Friedrich-Alexander-Universität
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Recurrent Neural Networks (RNN) for Modeling of Nonlinear Systems

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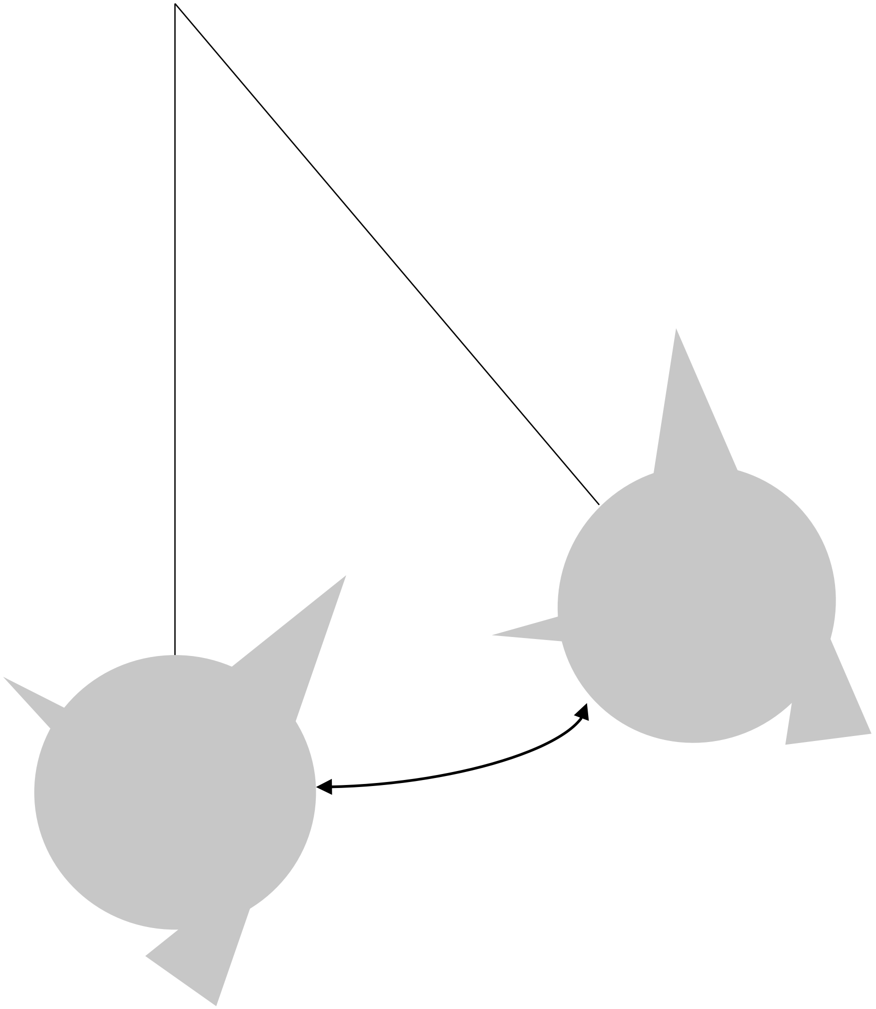
Lehrstuhl für Regelungstechnik

Motivation

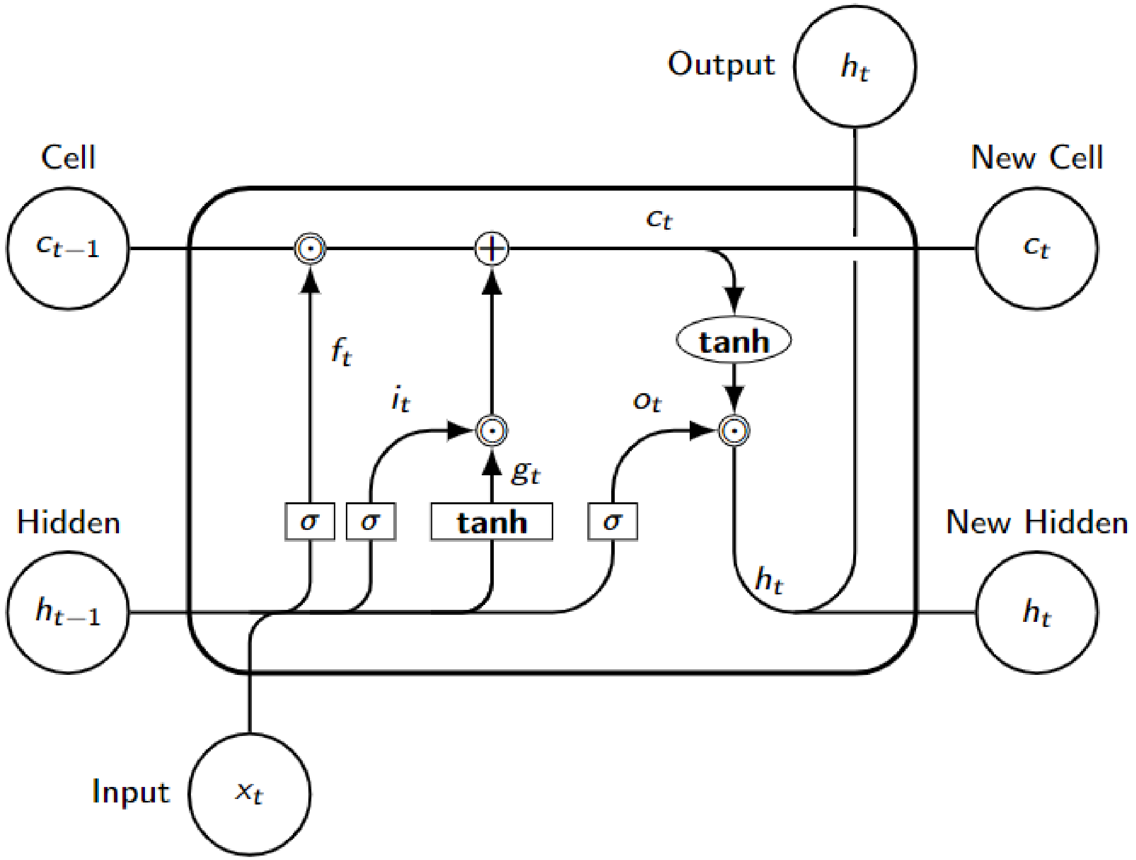
- Problem:
  - Complex System without analytic form
  - Example: Pendulum with unknown friction model
- Solution
  - Train a model with measured data
  - Prediction of future states with model



Methods

LSTM-Cell

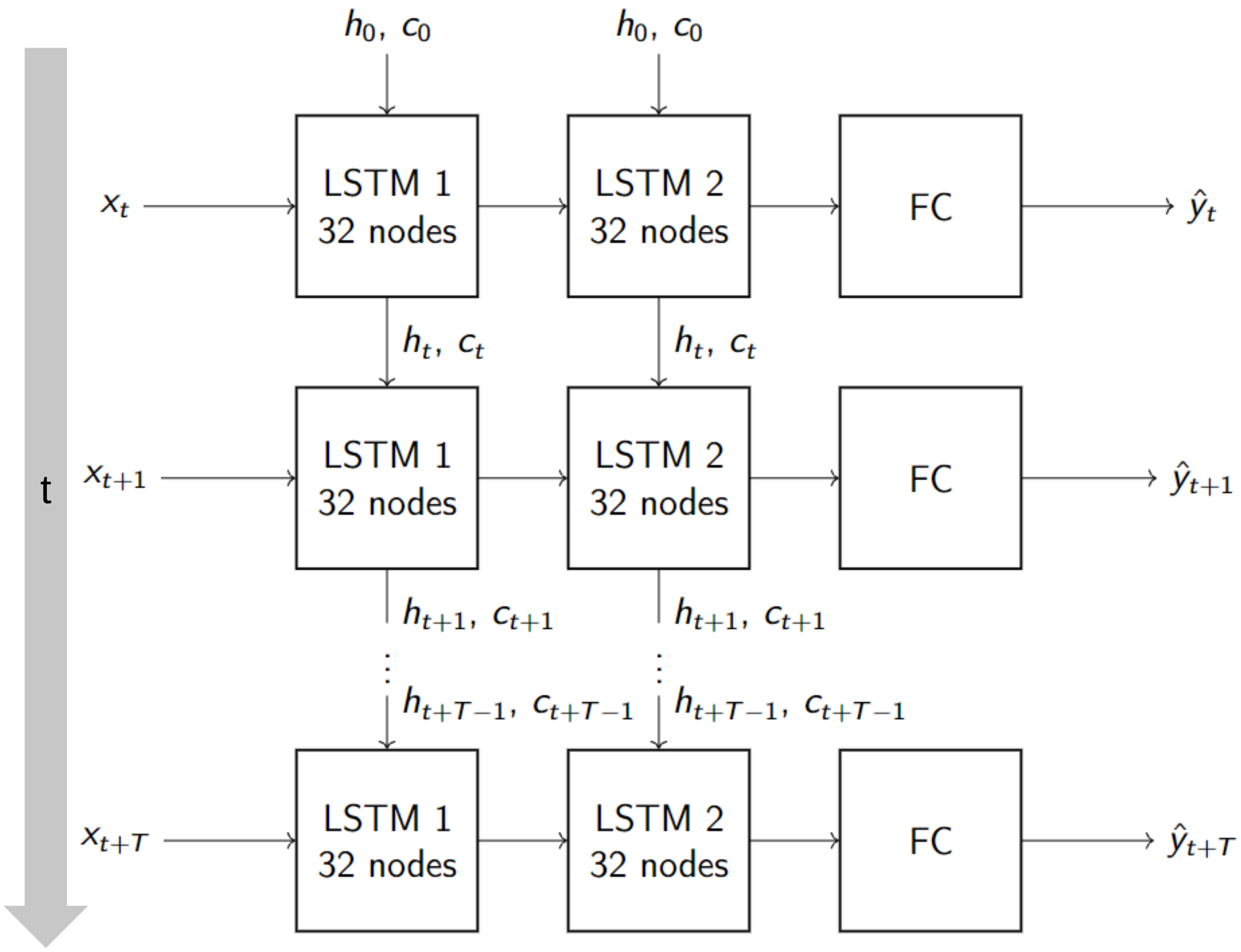
- Long-term dependency through cell state
- Passing of new initial value to next timestep internally
- Gates with trainable weights
- Tanh activation function



Adapted from (Yu, Y., Si, X., Hu, C., & Zhang, J., 2019)

Model

- Example of model with 32 nodes and 2 hidden layers
- Transfer of states to next time steps
- Same weights at all time steps of individual Cells
- Ability to predict indefinite future states
- Fixed number of timesteps for training



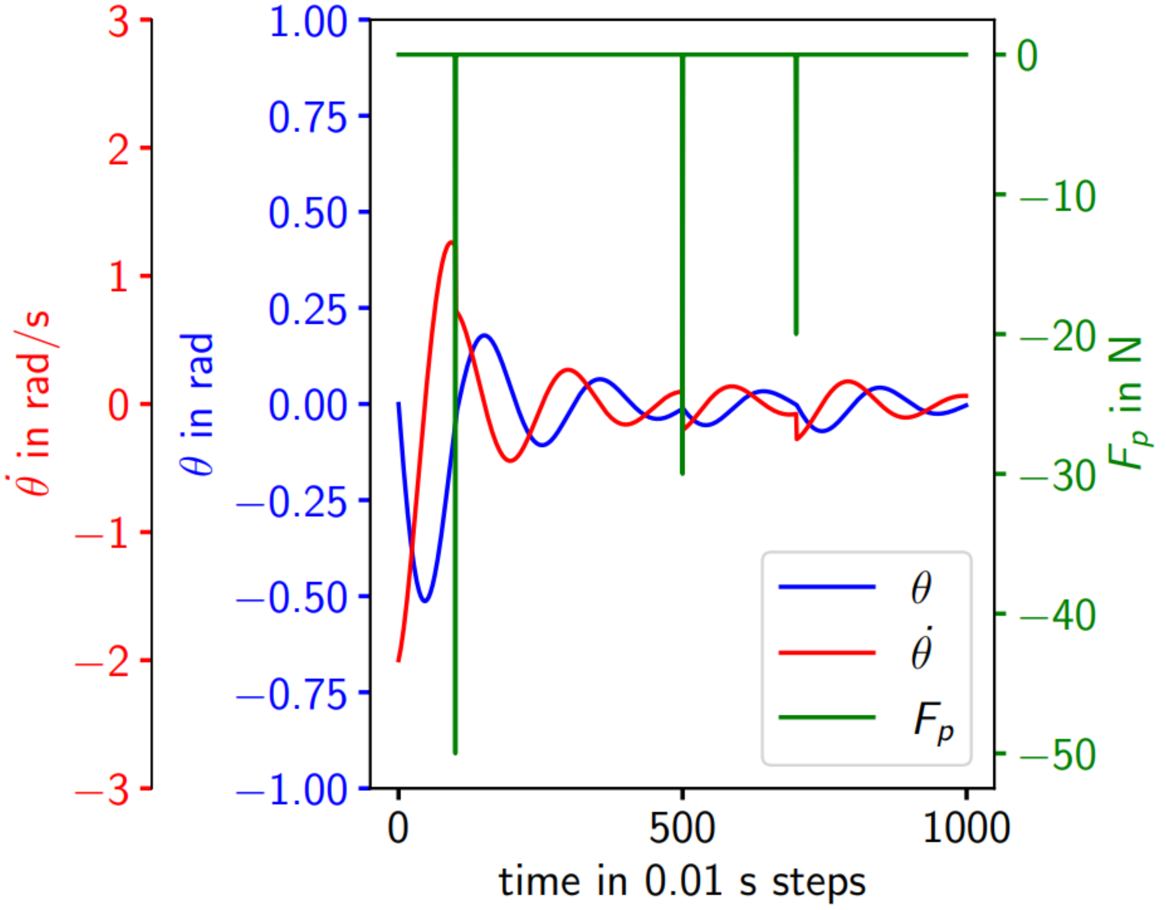
Dataset Creation

Pendulum-Complex Dataset

- Pushing a pendulum
- Dampened due to friction
- Varying shape of pushing force $F_p(t)$
- 200 different time series

$$\frac{d\theta}{dt} = \dot{\theta}$$

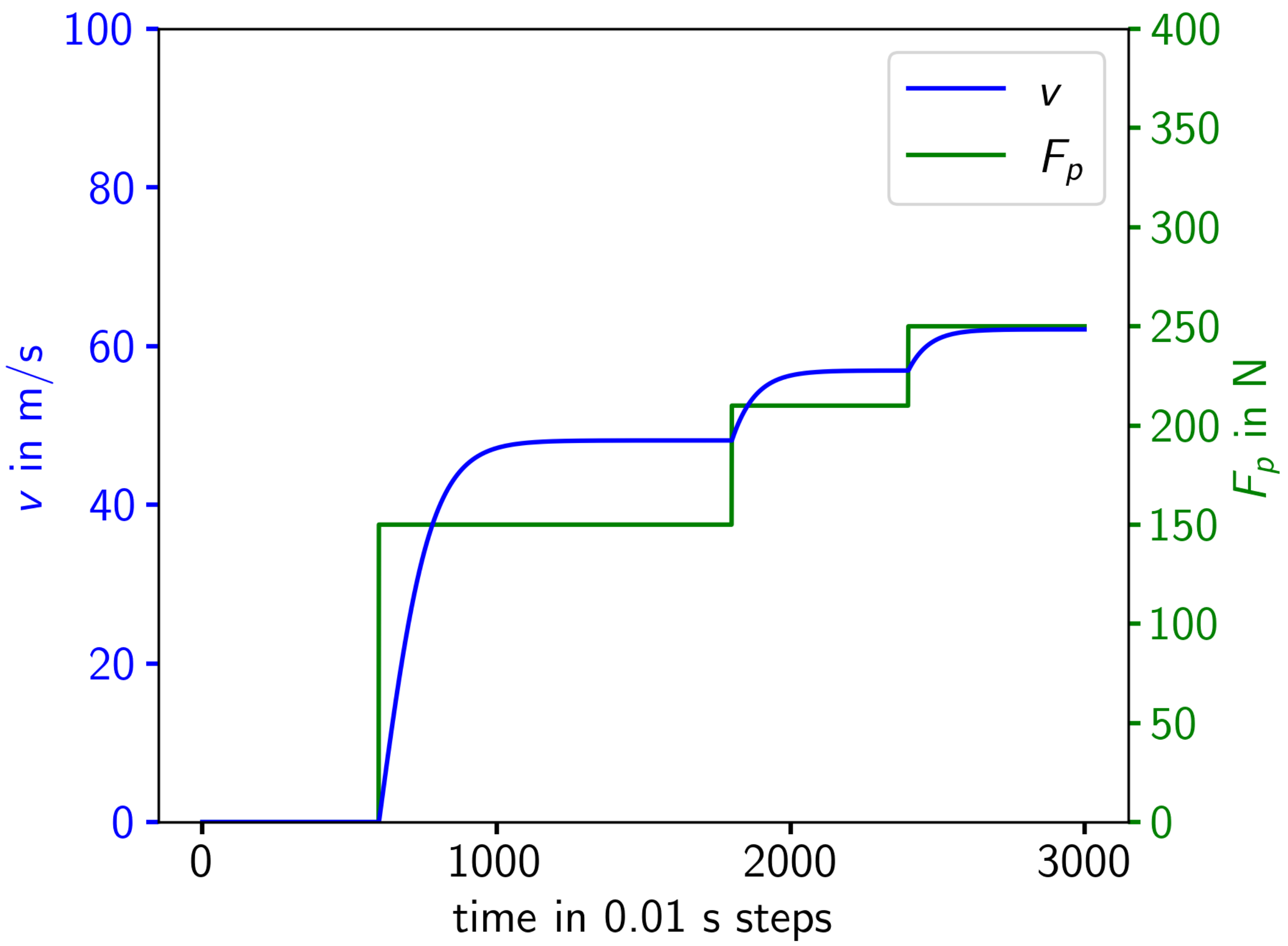
$$\frac{d\dot{\theta}}{dt} = \frac{F_p(t)}{ml}\cos(ft) - q\dot{\theta} - \frac{g}{l}\sin\theta$$



Drag-Complex Dataset

- Pushing force on object
- Drag on object
- Varying of shape of pushing force $F_p(t)$
- 40 different time series

$$\frac{dx}{dt} = v$$
$$\frac{dv}{dt} = F_p(t) - \frac{b}{m} v^2$$



Preparation for Training

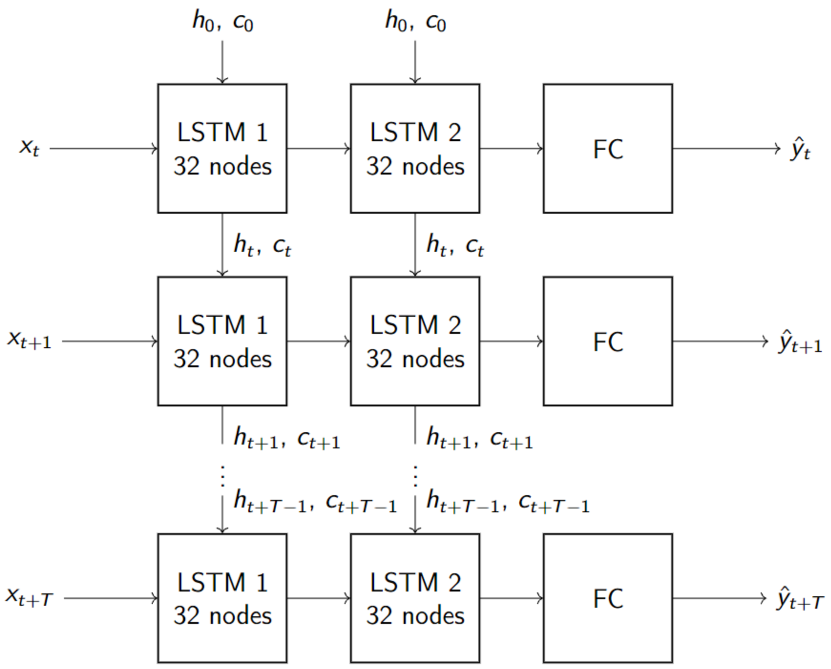
- Split into training, validation and test set
- Scaling of data with MinMaxScaler to [-1,1] to match LSTM output
- Transformation into correct format
  - Initial condition given at first timestep, zero afterwards
  - Force input given at every time step

$$X = \begin{bmatrix} \theta_t & \dot{\theta}_t & F_t \\ 0 & 0 & F_{t+1} \\ 0 & 0 & F_{t+2} \\ \vdots & \vdots & \vdots \\ 0 & 0 & F_{T-1} \end{bmatrix}$$

$$y = \begin{bmatrix} \theta_{t+1} & \dot{\theta}_{t+1} \\ \theta_{t+2} & \dot{\theta}_{t+2} \\ \theta_{t+3} & \dot{\theta}_{t+3} \\ \vdots & \vdots \\ \theta_T & \dot{\theta}_T \end{bmatrix}$$

Putting it together

- Calculation of loss from forward pass
- Update of weights through backpropagation and Adam optimizer



$$y = \begin{bmatrix} \theta_{t+1} & \dot{\theta}_{t+1} \\ \theta_{t+2} & \dot{\theta}_{t+2} \\ \theta_{t+3} & \dot{\theta}_{t+3} \\ \vdots & \vdots \\ \theta_T & \dot{\theta}_T \end{bmatrix}$$

$$X = \begin{bmatrix} \theta_t & \dot{\theta}_t & F_t \\ 0 & 0 & F_{t+1} \\ 0 & 0 & F_{t+2} \\ \vdots & \vdots & \vdots \\ 0 & 0 & F_{T-1} \end{bmatrix}$$

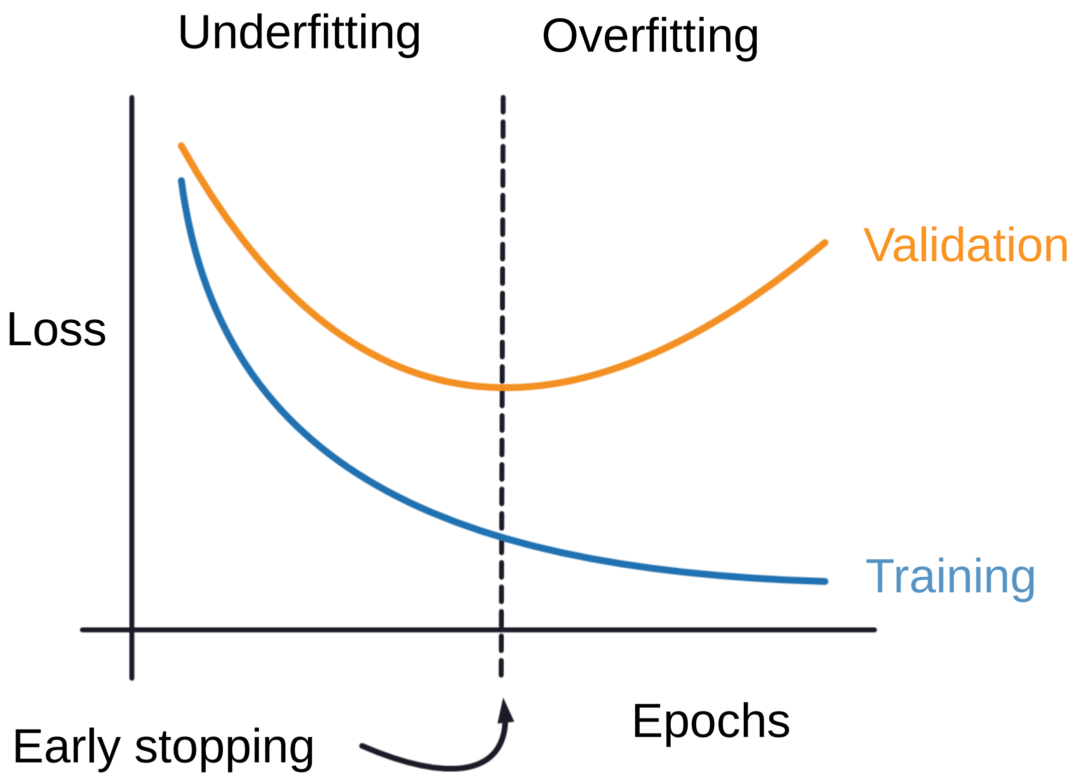
$$\min_W L(X, W, y)$$

$$L = \sum_{t=0}^{T} (\hat{y}_t - y_t)^2$$

Hyperparameter Search

Manual Search

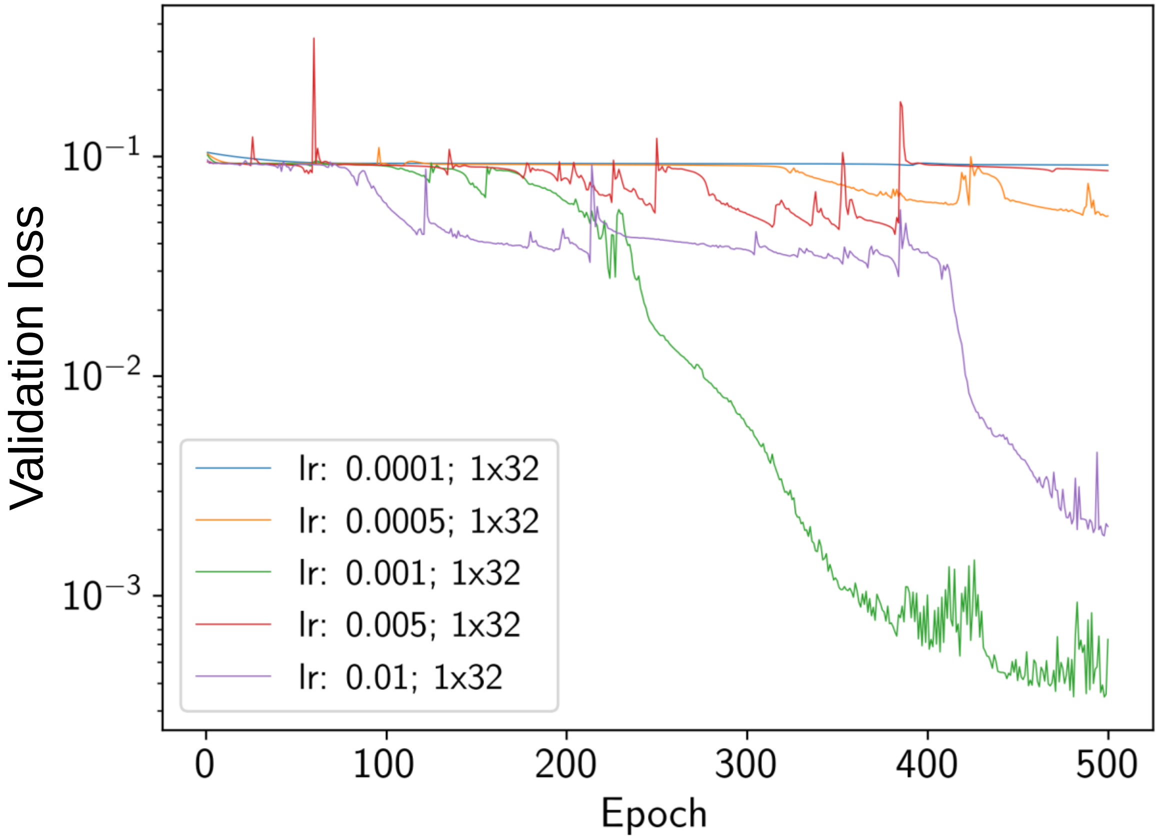
- Monitoring validation loss for early stopping
- Check hyperparameters one by one
- Take best parameter for next step
- Search order:
  1. Learning rate
  2. Number of layers
  3. Number of nodes in each layer
- Training for 500 epochs, then 5000 with best model

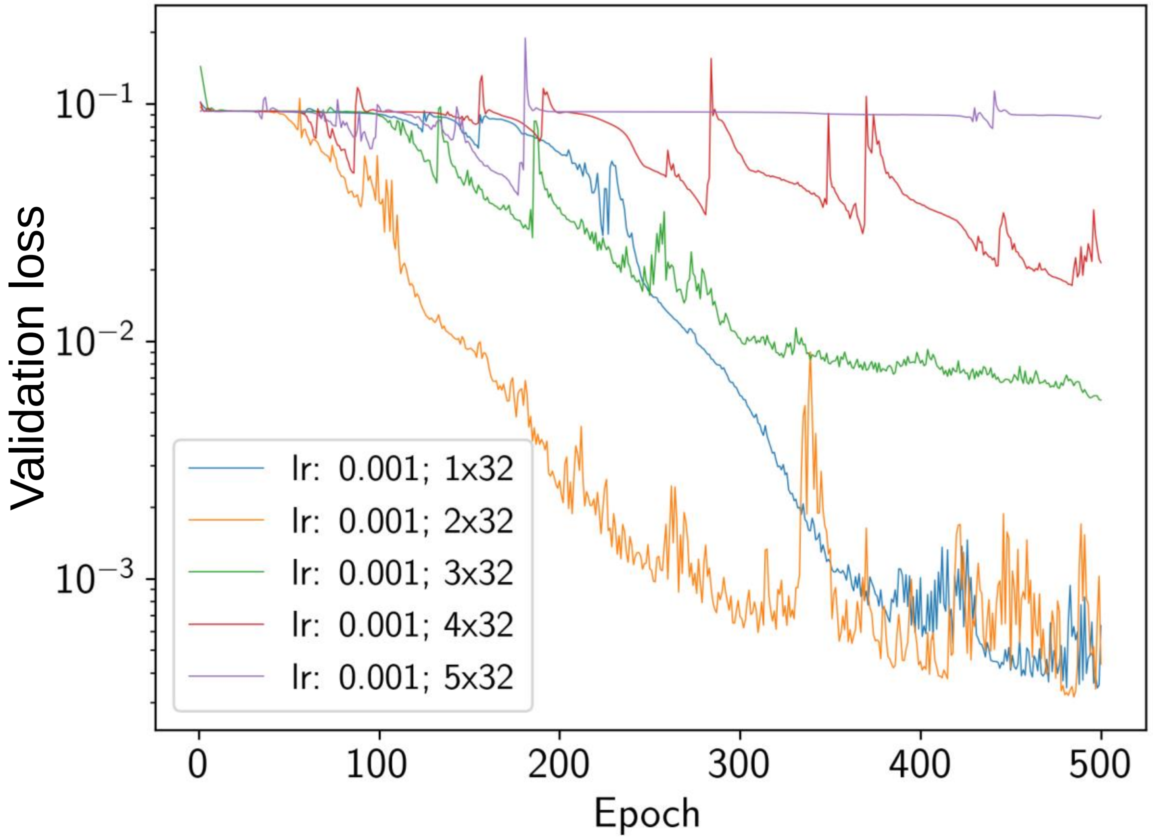


Adapted from (Ryan, H. & Alexis C. 2022)

Hyperparameter Search - Learning rate and layers

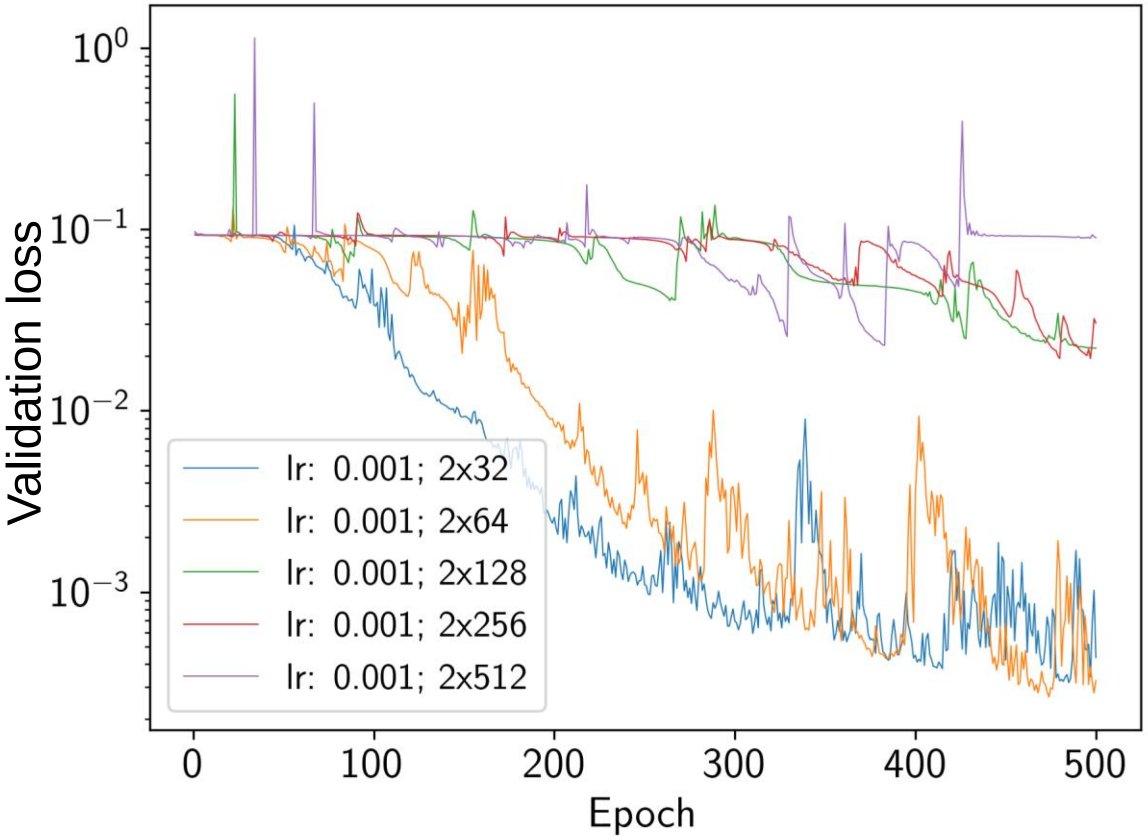
- Better training performance with higher learning rate and less layers

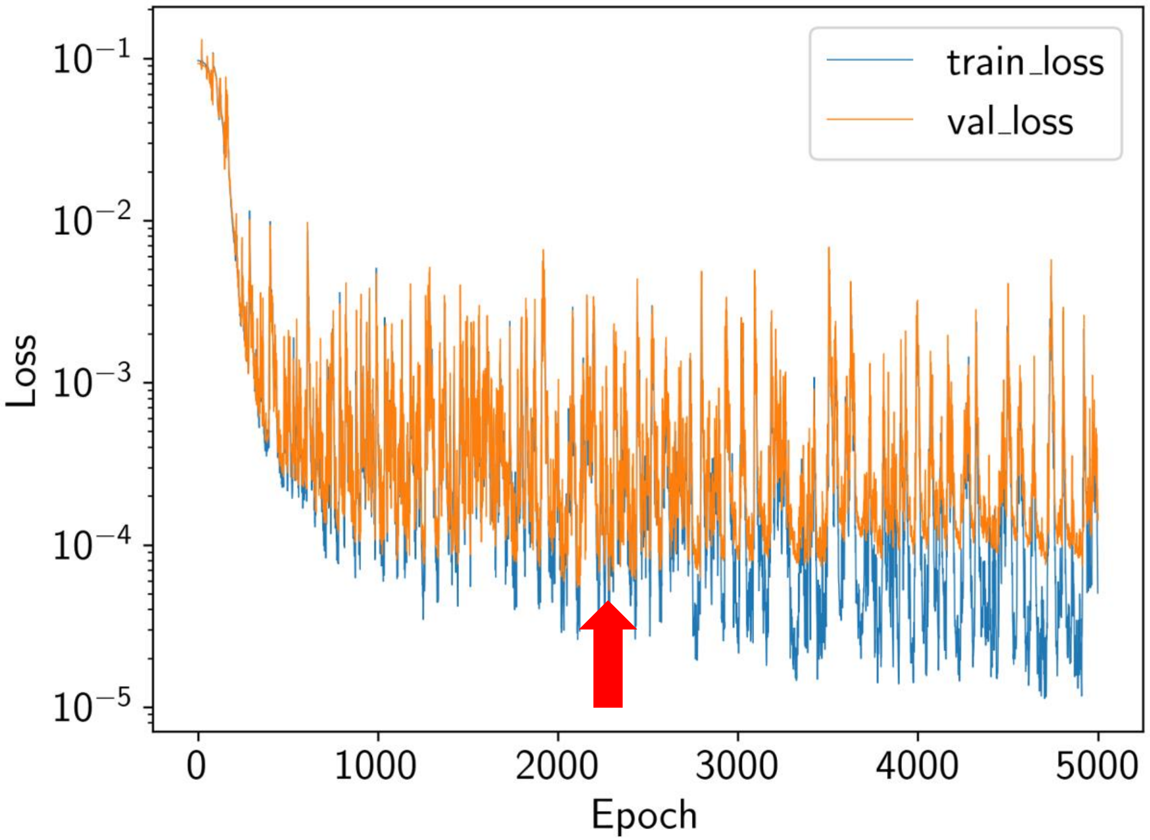




Hyperparameter Search - Nodes

- Higher training stability for smaller models and early stopping to save model with best validation loss

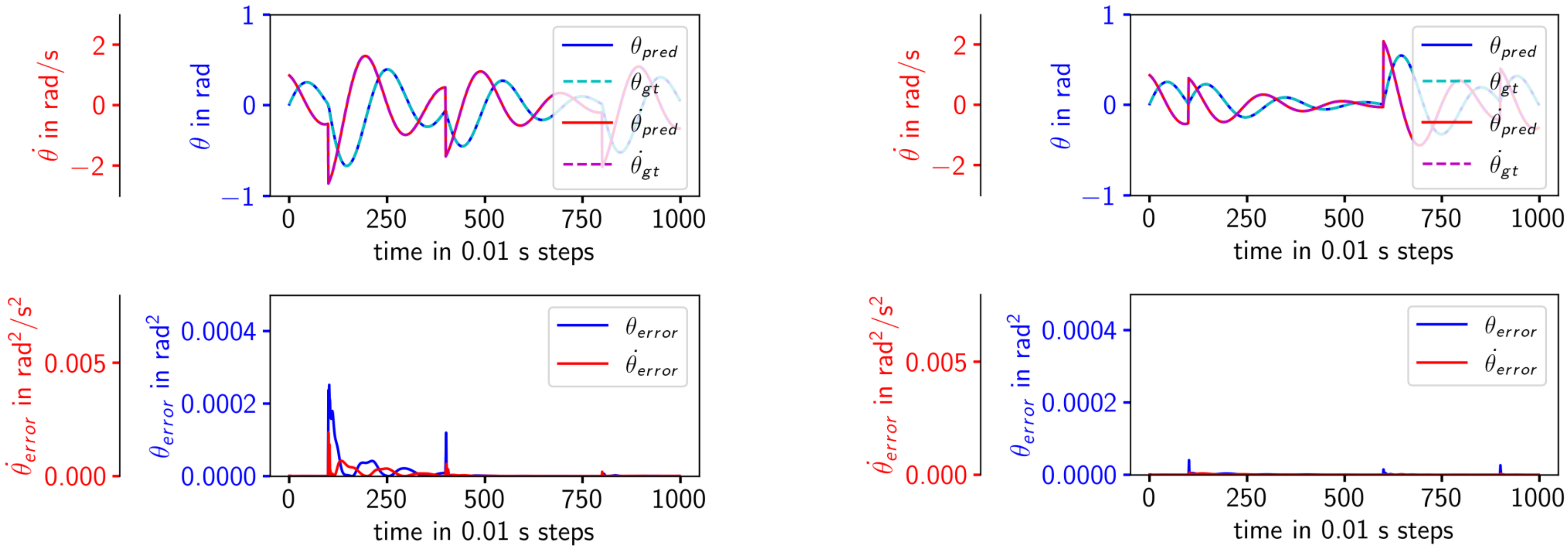




Evaluation and Conclusion

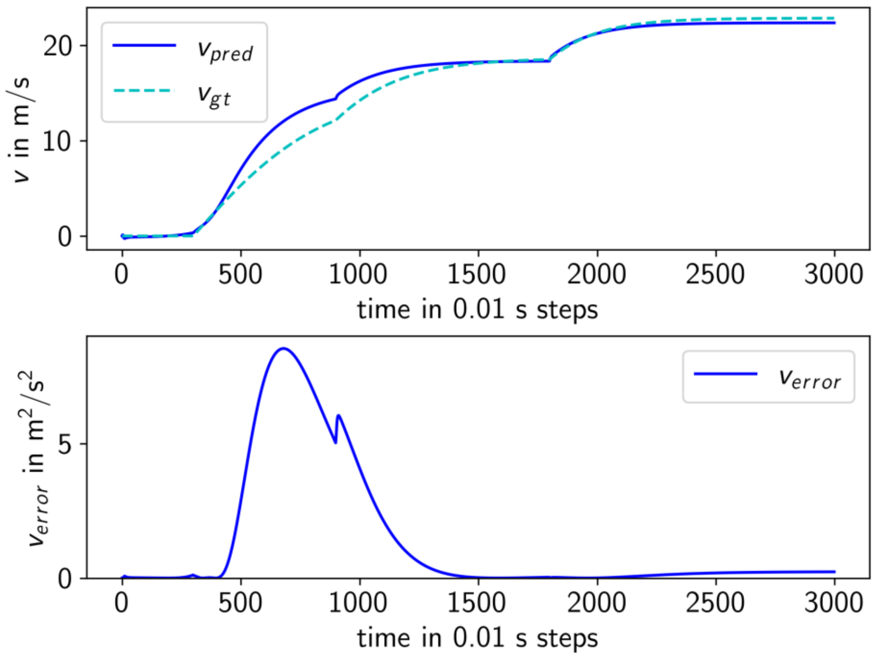
Evaluation - Pendulum

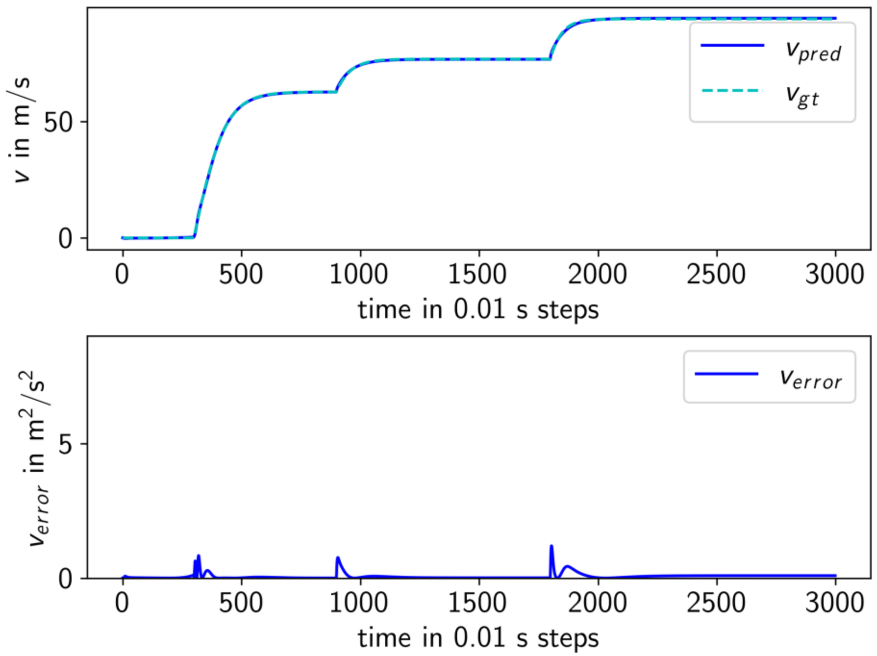
- Overall low error
- Small error spikes on input change



Evaluation - Drag

- Rare large error
- Error spikes on input change





Conclusion and Future Work

- Conclusion
  - Training difficulties with larger models
  - Error spikes on input change
  - Clear ability to predict nonlinear dynamic system
  - Recommendation:
    - 2 LSTM layers with 32/64 nodes and learning rate of 0.01 for similar systems
    - New hyperparameter search for different/more complex systems
- Future Work
  - Deeper look into training instabilities with larger models necessary
  - Testing for more complex ordinary differential equations
  - Analysis on what and how much training data is necessary to achieve good results

Thank You!

References

- Yu, Y., Si, X., Hu, C., & Zhang, J. (2019). A review of recurrent neural networks: LSTM cells and network architectures. *Neural Computation*, *31*(7), 1235–1270.
- Holbrook, R., & Cook, A. (2022, May 5). Overfitting and underfitting. Kaggle. Retrieved January 12, 2023, from https://www.kaggle.com/code/ryanholbrook/overfitting-and-underfitting/tutorial

Problem description

- Pushing force:
  - $F_p = ma$
- Air resistance (Drag):
  - $F_d = \frac{1}{2} \rho v^2 C_D A$
- Simplification:
  - $b = \frac{1}{2} \rho C_D A$
- Resulting force:
  - $F = F_p - F_d$

Problem description

- Resulting force:
  - $F = F_p - F_d$
- Express as second order differential equation:
  - $m\frac{d^2x}{dt^2} = F_p - b\frac{dx}{dt}$
- Transform into system of two first order equations:
  - $\frac{dx}{dt} = v$
  - $\frac{dv}{dt} = \frac{F_p - bv^2}{m}$
  - → use in odeint python function to get values of velocity and/or position