

JEE Main 2020 Paper - 4th September 2020 — Shift 1 (Maths)

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- 1) Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x \cos x + \sin x)$, $x > 0$. If $y(\pi) = \pi$, then $y''(\frac{\pi}{2}) + y(\frac{\pi}{2})$ is equal to
 - a) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$
 - b) $2 + \frac{\pi}{2}$
 - c) $1 + \frac{\pi}{2}$
 - d) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$
- 2) The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:
 - a) ${}^{51}C_7 - {}^{30}C_7$
 - b) ${}^{51}C_7 + {}^{30}C_7$
 - c) ${}^{50}C_7 - {}^{30}C_7$
 - d) ${}^{50}C_6 - {}^{30}C_6$
- 3) Let $[t]$ denote the greatest integer $\leq t$. Then the equation in x , $[x]^2 + 2[x + 2] - 7 = 0$ has:
 - a) exactly four integral solutions
 - b) infinitely many solutions
 - c) no integral solution
 - d) exactly two solutions
- 4) Let $\mathbf{P}(3, 3)$ be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at \mathbf{P} intersects the x-axis at $(9, 0)$ and e is its eccentricity, then the ordered pair (a^2, e^2) is equal to:
 - a) $(9, 3)$
 - b) $(\frac{9}{2}, 2)$
 - c) $(\frac{9}{2}, 3)$
 - d) $(\frac{3}{2}, 2)$
- 5) Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) be a given ellipse, the length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to
 - a) 135
 - b) 116
 - c) 126
 - d) 145
- 6) Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$. Then $f(3) - f(1)$ is equal to:
 - a) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
 - b) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$
 - c) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
 - d) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$
- 7) If $1 + (1 - 2^2 \cdot 1) + (1 - 4^2 \cdot 3) + (1 - 6^2 \cdot 5) + \dots + (1 - 20^2 \cdot 19) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to:
 - a) $(10, 97)$

- b) (11, 103)
 c) (11, 97)
 d) (10, 103)
- 8) The integral $\int \left(\frac{x}{x \sin x + \cos x} \right)^2 dx$ is equal to (where C is a constant of integration):
 a) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$
 b) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$
 c) $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$
 d) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$
- 9) Let $f(x) = |x - 2|$ and $g(x) = f(f(x))$, $x \in [0, 4]$. Then $\int_0^3 (g(x) - f(x)) dx$ is equal to:
 a) $\frac{1}{2}$
 b) 0
 c) 1
 d) $\frac{3}{2}$
- 10) Let x_0 be the point of local maxima of $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$, where $\vec{a} = x\hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$, and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at $x = x_0$ is:
 a) -22
 b) -4
 c) -30
 d) 14
- 11) A triangle ABC lying in the first quadrant has two vertices as $A(1, 2)$ and $B(3, 1)$. If $\angle BAC = 90^\circ$, and $\text{ar}(\triangle ABC)$ is $5\sqrt{5}$ sq units, then the abscissa of the vertex C is:
 a) $1 + \sqrt{5}$
 b) $1 + 2\sqrt{5}$
 c) $2\sqrt{5} - 1$
 d) $2 + \sqrt{5}$
- 12) Let f be a twice differentiable function on $(1, 6)$. If $f(2) = 8$, $f'(2) = 5$, $f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in [1, 6]$, then:
 a) $f(5) + f'(5) \geq 28$
 b) $f'(5) + f''(5) \leq 20$
 c) $f(5) \leq 10$
 d) $f(5) + f'(5) \leq 26$
- 13) Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is:
 a) 33 : 31
 b) 9 : 7
 c) 3 : 1
 d) 5 : 3
- 14) Let $u = \frac{2z+i}{z-ki}$, $z = x + iy$ and $k > 0$. If the curve represented by $\text{Re}(u) + \text{Im}(u) = 1$ intersects the y-axis at the points P and Q where $PQ=5$, then the value of k is:
 a) 4
 b) $\frac{1}{2}$
 c) 2
 d) $\frac{3}{2}$
- 15) If $A = \begin{pmatrix} \cos \theta & i \sin \theta \\ i \sin \theta & \cos \theta \end{pmatrix}$, $(\theta = \frac{\pi}{24})$ and $A^5 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, where $i = \sqrt{-1}$, then which one of the following is not true?

a) $a^2 - d^2 = 0$

b) $a^2 - c^2 = 1$

c) $0 \leq a^2 + b^2 \leq 1$

d) $a^2 - b^2 = \frac{1}{2}$