1 H-2.1

Bit stream: 11 00 10 00 11 11.

1.1 Bipolar 8-ASK

Bit stream: 110 010 001 111.

1.1.1 Natural Mapping

+5, -3, -5, +7

1.1.2 Gray Mapping

+1, -1, -5, +3

1.2 16-QAM

Bit stream: 1100 1000 1111.

1.2.1 Natural Mapping

+3-3j, +1-3j, +3+3j

1.2.2 Gray Mapping

+3+3j, +3-3j, +1+1j

2 H-2.2

In Figure 1 and Figure 2 the 4-QAM constellations are shown for natural and gray mapping respectively.
Bitstream: 11 00 10 00 11 11

2.1 Natural Mapping

+1-1j, -1-1j, +1+1j, -1-1j, +1-1j, +1-1j

2.2 Gray Mapping

+1+1j, -1-1j, +1-1j, -1-1j, +1+1j, +1+1j

3 H-2.3

Bitstream: 01 010
Assuming gray mapping.

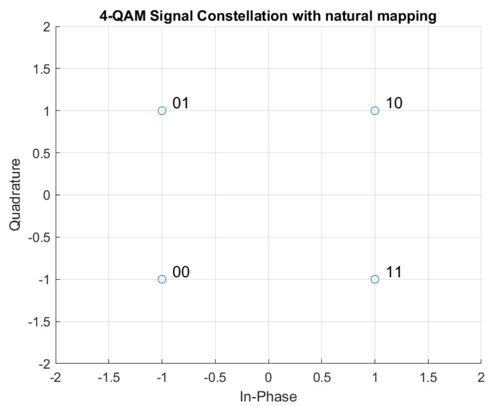

Figure 1: 4-QAM constellation with natural mapping

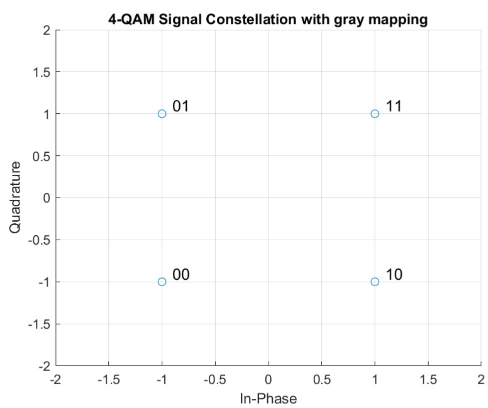

Figure 2: 4-QAM constellation with gray mapping

3.1 4-QAM

Symbol length is 2 bits, so we have 3 symbols. So we need to pad one 0 for the last symbol.
Zero padded bitstream: 01 01 0(0)
-1+1j, -1+1j, -1-1j

3.2 8-ASK

Symbol length is 3 bits, so we have 2 symbols. So we need to pad one 0 for the last symbol.
Zero padded bitstream: 010 10(0)
-1, +7

3.3 16-QAM

Symbol length is 4 bits, so we have 2 symbols. So we need to pad three 0 for the last symbol.
Zero padded bitstream: 0101 0(000)
-1+3j, -3-3j

4 H-2.4

$$s_{RF}(t) = \sqrt{2} Re\left\{s(t)e^{j2\pi f_c t}\right\} \tag{1}$$

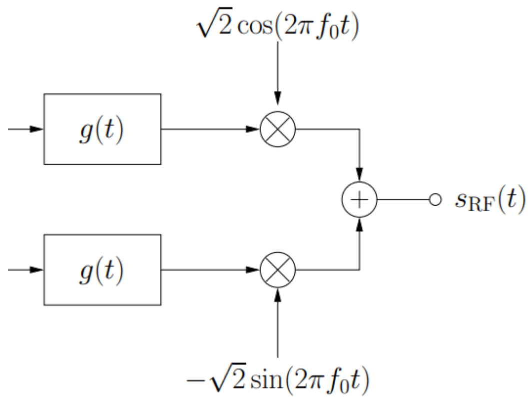

Figure 3: Modulation to carrier frequency

5 H-2.5

$0 \leq t \leq T$

$$\begin{aligned} s_{RF}(t) = \sqrt{2}Re\left\{s_1(t)e^{j2\pi f_c t}\right\} = \sqrt{2}Re\left\{3 - 5j[cos(2\pi f_c t) + jsin(2\pi f_c t)]\right\} = \\ = \sqrt{2}(3\cos(2\pi f_c t) + 5\sin(2\pi f_c t)) \end{aligned} \tag{2}$$

$T \leq t \leq 2T$

$$\begin{aligned} s_{RF}(t) = \sqrt{2}Re\left\{s_2(t)e^{j2\pi f_c t}\right\} = \sqrt{2}Re\left\{-1 - 1j[cos(2\pi f_c t) + jsin(2\pi f_c t)]\right\} = \\ = \sqrt{2}(-\cos(2\pi f_c t) + \sin(2\pi f_c t)) \end{aligned} \tag{3}$$

$2T \leq t \leq 3T$

$$s_{RF}(t) = \sqrt{2}Re\left\{s_3(t)e^{j2\pi f_c t}\right\} = \sqrt{2}Re\left\{7 + 3j[cos(2\pi f_c t) + jsin(2\pi f_c t)]\right\} = \\ = \sqrt{2}(7\cos(2\pi f_c t) - 3\sin(2\pi f_c t)) \tag{4}$$

6 H-2.6

```
total = 0;

for i = 1:length(x)
    re = real(x(i));
    im = imag(x(i));
    pow = re^2 + im^2;
    total = total + pow;
end

mean_power = total/length(x)
```

7 H-2.7

$$E_b = S_e T_b = S_e \frac{1}{f_s \log_2 M} \tag{5}$$

8 H-2.8

$$P_N = 2N_0/2f_s \tag{6}$$

9 H-2.9

10 H-2.10

11 H-2.11 and H-2.12

11.1 Natural Mapping

symbols: -3-1j, +3+1j, +1+3j noise: +1.5-1.7j, +0.2+1.1j, -1.1+0.5j symbolds with noise: -1.5-2.7j, +3.2+2.1j, -0.1+3.5j
estimated symbols: -1-3j, +3+3j, -1+3j estimated bit sequence: 0100 1111 0111 origianl sequence: 0001 1110 1011 errors: 5 BER = 5/12 = 0.42

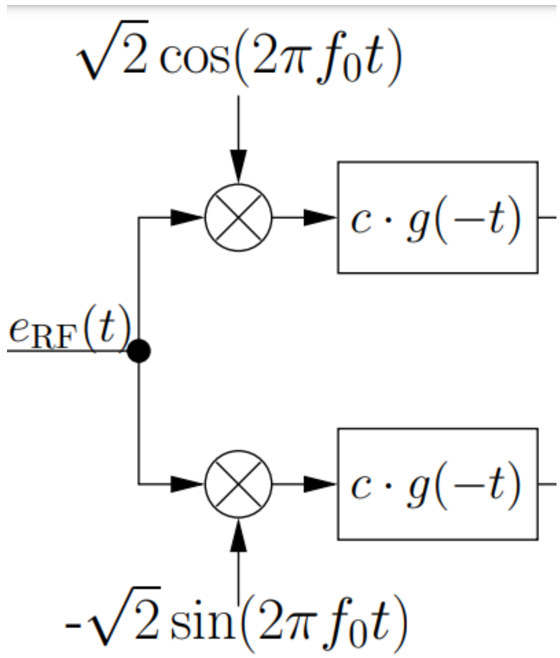

Figure 4: Demodulation to carrier frequency

11.2 Gray Mapping

symbols: -1-3j, +3+1j, +1-1j noise: +1.5-1.7j, +0.2+1.1j, -1.1+0.5j symbolds with noise: +0.5-4.7j, +3.2+2.1j, -0.1-0.5j
estimated symbols: +1-3j, +3+3j, -1-1j estimated bit sequence: 1001 1100 0011 origianl sequence: 0001 1110 1011 errors: 3 BER = 3/12 = 0.25

11.3 Conclusion

Gray mapping is better than natural mapping, as it has a lower error rate. This is because, if an error happens with gray mapping, it is more likely that less bits are wrong, as the neighbors always differ by one bit. This is not the case with natural mapping.

12 H-2.13

8-ASK

12.1 Natural Mapping

000, 001, 010, 011, 100, 101, 110, 111

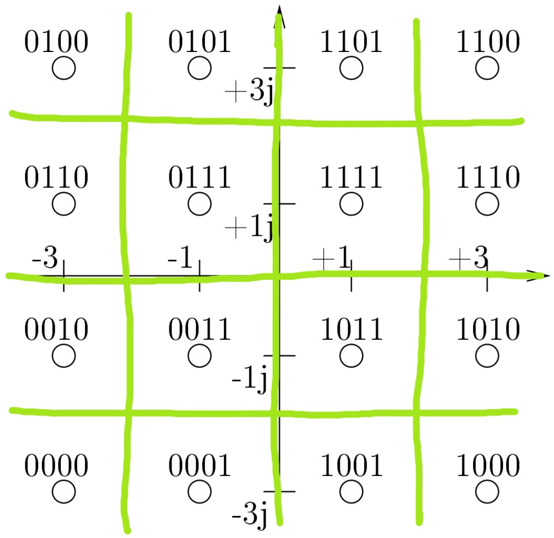

Figure 5: 16-QAM constallation with decision boundaries

Average bit error: $bit_{error} = \frac{1}{7}(1+2+1+3+1+2+1) = \frac{1}{7}11 = \frac{11}{7}$

12.2 Gray Mapping

000, 001, 011, 010, 110, 111, 101, 100
Average bit error: 1

13 H-2.14

Count the number of wrong bits in the sequence (or how many need to be changed to get the origial sequence), and divide by the total number of bits.

14 H-2.15

For a confidence level (CL) of 95%, the number of bits to test need to be 3×10^6, to have a BER lower than 10^{-6}.

$$N_{bits} = \frac{-\ln(1-CL)}{BER} = 3 \times 10^6 \quad (7)$$