**ESO 208A: Computational Methods in Engineering**
**Programming Assignment 4: *Ordinary Differential Equations***

**Due date: Sunday, 10th November 24:00 mid-night**

1) Write a computer program for solving IVPs.

**Input:** (i) Ordinary differential equation to be solved $\frac{\mathrm{d}y}{\mathrm{d}x} = f(x, y)$; (ii) initial values $x_0$ and $y_0$; (iii) final value $x_f$ (iv) interval size $h$; (v) maximum tolerance $tol$.

**Options:** The user should have the option of selecting one or more of the following –

(a) Euler Forward
(b) Euler Backward
(c) Trapezoidal
(d) 4th -order Adams-Bashforth
(e) 4th -order Adams-Moulton
(f) 4th -order Backward Difference Formulation (BDF)
(g) 4th Order Runge-Kutta

Obtaining analytical solution is optional.

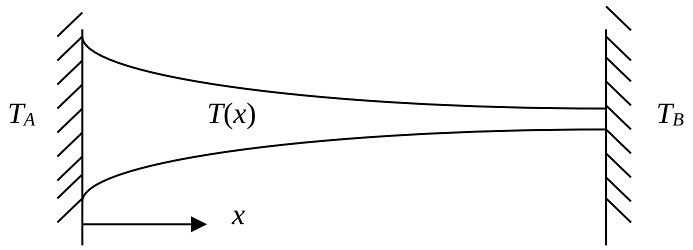
**Output:** The output from the program should be:

(a) A text file containing the values of $x_i$ and corresponding $y_i$;
(b) A figure showing $y$ vs $x$.

**Test Problem:** Provided at the end as well other problems in Tutorial 12.

2) Write a computer program to solve the following BVP:

The diagram shows a body of conical section fabricated from stainless steel immersed in air at zero temperature. It is of circular cross section that varies with *x*. The large end is located at *x* = 0 and is held at temperature $T_A = 5$. The small end is located at $x = L = 2$ and is insulated (i.e, the temperature gradient is zero).




Conservation of energy can be used to develop a heat balance equation at any cross section of the body. When the body is not insulated along its length and the system is at steady state, its temperature satisfies the following ODE:

$$\frac{d^2T}{dx^2} + a(x)\frac{dT}{dx} + b(x)T = f(x)$$

where, a(x), b(x), and f(x) are functions of the cross-sectional area, heat transfer coefficients, and the heat sinks inside the body. In the present case, they are given by

$$a(x) = -\frac{x+3}{x+1}, \qquad b(x) = \frac{x+3}{(x+1)^2}, \qquad \text{and} \qquad f(x) = 2(x+1) + 3b(x).$$

a) Discretize the above equation using 2nd order central difference approximation and formulate the set of linear simultaneous equations. Incorporate the boundary conditions such that the accuracy of the scheme is preserved. Use $\Delta x = 0.5$.

b) Solve the system of equations using Thomas Algorithm and draw the temperature profile indicating the values at the nodes.

In the program, keep the following user options:

- Grid size (*h*)
- Implementation options for the small end boundary: 2nd order Backward Difference or 2nd order Central Difference with Ghost Node

You may use the Thomas Algorithm program module that you wrote in Programming Assignment 2.

**Test Problem:** Problems in Tutorial 13.

**Due date: Sunday, November 10th, 2019, 24:00 midnight**

Make a single zip folder with all your program file(s) name it roll_number.zip (e.g., If your roll no. is 123456, the folder name should be 'P3_123456.zip'). The folder should include -

(i) All the computer program file(s), input file(s) and output file(s)

(ii) A PDF file of the plots and the solution of the test cases given in this assignment.

Send the zip file by e-mail to: eso208.sec*@gmail.com, where * is section number 1-10. Example: for section O5, it is eso208.sec5@gmail.com; for section O10, it is eso208.sec10@gmail.com

**Additional Test Data: Part 1:**

## Sample input file

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 5e^{-100(x-2)^2} - 0.5y$$

$$x_0 = 0.0$$

$$y_0 = 0.5$$

$$x_f = 4.0$$

$$h = 0.2$$

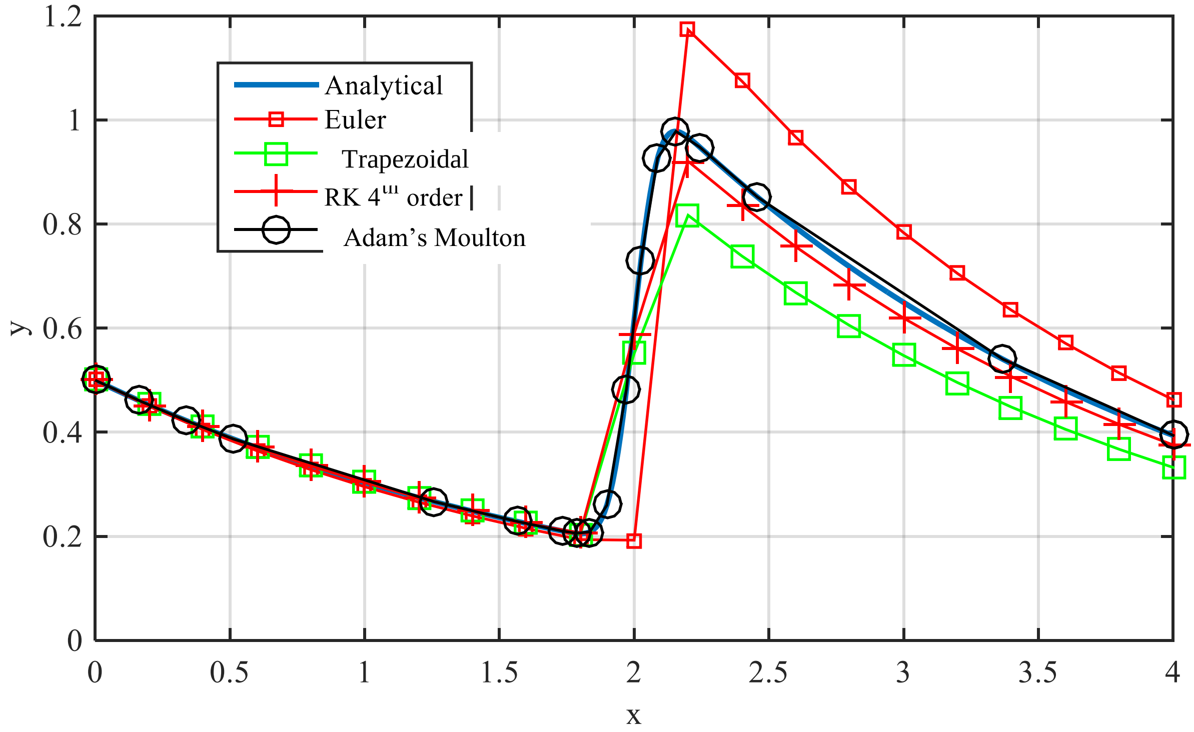
$$h_{\max} = 2.0$$

$$\alpha = 0.25$$

$$tol = 10^{-5}$$

## Sample output files

| x | y_analytical | y_eulerF | y_trapezoidal | y_RK4 |
|---|---|---|---|---|
| 0.00000 | 0.50000 | 0.50000 | 0.50000 | 0.50000 |
| 0.20000 | 0.45242 | 0.45000 | 0.45250 | 0.45242 |
| 0.40000 | 0.40937 | 0.40500 | 0.40951 | 0.40937 |
| 0.60000 | 0.37041 | 0.36450 | 0.37061 | 0.37041 |
| 0.80000 | 0.33516 | 0.32805 | 0.33540 | 0.33516 |
| 1.00000 | 0.30327 | 0.29525 | 0.30354 | 0.30327 |
| 1.20000 | 0.27441 | 0.26572 | 0.27470 | 0.27441 |
| 1.40000 | 0.24829 | 0.23915 | 0.24861 | 0.24829 |
| 1.60000 | 0.22466 | 0.21523 | 0.22499 | 0.22466 |
| 1.80000 | 0.20534 | 0.19371 | 0.20374 | 0.20642 |
| 2.00000 | 0.61483 | 0.19265 | 0.55135 | 0.58950 |
| 2.20000 | 0.96673 | 1.17339 | 0.81685 | 0.92054 |
| 2.40000 | 0.87663 | 1.07437 | 0.73845 | 0.83578 |
| 2.60000 | 0.79321 | 0.96693 | 0.66830 | 0.75625 |
| 2.80000 | 0.71773 | 0.87024 | 0.60481 | 0.68428 |
| 3.00000 | 0.64942 | 0.78321 | 0.54736 | 0.61916 |
| 3.20000 | 0.58762 | 0.70489 | 0.49536 | 0.56024 |
| 3.40000 | 0.53170 | 0.63440 | 0.44830 | 0.50693 |
| 3.60000 | 0.48111 | 0.57096 | 0.40571 | 0.45869 |
| 3.80000 | 0.43532 | 0.51387 | 0.36717 | 0.41504 |
| 4.00000 | 0.39390 | 0.46248 | 0.33229 | 0.37554 |



Analytical
Euler
Trapezoidal
RK 4th order
Adam's Moulton
1.2
1
0.8
0.6
0.4
0.2
0
y
0
0.5
1
1.5
2
2.5
3
3.5
4
x