

# 47MapReduce and PageRank

## Question 1:

Suppose our input data to a map-reduce operation consists of integer values (the keys are not important). The map function takes an integer  $i$  and produces the list of pairs  $(p, i)$  such that  $p$  is a prime divisor of  $i$ . For example,  $\text{map}(12) = [(2, 12), (3, 12)]$ .

The reduce function is addition. That is,  $\text{reduce}(p, [i_1, i_2, \dots, i_k])$  is  $(p, i_1 + i_2 + \dots + i_k)$ .

Compute the output, if the input is the set of integers 15, 21, 24, 30, 49.

prime no: 2, 3, 5, 7, 11, .....

15: [3, 15], [5, 15]

21: [3, 21], [7, 21]

24: [2, 24], [3, 24]

30: [2, 30], [3, 30], [5, 30]

49: [7, 49]

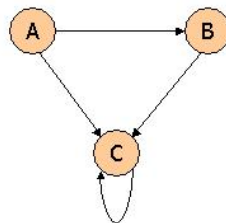
by combining all common elements part i.e compare left element and add rightmost element of that to get the solution.

[2, (24+30)], [3, (15+21+24+30)], [5, (15+30)], [7, (21+49)]

so, Output is ([2, 54], [3, 90], [5, 45], [7, 70])

## Question 2:

Consider three Web pages with the following links:



Suppose we compute PageRank with a  $\beta$  of 0.7, and we introduce the additional constraint that the sum of the PageRanks of the three pages must be 3, to handle the problem that otherwise any multiple of a solution will also be a solution. Compute the PageRanks  $a$ ,  $b$ , and  $c$  of the three pages A, B, and C, respectively.

Value of  $a$ ,  $b$ , or  $c$  as we iterate are:  $a$

All PageRank is multiplied by .7 before distribution, and .3 is then added to each new PageRank.

$$a = \beta(0) + (1-\beta) \rightarrow .3$$

$$b = \beta(a/2) + (1 - \beta) \rightarrow .7(a/2) + .3$$

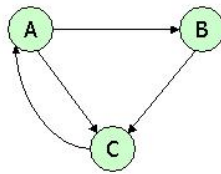
$$c = \beta(a/2 + b + c) + (1 - \beta) \rightarrow .7(a/2 + b + c) + .3$$

That immediately tells us  $a = .3$ . We can then use the second equation to discover  $b = .7(.3/2) + .3 = .405$ . Finally, the third equation simplifies to  $c = .7(.555 + c) + .3$ , or  $.3c = .6885$ . From this equation we get  $c = 2.295$

To compute the subs of each two of the variables:

$$a + b = .705, a + c = 2.595, \text{ and } b + c = 2.7$$

### Question 3:



Suppose we compute PageRank with  $\beta=0.85$ . Write the equations for the PageRanks  $a$ ,  $b$ , and  $c$  of the three pages A, B, and C, respectively.

We Know,

$$a = \beta * c + (1 - \beta) 1/3$$

$$b = \beta * a/2 + (1 - \beta) 1/3$$

$$c = \beta * (a/2 + b) + (1 - \beta)1/3$$

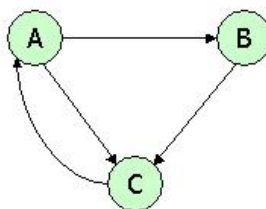
Here,

$$a = 0.85*c + (1 - 0.85) 1/3, a = 0.85c + 0.05$$

$$b = 0.85*0.5*a + 0.05, b = 0.425a + 0.05$$

$$c = 0.85*[0.5*a + b] + 0.05, c = 0.425a + 0.85b + 0.05$$

### Question 4:



Assuming no "taxation," compute the PageRanks  $a$ ,  $b$ , and  $c$  of the three pages A, B, and C, using iteration, starting with the "0th" iteration where all three pages have rank  $a = b = c = 1$ . Compute as far as the 5th iteration, and also determine what the PageRanks are in the limit

$$a = c$$

$$b = a/2$$

$$c = a/2 + b$$

$$\text{At 0th iteration: } a = 1; b = 1; c = 1$$

$$\text{At 1st iteration: } a = c = 1; b = 1/2; c = 1/2 + 1 = 3/2$$

$$\text{At 2nd iteration: } a = c = 3/2; b = a/2 = 1/2; c = 1/2 + 1/2 = 1$$

$$\text{At 3rd iteration: } a = c = 1; b = a/2 = (3/2)/2; c = 3/4 + 1/2 = 5/4$$

$$\text{At 4th iteration: } a = c = 5/4; b = a/2 = 1/2; c = 5/4$$

$$\text{At 5th iteration: } a = 5/4; b = 5/8; c = 9/8$$

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