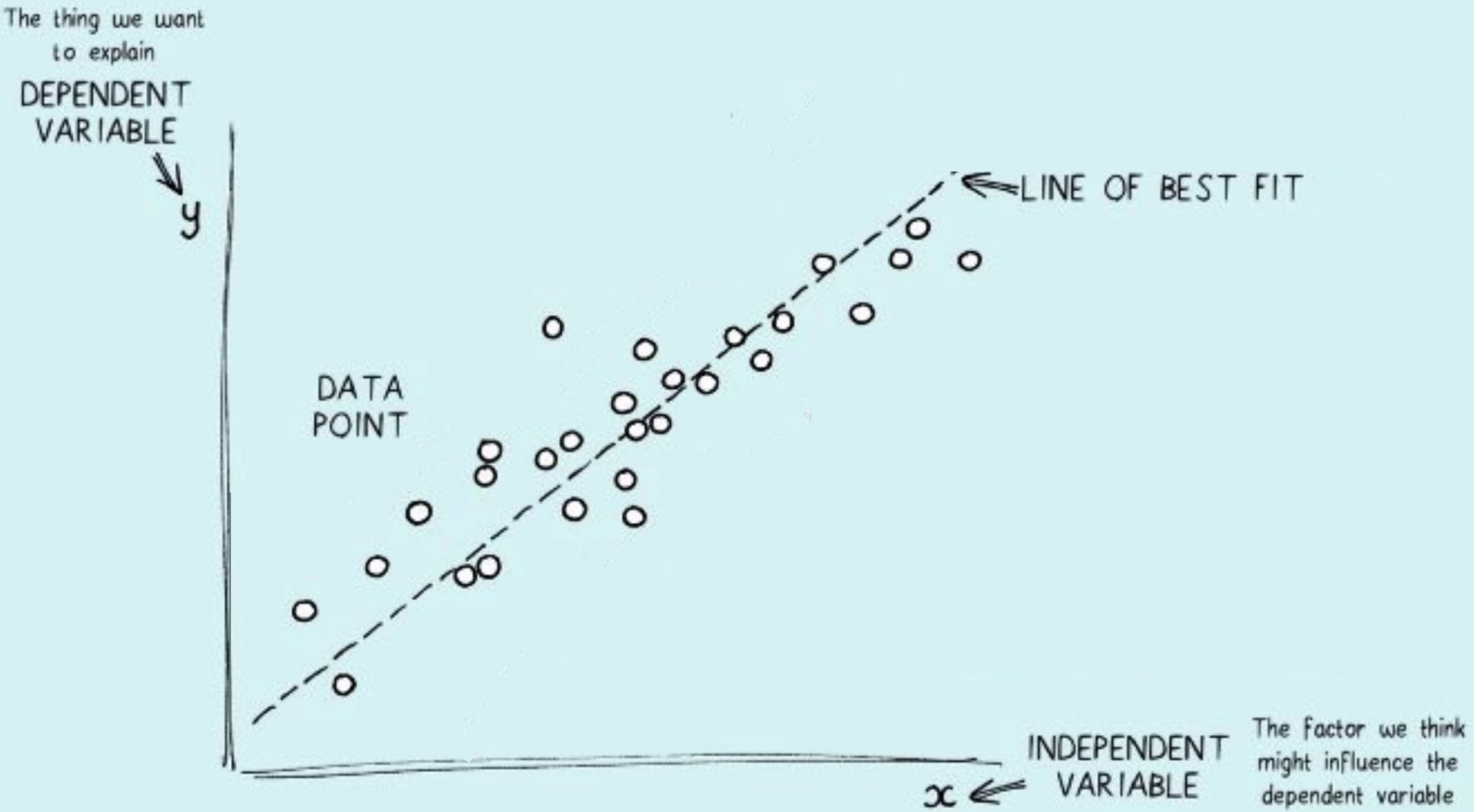


# Assumption of Independence

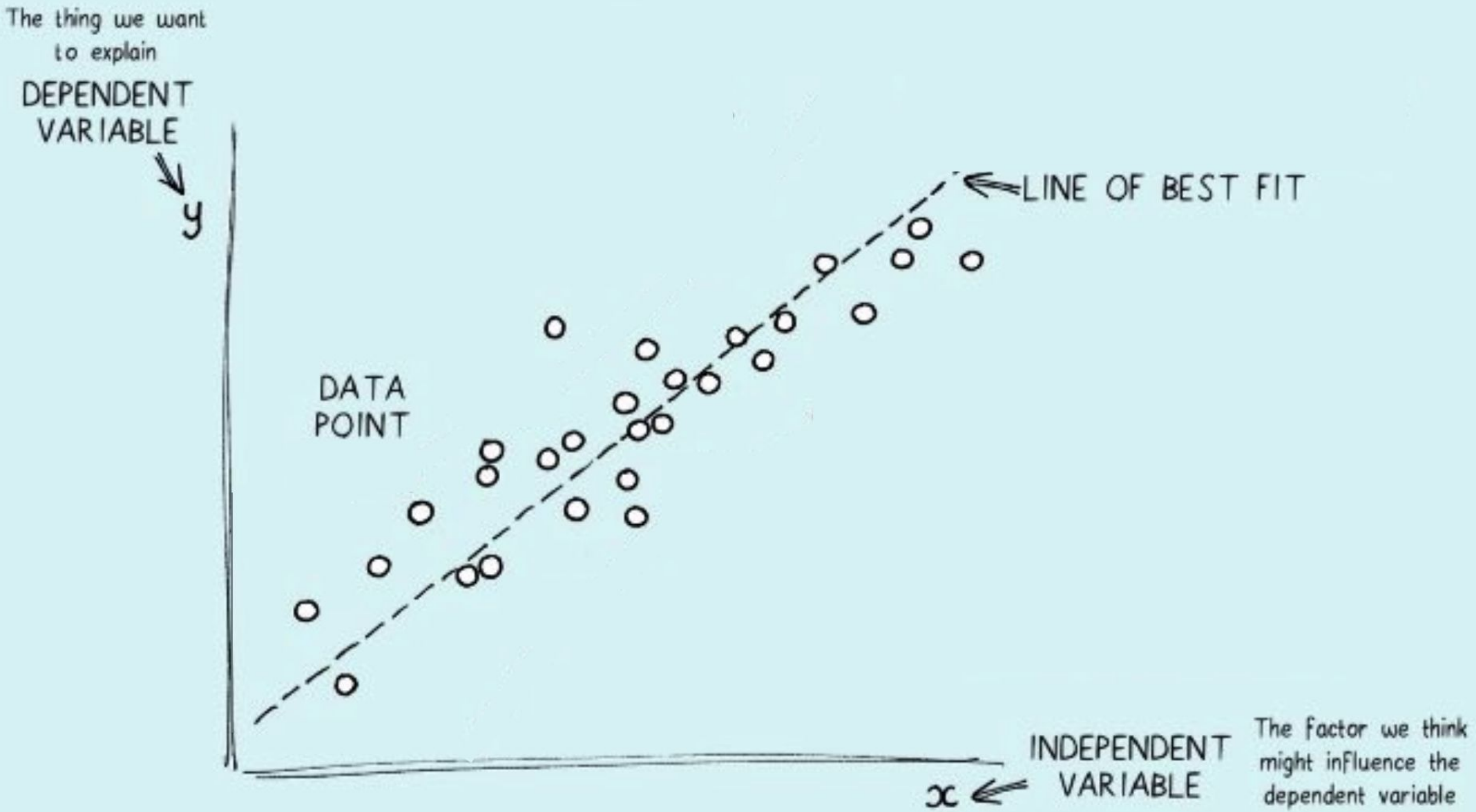
- Autocorrelation: correlation between the residuals. This would violate an assumption of independence



# Linear Regression Q & A



# Linear Regression Appendix



# Pearson Correlation Coefficient

$$\rho_{X,Y} = \frac{cov(X,Y)}{\sigma_X \sigma_Y} = \frac{\frac{1}{n}\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\frac{\Sigma(x-\bar{x})^2}{n}}\sqrt{\frac{\Sigma(y-\bar{y})^2}{n}}} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\Sigma(x-\bar{x})^2}\sqrt{\Sigma(y-\bar{y})^2}}$$

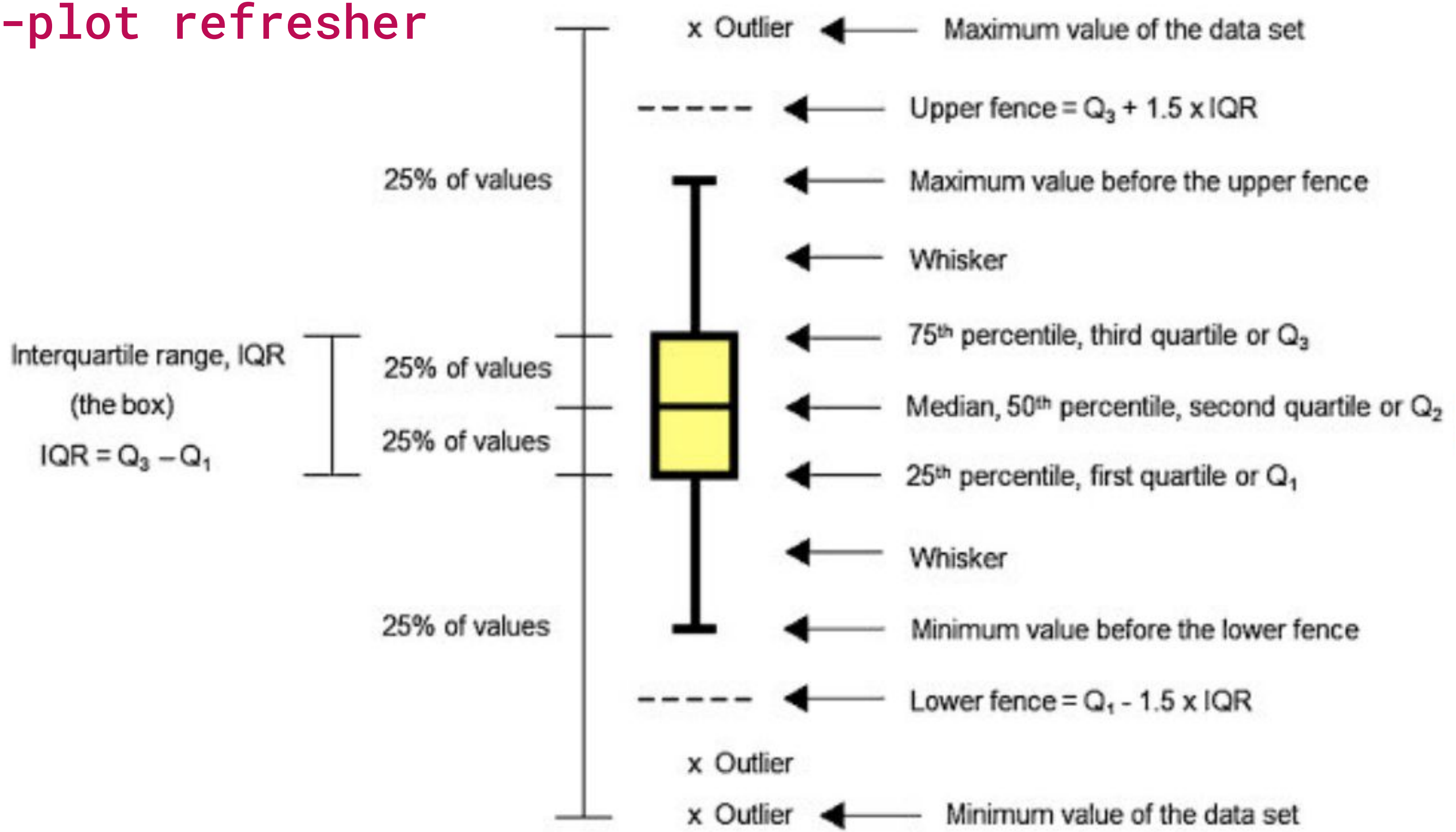
$\rho =$ Greek letter "rho"

$cov =$ covariance

$\sigma =$ standard deviation

$\bar{x} =$ mean of X

# Box-plot refresher



# Line of Best fit

The sum of the distances (error) above the mean has the same absolute value as the distances (error) below the mean.

| Error (E) | Square Error (SE) |
|---|---|
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