BINARY NUMBERS

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Why binary?

- Computers are built using digital circuits
    - Inputs and outputs can have only two values
    - 1 (high voltage) or 0 (low voltage)
- How to represent and do the computation in binary numbers.
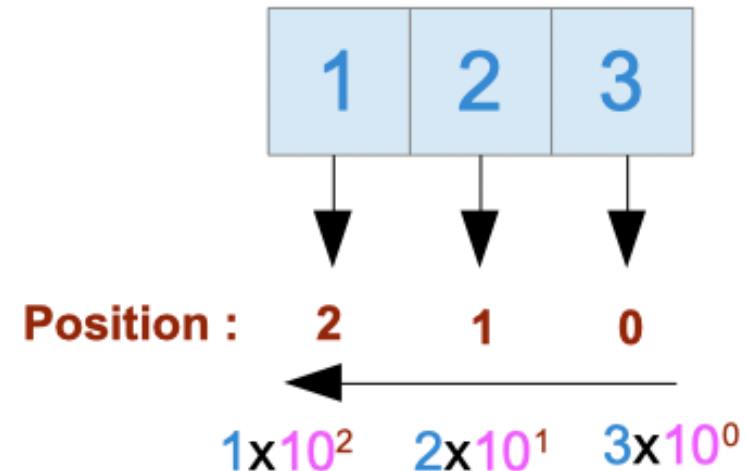- Base 10 : Decimal
- Base 2 : Binary

Decimal System

- Digits can be 0,1,2,3,4,5,6,7,8,9
- Each digit represents a power of 10

123

1 Hundreds
2 Tens
3 Ones

Base 10

1	2	3

Position : 2 1 0

1×10^2 2×10^1 3×10^0

Binary System

- Digits can be 0,1
- Each digit represents a power of 2

Base 2

1	0	1

Position: 2 1 0

$1\times2^2 \quad 0\times2^1 \quad 1\times2^0$

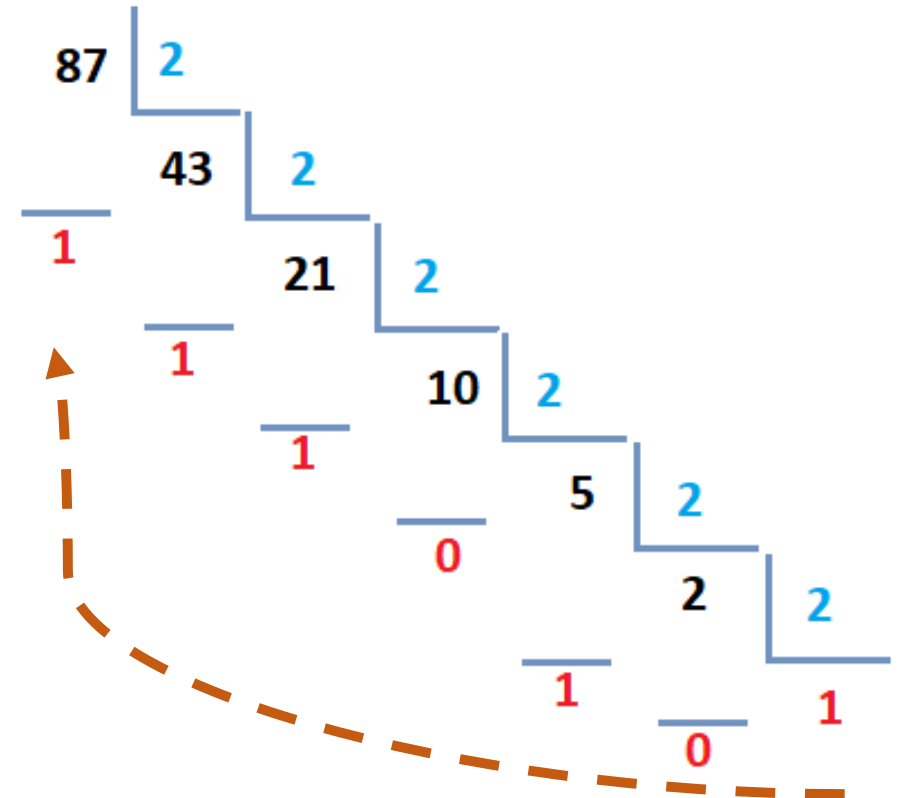$= 4 + 0 + 1 = 5$ (decimal)

digits position 5 4 3 2 1 0

binary 100010

$32 = 1\times2^5$
$0 = 0\times2^4$
$0 = 0\times2^3$
$0 = 0\times2^2$
$2 = 1\times2^1$
$0 = 0\times2^0$

decimal $32 + 0 + 0 + 0 + 2 + 0 = 34$

Decimal to Binary
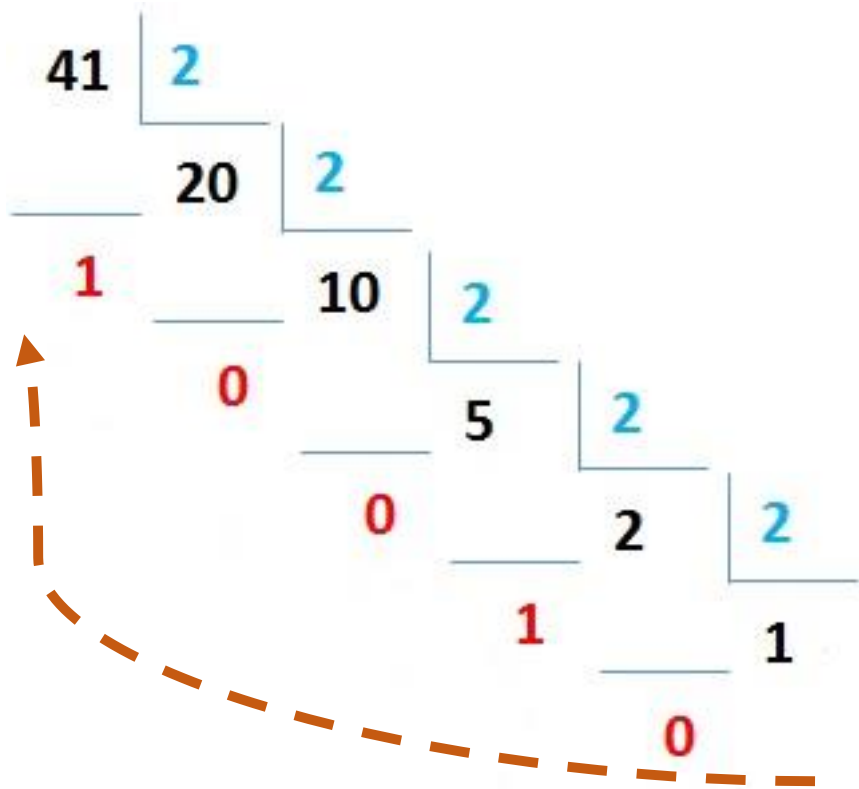
$(41)_{10} = (101001)_2$

$(87)_{10} = (1010111)_2$

Basic Arithmetic Operation: Addition

- **Decimal Addition**

Add 3758 to 4657:

$$
\begin{array}{r}
3\,7\,5\,8 \\
+\ \ 4\,6\,5\,7 \\
\hline
\end{array}
$$

Basic Arithmetic Operation: Addition

- **Decimal Addition**

Add 3758 to 4657:

1) Add $8 + 7 = 15$

Write down 5, carry 1

$$
\begin{array}{r}
1 \\
3\,7\,5\,8 \\
+\ \ 4\,6\,5\,7 \\
\hline
5
\end{array}
$$

Basic Arithmetic Operation: Addition

- **Decimal Addition**

Add 3758 to 4657:

1 1

3 7 5 8

+ 4 6 5 7
_____

1 5

1) Add $8 + 7 = 15$

  Write down 5, carry 1

2) Add $5 + 5 + 1 = 11$

  Write down 1, carry 1

Basic Arithmetic Operation: Addition

- **Decimal Addition**

Add 3758 to 4657:

$$\begin{array}{r} 1\ 1\ \ \ \\ 3\ 7\ 5\ 8 \\ +\ \ 4\ 6\ 5\ 7 \\ \hline 4\ 1\ 5 \end{array}$$

1) Add $8 + 7 = 15$
   Write down 5, carry 1

2) Add $5 + 5 + 1 = 11$
   Write down 1, carry 1

3) Add $7 + 6 + 1 = 14$
   Write down 4, carry 1

Basic Arithmetic Operation: Addition

- **Decimal Addition**

Add 3758 to 4657:

$$
\begin{array}{r}
\color{red}{1\ 1\ 1} \\
3\ 7\ 5\ 8 \\
+\ \ 4\ 6\ 5\ 7 \\
\hline
8\ 4\ 1\ 5
\end{array}
$$

1) Add $8 + 7 = 15$

   Write down 5, carry 1

2) Add $5 + 5 + 1 = 11$

   Write down 1, carry 1

3) Add $7 + 6 + 1 = 14$

   Write down 4, carry 1

4) Add $3 + 4 + 1 = 8$

   Write down 8

Basic Arithmetic Operation: Addition

- **Binary Addition**

Rules:

- $0 + 0$ $= 0$
- $0 + 1$ $= 1$
- $1 + 0$ $= 1$ (just like in decimal)

- $1 + 1$ $= 2_{10}$
  $= 10_2 = 0$ with 1 to carry

- $1 + 1 + 1$ $= 3_{10}$
  $= 11_2 = 1$ with 1 to carry

Basic Arithmetic Operation: Addition

- **Binary Addition**

Example : Add

binary **110111** to **11100**

```
    1 1 0 1 1 1
+   0 1 1 1 0 0
  _____
```

Basic Arithmetic Operation: Addition

- **Binary Addition**

Example : Add

binary **110111** to **11100**

1) Add **1 + 0 = 1**
Write **1**

```
    1  1  0  1  1  1
 +  0  1  1  1  0  0
 _____
                      1
```

Basic Arithmetic Operation: Addition

- **Binary Addition**

Example : Add

binary **110111** to **11100**

1) Add **1 + 0 = 1**
   Write **1**

2) Add **1 + 0 = 1**
   Write **1**

```
   1 1 0 1 1 1
 +   0 1 1 1 0 0
 _____
             1 1
```

Basic Arithmetic Operation: Addition

• Binary Addition

Example : Add

binary **110111** to **11100**

$$
\begin{array}{ccccccc}
 & & 1 & & & & \\
 & 1 & 1 & 0 & 1 & 1 & 1 \\
+ & 0 & 1 & 1 & 1 & 0 & 0 \\
\hline
 & & & & 0 & 1 & 1 \\
\end{array}
$$

1) Add **1 + 0 = 1**
   Write **1**

2) Add **1 + 0 = 1**
   Write **1**

3) Add **1 + 1 = 2 (10** in binary)
   Write **0**, carry **1**

Basic Arithmetic Operation: Addition

• **Binary Addition**

Example : Add

binary **110111** to **11100**

```
      1 1
    1 1 0 1 1 1
  +   0 1 1 1 0 0
  _____
            0 0 1 1
```

1) Add **1 + 0 = 1**
   Write **1**

2) Add **1 + 0 = 1**
   Write **1**

3) Add **1 + 1 = 2 (10** in binary**)**
   Write **0**, carry **1**

4) Add **1 + 0 + 1 = 2**
   Write **0**, carry **1**

Basic Arithmetic Operation: Addition

- **Binary Addition**

Example : Add

binary **110111** to **11100**

```
    1 1 1
  1 1 0 1 1 1
+ 0 1 1 1 0 0
─────────────
  1 0 0 1 1
```

1) Add **1 + 0 = 1**
    Write **1**

2) Add **1 + 0 = 1**
    Write **1**

3) Add **1 + 1 = 2** (**10** in binary)
    Write **0**, carry **1**

4) Add **1 + 0 + 1 = 2**
    Write **0**, carry **1**

5) Add **1 + 1 + 1 = 3** (**11** in binary)
    Write **1**, carry **1**

Basic Arithmetic Operation: Addition

- **Binary Addition**

Example : Add

binary **110111** to **11100**

```
  1 1 1 1
    1 1 0 1 1 1
+   0 1 1 1 0 0
─────────────────
    0 1 0 0 1 1
```

1) Add **1 + 0 = 1**
   Write **1**

2) Add **1 + 0 = 1**
   Write **1**

3) Add **1 + 1 = 2** (**10** in binary)
   Write **0**, carry **1**

4) Add **1+ 0 + 1 = 2**
   Write **0**, carry **1**

5) Add **1 + 1 + 1 = 3** (**11** in binary)
   Write **1**, carry **1**

6) Add **1 + 1 + 0 = 2**
   Write **0**, carry **1**

Basic Arithmetic Operation: Addition

- **Binary Addition**

Example : Add

binary **110111** to **11100**

```
  1 1 1 1
    1 1 0 1 1 1
+   0 1 1 1 0 0
  _____
  1 0 1 0 0 1 1
```

1) Add **1 + 0 = 1**
   Write **1**

2) Add **1 + 0 = 1**
   Write **1**

3) Add **1 + 1 = 2** (**10** in binary)
   Write **0**, carry **1**

4) Add **1+ 0 + 1 = 2**
   Write **0**, carry **1**

5) Add **1 + 1 + 1 = 3** (**11** in binary)
   Write **1**, carry **1**

6) Add **1 + 1 + 0 = 2**
   Write **0**, carry **1**

7) Bring down the carried **1**
   Write **1**