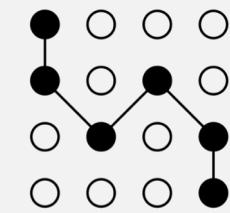
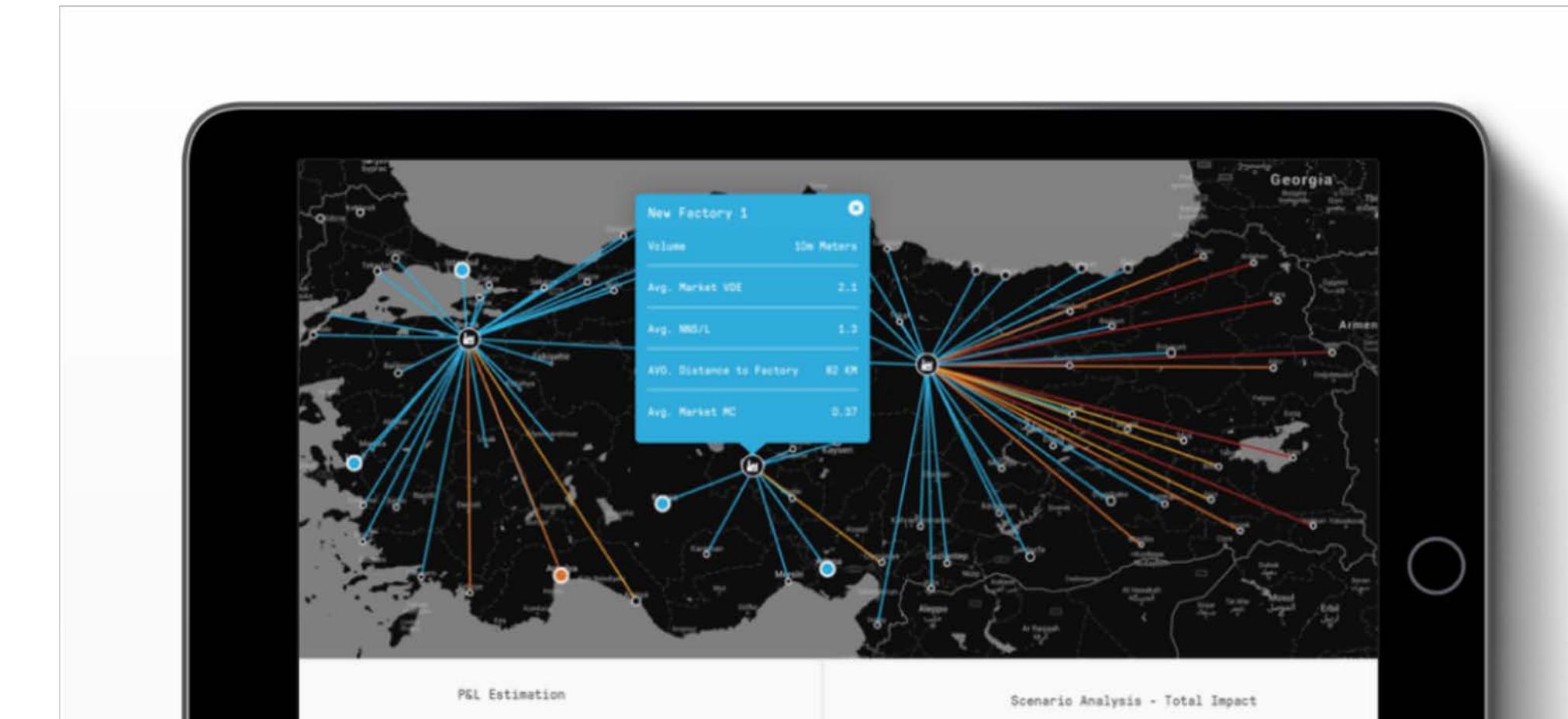




# USING PULP AND SOLVERS FOR BUSINESS ANALYTICS

NOVEMBER 29, 2019

At MOKA, we build intelligent planning software that allows organizations to make smarter, faster, strategic decisions.



**Unifies strategic data model,**  
automates & speeds analysis,  
and addresses data gaps



**Delivers a lasting organizational**  
capability. Enterprise-ready, secure &  
light-weight for fast deployment



**Predictive & prescriptive**  
analytics drive insights &  
evaluate complex scenarios

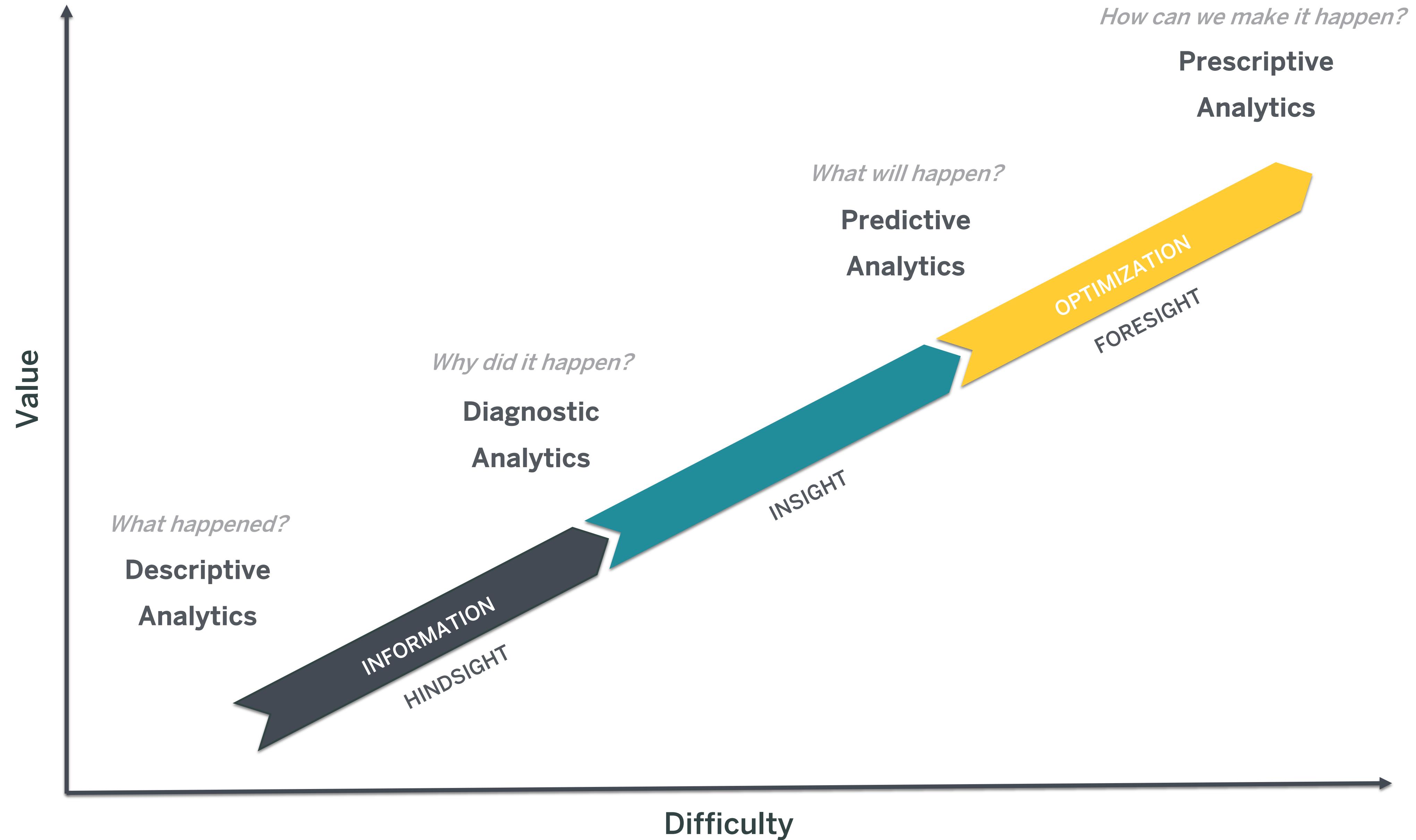


**Encode and distribute high-quality**  
strategy frameworks across your  
entire organization

The Winning Model powers strategic planning for manufacturing business models – from CPG to automotive to industrials.  
Ready to go out-of-the box, we tailor the core modules and intelligence engine to a client's specific business.



# BUSINESS ANALYTICS VALUE LADDER



# Talk Outline

1

What is Linear  
Programming?

2

Using PuLP for Macro  
Capacity Planning

3

Python Solver  
Landscape

4

Best Practices for Using  
Solvers for Analytics

# LINEAR PROGRAMMING

## Matrix Form

minimize

$$\mathbf{c}^T \mathbf{x}$$

subject to

$$A\mathbf{x} \leq \mathbf{b}$$

and

$$\mathbf{x} \geq \mathbf{0}$$

## Standard Form

$$f(x_1, x_2, \dots) = c_1x_1 + c_2x_2 + \dots$$

$$a_{11}x_1 + a_{12}x_2 + \dots \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \dots \leq b_2$$

$$a_{31}x_1 + a_{32}x_2 + \dots \leq b_3$$

...

$$x_1 \geq 0$$

$$x_2 \geq 0$$

...

Trick #1: negate all the constants to maximize a function

Trick #2: re-arrange expressions (e.g., subtract from both sides) to isolate constant on the right side

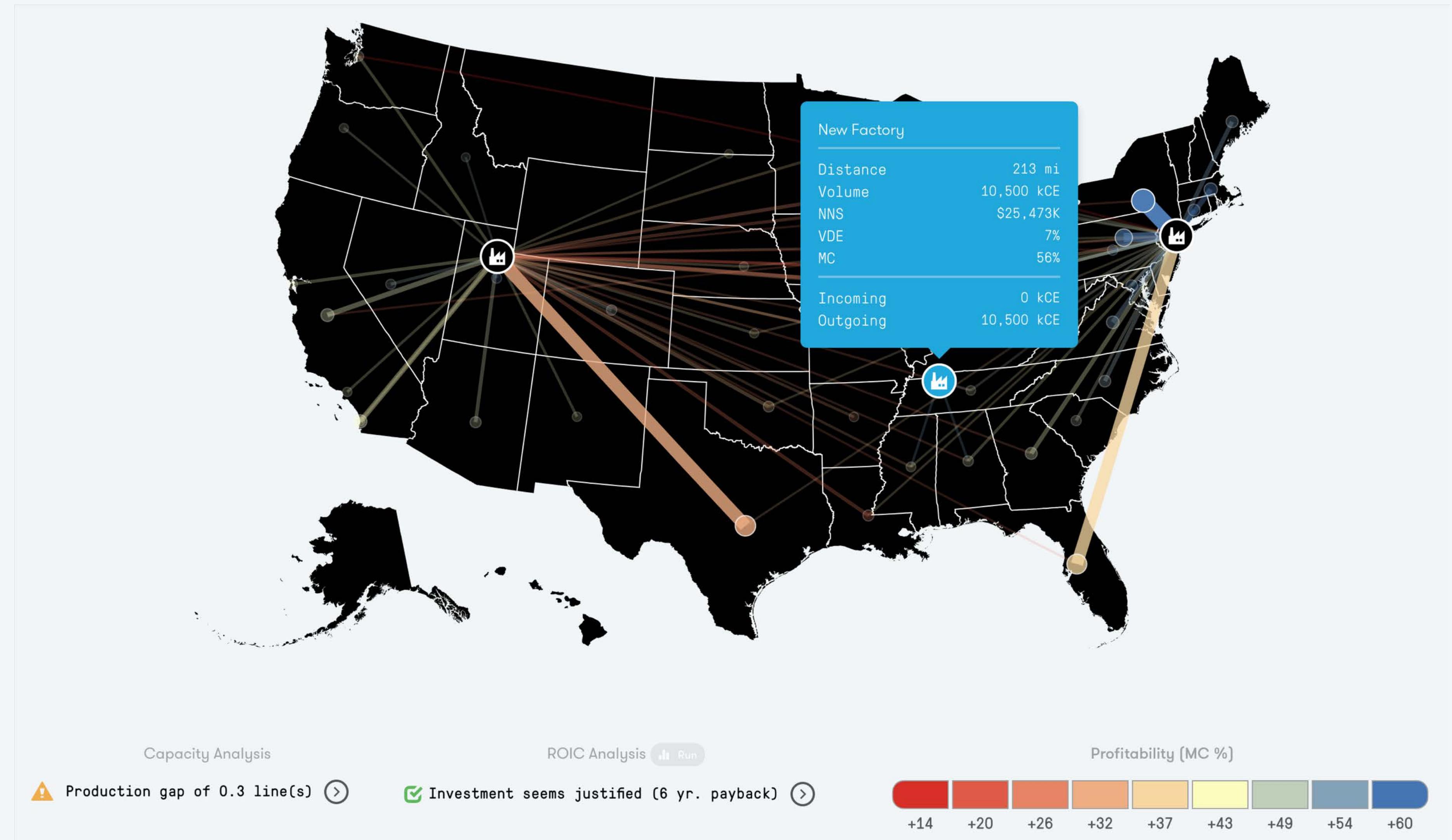
Trick #3: add slack/dummy variables to encode other relationships, e.g., strict equality

*Many tools, including PuLP, can apply these tricks automatically to allow a more natural expression*



## MACRO CAPACITY PLANNING: BUSINESS QUESTIONS

- Where are the inefficiencies in our supply chain?
- Will we have enough production capacity if demand grows?
- Is our supply chain positioned to profitably capture growth opportunities?
- Should we open a new warehouse and/or factory? If so, where?



With the Winning Model's Network Strategy Module, Network Planners and CEOs  
can evaluate network strategy and investment opportunities instantaneously

# Jupyter Walkthrough

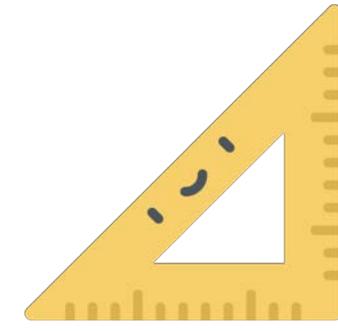
<https://github.com/moka-analytics/engineering>

# PYTHON SOLVER LANDSCAPE

<sup>1</sup> Not all paradigms shown. [Wikipedia](#) is a good resource for exploring various paradigms

Paradigm <sup>1</sup>	Problem Form	Example Use Cases	Example Python Packages
<b>Mathematical/Numerical</b>			
<b>Linear Programming</b>	$\begin{array}{ll} \text{minimize} & \mathbf{c}^T \mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b} \\ \text{and} & \mathbf{x} \geq \mathbf{0} \end{array}$	<ul style="list-style-type: none"> <li>Supply chain optimization</li> <li>Production planning</li> </ul>	<ul style="list-style-type: none"> <li><a href="#">PuLP</a>: interface to linear and mixed-integer solvers</li> <li><a href="#">MIPCL</a>: commercial mixed-integer programming</li> <li><a href="#">GLOP</a>: Google's LP-only solver</li> </ul>
<b>Integer Programming</b>	$\begin{array}{ll} \text{maximize} & \mathbf{c}^T \mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b}, \\ & \mathbf{x} \geq \mathbf{0}, \\ \text{and} & \mathbf{x} \in \mathbb{Z}^n, \end{array}$	<ul style="list-style-type: none"> <li>Minimize interference across cellular network</li> <li>Bus scheduling/Vehicle routing</li> <li>"Knapsack" problem</li> </ul>	
<b>Quadratic Programming</b>	$\begin{array}{ll} \text{minimize} & \frac{1}{2} \mathbf{x}^T Q \mathbf{x} + \mathbf{c}^T \mathbf{x} \\ \text{subject to} & A\mathbf{x} \leq \mathbf{b}, \end{array}$	<ul style="list-style-type: none"> <li>Financial portfolio optimization</li> <li>Image/signals processing</li> <li>Least-Squares regression</li> </ul>	<ul style="list-style-type: none"> <li><a href="#">qpsolvers</a>: unified interface around quadratic solvers</li> <li><a href="#">quadprog</a>: implementation of the Goldfarb/Idnani dual algorithm</li> </ul>
<b>Convex Optimization</b>	$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & g_i(x) \leq 0, \quad i = 1, \dots, m \\ & f, g_1 \dots g_m : \mathbb{R}^n \rightarrow \mathbb{R} \text{ are all convex} \end{array}$	<ul style="list-style-type: none"> <li>Training ML models</li> <li>Linear/Quadratic are a special case of Convex</li> </ul>	<ul style="list-style-type: none"> <li><a href="#">cvxpy</a></li> <li><a href="#">cvxopt</a></li> </ul>
<b>Non-Linear Programming</b>	$\begin{array}{ll} \text{minimize} & f(x) \\ \text{subject to} & g_i(x) \leq 0 \text{ for each } i \in \{1, \dots, m\} \\ & h_j(x) = 0 \text{ for each } j \in \{1, \dots, p\} \\ & x \in X. \end{array}$		<ul style="list-style-type: none"> <li><a href="#">pyOpt</a></li> </ul>
<b>Multi-Paradigm</b>			<ul style="list-style-type: none"> <li><a href="#">Pyomo</a>: multi-paradigm interface to multiple solvers</li> <li><a href="#">Google OR-Tools</a>: Google's operations research tools</li> <li><a href="#">Gurobi</a>: commercial optimizer supporting multiple languages</li> <li><a href="#">SciPy</a>: its optimize package contains numerous solvers</li> </ul>
<b>Logical/Constraint</b>			
<b>Logic Programming</b>	$\begin{array}{l} P(x). \\ ?- Q(x). \end{array}$	<ul style="list-style-type: none"> <li>Expert system AI</li> <li>Knowledge graph inference</li> <li>Natural Language Processing (NLP)</li> </ul>	<ul style="list-style-type: none"> <li><a href="#">logpy</a> (aka kanren)</li> <li><a href="#">PySWIP</a>: SWI-Prolog bridge</li> <li><a href="#">ErgoAI</a>: via a Java bridge</li> <li><a href="#">Pyke</a>: pure Python</li> </ul>
<b>Satisfiability Modulo Theories (SMT)</b>	$\exists x. P(x)$	<ul style="list-style-type: none"> <li>Static software analysis</li> <li>Program synthesis</li> <li>Scheduling</li> </ul>	<ul style="list-style-type: none"> <li><a href="#">pySMT</a>: interface to SMT-LIB format supporting all major solvers</li> <li><a href="#">Z3</a>: Microsoft's SMT solver with support for multiple</li> <li><a href="#">CP-SAT</a>: Google's constraint programming solver</li> </ul>

# BEST PRACTICES FOR USING SOLVERS FOR ANALYTICS



Avoid false precision



Know when to tradeoff speed vs. precision



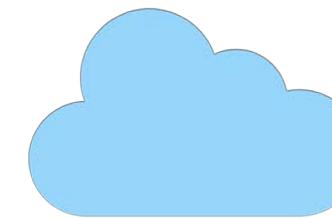
Educate users about what's being optimized, e.g., assumptions and limitations



Support user intuition with analytics and visualizations



Don't be afraid to run a solver multiple times



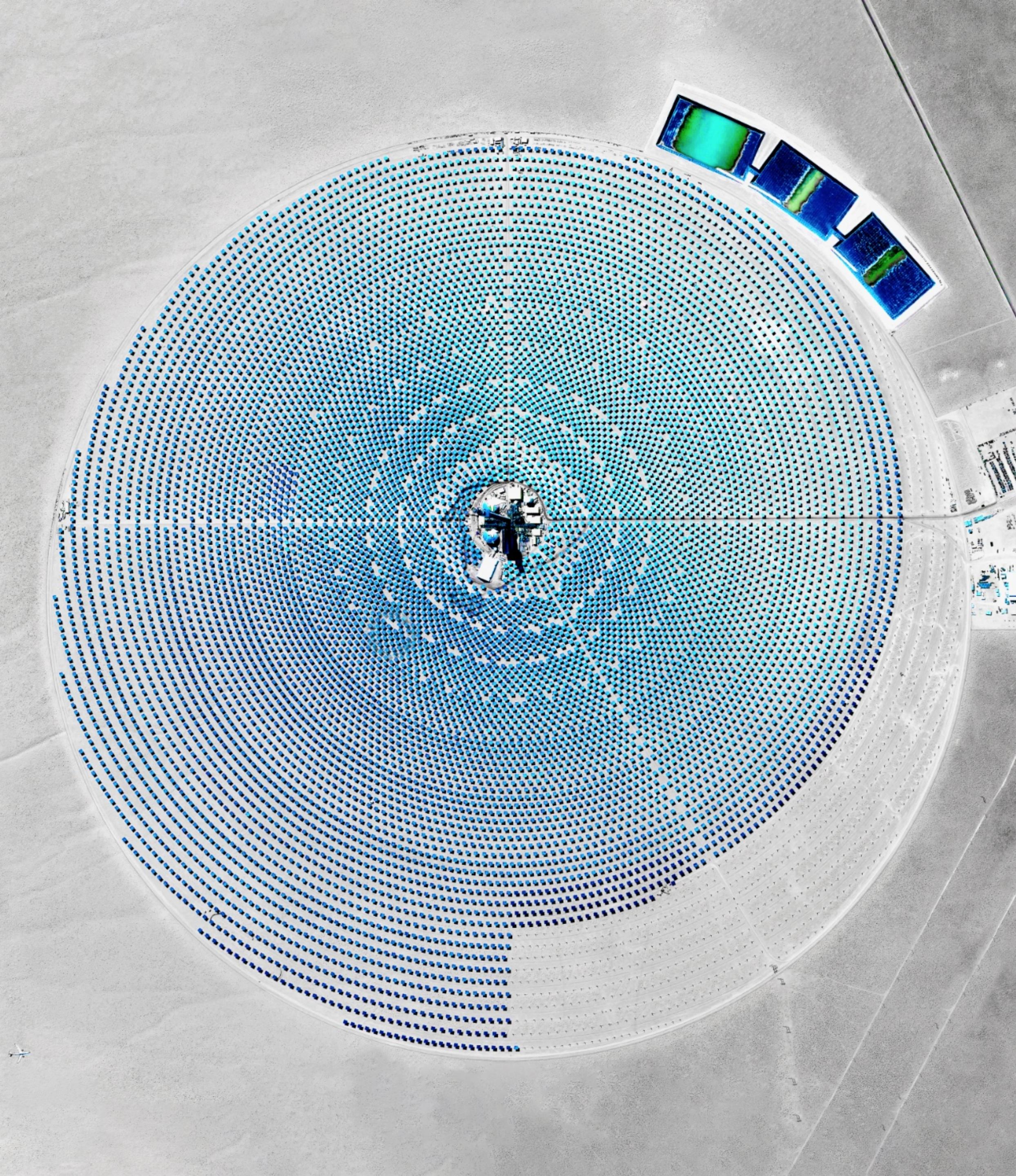
Get creative with elastic computing resources



Benchmark optimizers to find best for your workload



Profile your Python problem creation code



## KEY TAKEAWAYS

- Important business problems can be framed as optimization problems
- There's a Python interface for every kind of problem
- Augment solver output with complementary analyses and visualizations

# We're Hiring!

- Senior Full Stack Engineer
- Senior Front-End Engineer
- Data Integration Engineer
- Vice President, Deployment Strategy
- Business Project Manager

Come find me after the talk, or email me at [todd@moka.nyc](mailto:todd@moka.nyc)

**We offer \$\$\$ for successful referrals**

Slides and notebook available on GitHub:

<https://github.com/moka-analytics/engineering>

Learn more about MOKA Analytics at

<https://www.moka.nyc>

Find more of my talks and articles at

<https://toddschiller.com/>

# Appendix

# THREE PYTHON SOLVER ARCHITECTURES

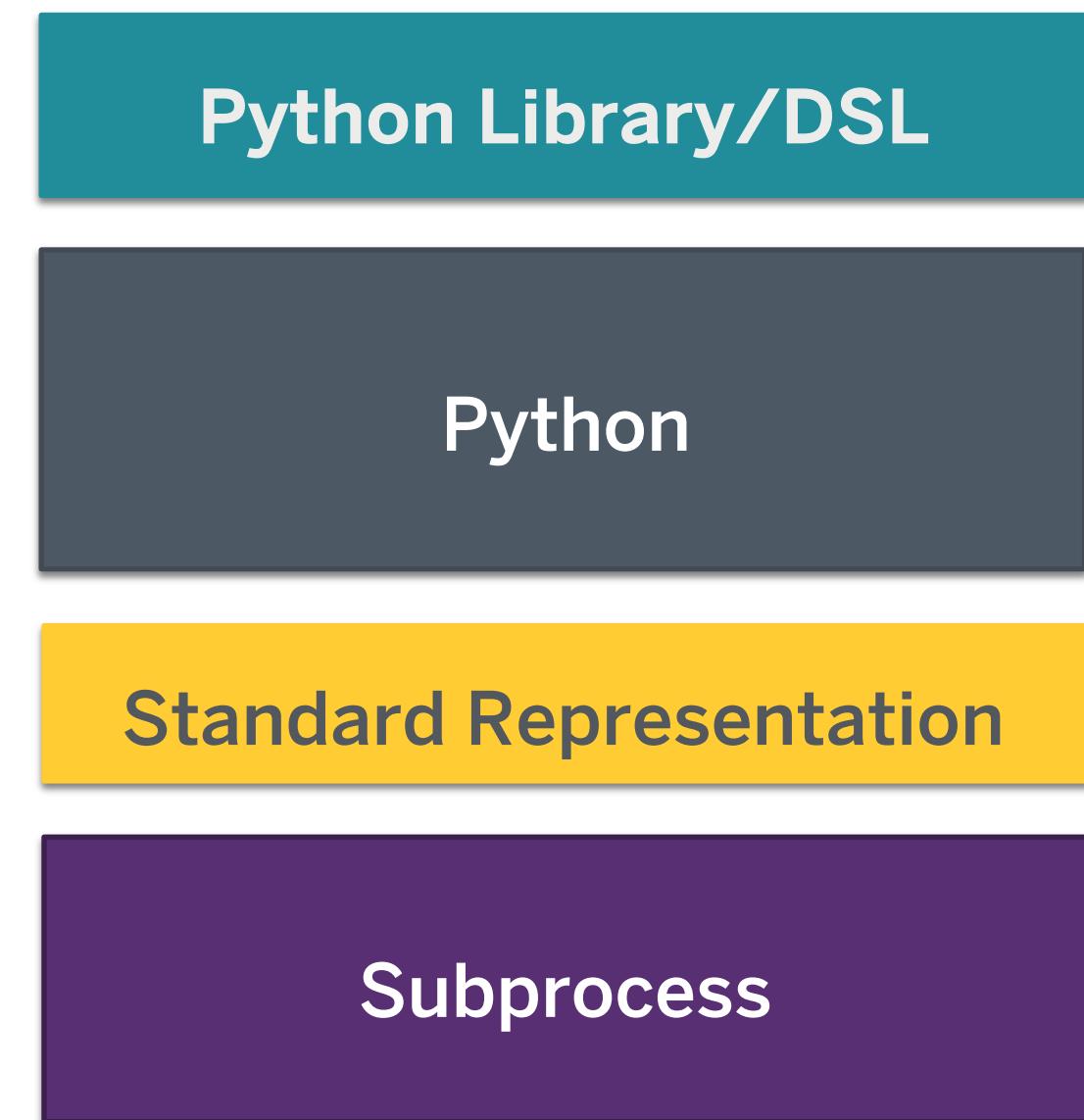
Pure Python



Python w/ C



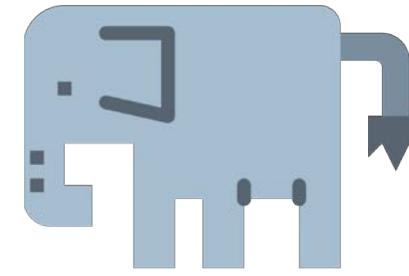
Python w/ External Executable



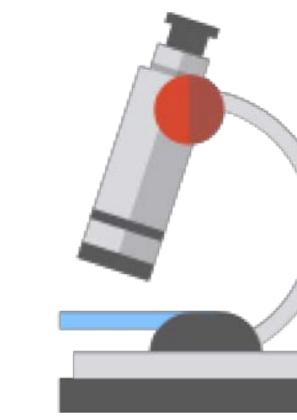
Can easily swap solvers. But pay a speed penalty for (de)serialization

DSL = Domain Specific Language

# TACTICS FOR SUPPORTING INTUITION BEHIND OPTIMIZATION RESULTS



Focus on major drivers /  
points of sensitivity



Show local trade-offs



Leverage existing descriptive  
and diagnostic analytics



Show “diff” of output as user  
tweaks assumptions