JEE Main 2020 Paper - 4th September 2020 — Shift 1 (Maths)

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1) Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x\cos x + \sin x), x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to
   a) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$
   b) $2 + \frac{\pi}{2}$
   c) $1 + \frac{\pi}{2}$
   d) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$
2) The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:
   a) ${}^{51}C_7 - {}^{30}C_7$
   b) ${}^{51}C_7 + {}^{30}C_7$
   c) ${}^{50}C_7 - {}^{30}C_7$
   d) ${}^{50}C_6 - {}^{30}C_6$
3) Let $\lfloor t \rfloor$ denote the greatest integer $\leq t$. Then the equation in $x, \lfloor x \rfloor^2 + 2\lfloor x + 2 \rfloor - 7 = 0$ has:
   a) exactly four integral solutions
   b) infinitely many solutions
   c) no integral solution
   d) exactly two solutions
4) Let $\mathbf{P}(3,3)$ be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at $\mathbf{P}$ intersects the x-axis at $(9,0)$ and e is its eccentricity, then the ordered pair $\left(a^2, e^2\right)$ is equal to:
   a) $(9,3)$
   b) $\left(\frac{9}{2}, 2\right)$
   c) $\left(\frac{9}{2}, 3\right)$
   d) $\left(\frac{3}{2}, 2\right)$

5) Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ be a given ellipse, the length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to

   a) 135
   b) 116
   c) 126
   d) 145

6) Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2}\, dx$. Then $f(3) - f(1)$ is equal to:

   a) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
   b) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$
   c) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
   d) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

7) If $1 + \left(1 - 2^2 \cdot 1\right) + \left(1 - 4^2 \cdot 3\right) + \left(1 - 6^2 \cdot 5\right) + \cdots + \left(1 - 20^2 \cdot 19\right) = \alpha - 220\beta$, then an ordered pair (α, β) is equal to:

   a) $(10, 97)$
   b) $(11, 103)$
   c) $(11, 97)$
   d) $(10, 103)$

8) The integral $\int \left(\frac{x}{x \sin x + \cos x}\right)^2 dx$ is equal to (where C is a constant of integration):

   a) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + C$
   b) $\sec x + \frac{x \tan x}{x \sin x + \cos x} + C$
   c) $\sec x - \frac{x \tan x}{x \sin x + \cos x} + C$
   d) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + C$

9) Let $f(x) = |x - 2|$ and $g(x) = f(f(x)), x \in [0, 4]$. Then $\int_0^3 (g(x) - f(x))\, dx$ is equal to:

   a) $\frac{1}{2}$
   b) 0
   c) 1
   d) $\frac{3}{2}$

10) Let x_0 be the point of local maxima of $f(x) = \overrightarrow{a} \cdot \left(\overrightarrow{b} \times \overrightarrow{c}\right)$, where $\overrightarrow{a} = x\hat{i} + 2\hat{j} + 3\hat{k}, \overrightarrow{b} = -2\hat{i} + x\hat{j} - \hat{k}$, and $\overrightarrow{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\overrightarrow{a} \cdot \overrightarrow{b} + \overrightarrow{b} \cdot \overrightarrow{c} + \overrightarrow{c} \cdot \overrightarrow{a}$ at $x = x_0$ is:

a) -22

b) -4

c) -30

d) 14

11) A triangle ABC lying in the first quadrant has two vertices as $\mathbf{A}(1,2)$ and $\mathbf{B}(3,1)$. If $\angle BAC = 90°$, and ar($\triangle ABC$) is $5\sqrt{5}$ s units, then the abscissa of the vertex $\mathbf{C}$ is:

a) $1 + \sqrt{5}$

b) $1 + 2\sqrt{5}$

c) $2\sqrt{5} - 1$

d) $2 + \sqrt{5}$

12) Let f be a twice differentiable function on $(1,6)$. If $f(2) = 8, f'(2) = 5, f'(x) \geq 1$ and $f''(x) \geq 4$, for all $x \in [1,6]$, then:

a) $f(5) + f'(5) \geq 28$

b) $f'(5) + f''(5) \leq 20$

c) $f(5) \leq 10$

d) $f(5) + f'(5) \leq 26$

13) Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is:

a) 33 :31

b) 9 :7

c) 3 :1

d) 5 :3

14) Let $u = \frac{2z+i}{z-ki}, z = x + iy$ and $k > 0$. If the curve represented by $Re(u) + Im(u) = 1$ intersects the y-axis at the points $\mathbf{P}$ and $\mathbf{Q}$ where $PQ = 5$, then the value of k is:

a) 4

b) $\frac{1}{2}$

c) 2

d) $\frac{3}{2}$

15) If $A = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}, \left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,where $i = \sqrt{-1}$,then which one of the following is not true?

a) $a^2 - d^2 = 0$

b) $a^2 - c^2 = 1$

c) $0 \leq a^2 + b^2 \leq 1$

d) $a^2 - b^2 = \frac{1}{2}$