Computer Controlled Systems
Control Design Projects

2022.11.30

Contents

1 Task description

In this document, you can find some models for your Computer Controlled Systems assignment. To avoid any misunderstandings, please read this task description carefully.

All models below are linear time-invariant systems, resulting from real-world engineering problems, with different number of input, output and state variables. Your task is the analysis and control of one of these systems, based on the guidlines below. Choosing a suitable initial state, input, control objective (eg. stabilization, reference tracking), etc. for your model is part of the task. Example parameters were given (or already substituted into the equations) for all models - you can use these for simulations if you want.

All figures and calculations should be done in MATLAB (hint: there is no need to reinvent the wheel - there are plenty of useful functions and toolboxes for system control and symbolic calculations which you can use. Google them, look at tutorials, try to understand how you can make use of them). As some of the models are quite complicated, there is no need to calculate anything by hand. The MATLAB code should be submitted as part of the assignment, so please clean it up when you are finished and try to make it understandable.

Besides the code, a report should be submitted in PDF format, where you present and explain your work (you can include figures, formulas, results - anything you feel necessary).

In your work, you should cover the following topics.

Analysis

- State-space model
- Input-output model: transfer function, impulse response function
- Response of the system for a given input
- Stability (Lyapunov as well!)
- Controllability, observability
- Discretization

Control You should choose 3 of the 5 points below:

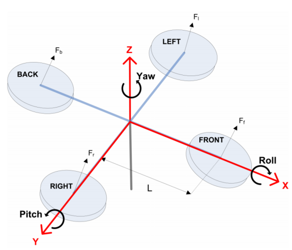
- PID
- Pole placement, State estimator
- LQR control (continuous time)
- LQR control (discrete time)
- Deadbeat (discrete time)

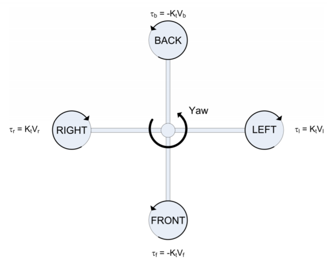
As it was advertised, you can work in pairs. However, the number of pairs or individuals doing the same model is limited to 3. You can reserve your place for a model in this google spreadsheet.

If you have any questions or problems, please feel free to contact Balázs Csutak or Mihály Vághy. Submissions should be made through the moodle system.

2 3-DOF Hover

The 3 DOF Hover experiment provides an economical test bed to understand and develop control laws for flight dynamics and control of vehicles with vertical lift off. The hover consists of a planar round frame with four propellers. The frame is mounted on a three degrees of freedom pivot joint that enables the body to rotate about the roll, pitch and yaw axes. The propellers are driven by four DC motors that are mounted at the vertices of the frame. The propellers generate a lift force that can be used to directly control the pitch and roll angles. Two of the propellers are counter-rotating, so that the total torque in the system is balanced when the thrust of the four propellers is approximately equal.





Let us define the variables as follows:

$$\text{State vector: } x^T = (\theta_y\ \theta_p\ \theta_r\ \omega_y\ \omega_p\ \omega_r)$$
$$\text{Input: } u^T = (V_f\ V_b\ V_r\ V_l)$$
$$\text{Output: } t^T = (\theta_y\ \theta_p\ \theta_r)$$

where $\omega_y = \dot{\theta}_y, \omega_p = \dot{\theta}_p, \omega_r = \dot{\theta}_r$.

The equations describing the system dynamics can be written as:

$$\dot{\omega}_y = -\frac{K_t}{J_y}V_f - \frac{K_t}{J_y}V_b + \frac{K_t}{J_y}V_r + \frac{K_t}{J_y}V_l$$
$$\dot{\omega}_p = \frac{LK_f}{J_p}V_f - \frac{LK_f}{J_p}V_b$$
$$\dot{\omega}_r = \frac{LK_f}{J_r}V_r - \frac{LK_f}{J_r}V_l$$

Using the above information, the state-space model can be easily derived. Typical parameters for the model:

K_t	Torque thrust constant of motor/propeller	0.0036	$N \cdot m/V$
K_f	Force-thrust constant of motor/propeller	0.1188	N/V
L	Distance between pivot to each motor	0.197	cm
J_y	Equivalent moment of inertia about yaw axis	0.110	$kg \cdot m^2$
J_p	Equivalent moment of inertia about pitch axis	0.0552	$kg \cdot m^2$
J_r	Equivalent moment of inertia about roll axis	0.0552	$kg \cdot m^2$

3 Home heating

Warning! Not as easy as it seems!

Let us consider a typical family house with three compartments: the basement, the main living area and the attic. To improve the efficiency of heating, we want to build a model describing how heat is transferred from one part to the another.

The main living area is surrounded with insulator layer, but the attic area has walls and ceiling without insulation. The walls and floor in the basement are insulated by the ground. The basement ceiling is insulated by air space in the joists, a layer of flooring on the main floor and a layer of drywall in the basement. The temperatures of the basement, main area and attic are considered $x_1(t)$, $x_2(t)$, $x_3(t)$.

The temperature of the ground is assumed to be constant T_{ground}, while the outside temperature on average is assumed to be another constant T_{out}. There is a heater in the main area, using which the temperature rate can be controlled. We consider the temperature of the main area as output.

To describe the heat transfers, we use Newton's cooling law:

$$\text{Temperature change rate} \;=\; k \cdot \text{Temperature difference}$$

Which means, we have the following equations:

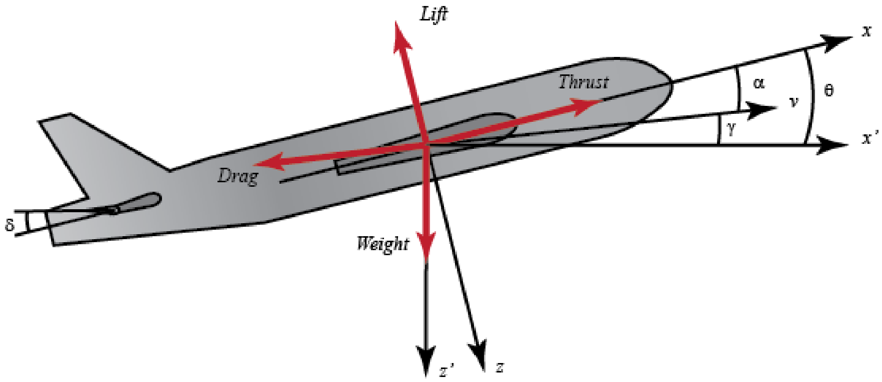
$$\begin{aligned}
\dot{x}_1(t) &= k_0(T_{ground} - x_1(t)) + k_1(x_2(t) - x_1(t)) \\
\dot{x}_2(t) &= k_1(x_1(t) - x_2(t)) + k_2(T_{out} - x_2(t)) + k_3(x_3(t) - x_2(t)) + u(t) \\
\dot{x}_3(t) &= k_3(x_2(t) - x_3(t)) + k_4(T_{out} - x_3(t))
\end{aligned}$$

where k_1, k_2, k_3, k_4 are constants resulting from the different insulations between the compartments. A typical parametrization would be $k_0 = 0.5, k_1 = 0.5, k_2 = 0.25, k_3 = 0.25, k_4 = 0.5$. For winter, $T_{ground} = 45$ and $T_{out} = 35$ Fahrenheit can be considered.

4 Autopilot for aircraft's pitch

The equations governing the motion of an aircraft are a very complicated set of six nonlinear coupled differential equations. However, under certain assumptions, they can be decoupled and linearized into longitudinal and lateral equations. In this example, we take a look at the aircraft pitch governed by the longitudinal dynamics.

The basic coordinate axes and forces acting on an aircraft are shown in the figure given below.



We will assume that the aircraft is in steady-cruise at constant altitude and velocity; thus, the thrust, drag, weight and lift forces balance each other in the x- and y-directions. We will also assume that a change in pitch angle will not change the speed of the aircraft under any circumstance (unrealistic but simplifies the problem a bit). Under these assumptions, the longitudinal equations of motion for the aircraft can be written as follows:

$$
\begin{aligned}
\dot{\alpha} &= \mu\Omega\sigma\left(-(C_L + C_D)\alpha + \frac{1}{\mu - C_L}q - (C_W \sin\gamma)\theta + C_L\right) \\
\dot{q} &= \frac{\mu\Omega}{2i_{yy}}\left(\big(C_M - \mu(C_L + C_D)\big)\alpha + \big(C_M + \sigma C_M(1 - \mu C_L)\big)q + (\mu C_W \sin\gamma)\delta\right) \\
\dot{\theta} &= \Omega q
\end{aligned}
$$

If you are interested, you can refer to any aircraft-related textbooks for the explanation of how to derive these equations. For this system, the input will be the elevator deflection angle δ and the output will be the pitch angle θ of the aircraft.

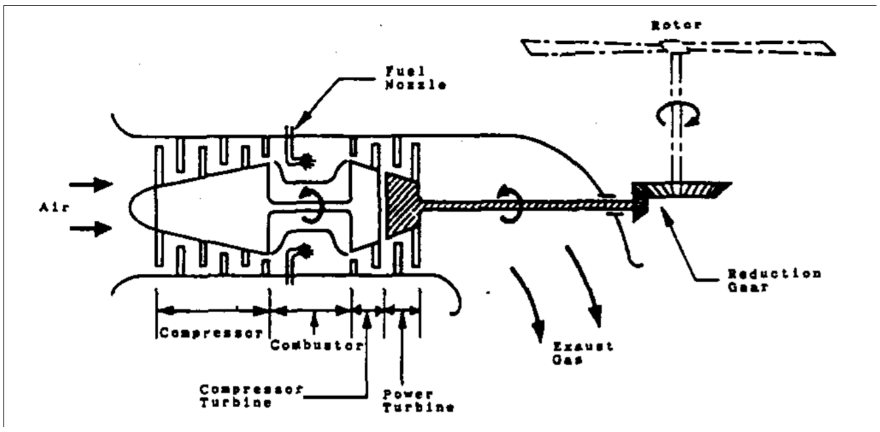
Unfortunately, the exact parameter values for any real aircraft are confidential, however parameters for a simplified, linearized model are available for one of Boeing's commercial jets:

$$
\begin{aligned}
\dot{\alpha} &= -0.313\alpha + 56.7q + 0.232\delta \\
\dot{q} &= -0.0139\alpha - 0.426q + 0.0203\delta \\
\dot{\theta} &= 56.7q
\end{aligned}
$$

5 Gasturbine engine for helicopters

Gasturbine engines for helicopters are subject to significant torque disturbance due to pilot's manipulation of collective pitch angle. Engine dynamics significantly vary with engine operating conditions. In order to cope with the disturbance and the dynamics change, conventional controls use multiple gain and gain-scheduling techniques, respectively. A lot of simulations, engine tests, and flight tests are necessary to validate the control laws.

In this example, we will look at a simplified model of a turboshaft engine. The figure below shows a typical turboshaft engine, which consists of a compressor, a combustor, a HP turbine, and a power turbine. The compressor and the HP turbine are connected by a shaft. Airflow is continuously compressed in the compressor and burned in the burner. Resulting high pressure and high temperature gas drives the HP turbine and then the power turbine which is connected to a helicopter rotor via a speed reduction gear. A pilot controls the rotor lift by manipulating the rotor blade pitch angle.



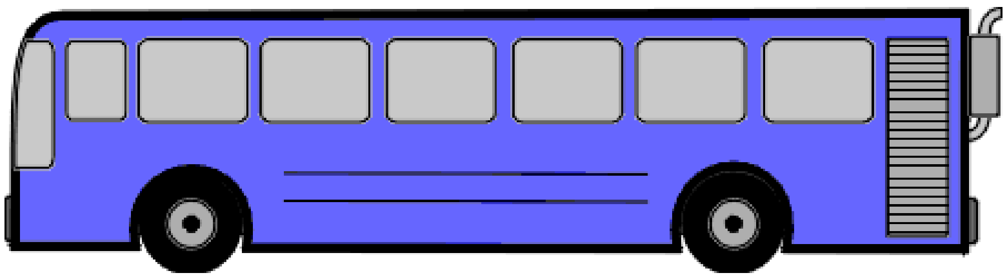
To design controllers, it is necessary to have models of engine dynamics from the shaft torque to the power turbine speed G_{tn}, and from the fuel flow (WF) to the power turbine speed (NF), G_{wn}. The resulting dynamic system has two inputs, three state variables and two outputs, ie. $x^T = (x_1\ x_2\ x_3)$, $u^T = (u_1\ u_2)$, $y^T = (y_1\ y_2)$. Dynamics of the system, depending on the compressor speed θ can be written as:

$$\dot{x} = (A_0 + A_1\theta + A_2\theta^2)x + (B_0 + B_1\theta + B_2\theta^2)u \tag{1}$$

$$A_0 = \begin{pmatrix} -4.365 & -0.6723 & -0.3363 \\ 7.088 & -6.557 & -4.601 \\ -2.41 & 7.584 & -14.31 \end{pmatrix},\ A_1 = \begin{pmatrix} -0.5608 & 0.8553 & 0.5892 \\ 2.5333 & -1.0398 & -7.7373 \\ 3.1917 & 1.7971 & -2.5887 \end{pmatrix},$$
$$A_2 = \begin{pmatrix} 0.6698 & -1.375 & -0.9909 \\ -2.8963 & -1.5292 & 10.516 \\ -3.5777 & 2.8389 & 1.9087 \end{pmatrix},\ B_0 = \begin{pmatrix} 2.374 & 0.7485 \\ 1.366 & 3.444 \\ 0.9416 & -9.619 \end{pmatrix},$$
$$B_1 = \begin{pmatrix} -0.1602 & -0.3521 \\ 0.1162 & -2.4839 \\ -0.1106 & -4.6057 \end{pmatrix},\ B_2 = \begin{pmatrix} 0.1562 & 0.1306 \\ -0.4958 & 4.0379 \\ -0.0306 & 0.8947 \end{pmatrix},\ C = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

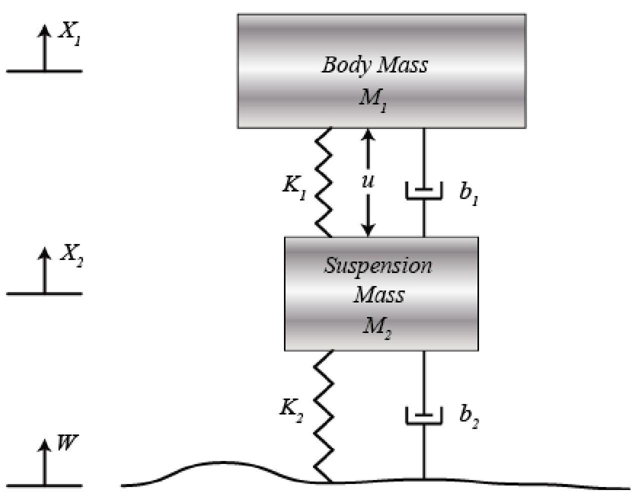
For this task, we take a look at the linear system model valid at $\theta = 0.73$ (ie. the compressor speed is 73% of its maximum). The state-space model can be easily written by simple substitution of the $A_0, A_1, A_2, B_0, B_1, B_2$ matrices into Equation (1).

6 Suspension system of a bus



Designing an automotive suspension system is an interesting and challenging control problem. When the suspension system is designed, a 1/4 model (one of the four wheels) is used to simplify the problem to a 1-D multiple spring-damper system. A diagram of this system is shown below. This model is for an active suspension system where an actuator is included that is able to generate the control force u to control the motion of the bus body.

Model of Bus Suspension System (1/4 Bus)



The equations for the above model can be written as:

$$
\begin{aligned}
m_1\ddot{x}_1 &= -b_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) + u \\
m_2\ddot{x}_2 &= b_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) + b_2(\dot{W} - \dot{x}_2) + k_2(W - x_2) - u
\end{aligned}
$$

where W is a disturbance input representing the surface of the road (see figure).

By introducing a variable $\dot{x}_1 = v_1$ and $\dot{x}_2 = v_2$, the state-space equation with four state variables, two inputs, and one output ($y = x_1 - x_2$) can be easily derived.

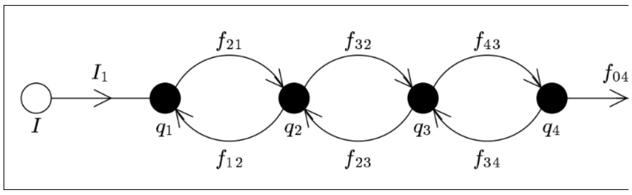
Typical parameters values:

(m_1)	1/4 bus body mass	2500 kg
(m_2)	suspension mass	320 kg
(k_1)	spring constant of suspension system	80,000 N/m
(k_2)	spring constant of wheel and tire	500,000 N/m
(b_1)	damping constant of suspension system	350 $N \cdot s/m$
(b_2)	damping constant of wheel and tire	15,020 $N \cdot s/m$

7 Compartmental model

A multi-compartment model is a type of mathematical model used for describing the way materials or energies are transmitted among the compartments of a system, each compartment being assumed to be a homogeneous entity within which the modelled substances are equivalent. These type of models have a wide-range of applications in the field of engineering, being suitable for numerous chemical or biological systems. Recently, compartmental models got big publicity, as a common way for modeling epidemic processes (eg. SIR, SEIR, SIER, etc.).

In this setup, the state variables represent the quantities of substances being in each of the compartments. As an example, let us consider the following model:



Here, we have 4 compartments noted by $q_i,\ i = 1...4$, so the model will have the state vector $x^T = (x_1\ x_2\ x_3\ x_4)$. The input is the amount flowing into the compartments, so in this case we have a single-input system. Let us consider a single output, $y = x_4$. It must be noted, that substances are flowing out from the system as well (f_{04}), so without input the total amount in the system decreases.

The system equations can be written by taking the inbound and outbound flows at each compartment into consideration. In this example, we assume, that the amount flowing out of a compartment depends only on the amount in it and the respective flow parameter (eg. the amount flowing from x_2 to x_3 is the function of x_2 and the parameter f_{32}). Based on this, the equations for the first two compartments:

$$\dot{x}_1 = -f_{21} \cdot x_1 + f_{12} \cdot x_2 + u$$
$$\dot{x}_2 = f_{21} \cdot x_1 + f_{23} \cdot x_3 - f_{32} \cdot x_2 - f_{12} \cdot x_2$$

Using this information, the state-space model can be easily derived.

Parameters for the system (example):

$$f_{12} = 0.15 \qquad f_{21} = 0.6 \qquad f_{23} = 0.25 \qquad f_{32} = 0.7$$
$$f_{34} = 0.05 \qquad f_{43} = 0.5 \qquad f_{40} = 0.3 \qquad u = 0.8$$

8 Quanser - Magnetic levitation

The Quanser Magnetic Levitation device is a single degree of freedom electromagnet-based system that allows users to levitate a ball vertically up and down. The overhead electromagnet generates an attractive force on the metal ball that initially sits on the post. The position of the ball is measured using a photo-sensitive sensor embedded inside the post. The system also includes a current sensor to measure the current inside the electromagnet's coil.

The project will be specified later. Until that, please watch a short demo here.

9 Quanser - QUBE Servo 2

The project will be specified later. Until that, please watch a short demo here.

10 Quanser - Rotary flexible link

The project will be specified later. Until that, please watch a short demo here.

11 Quanser - Ball and beam system

The project will be specified later. Until that, please watch a short demo here.