

matgeo-9.3.18

EE22BTECH11009 - Mokshith Kumar

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Question

Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$. 9.3.18

Solution:

The given circle can be expressed as conics with parameters,

$$V = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, u = 0, f = -9. \quad (1)$$

The line parameters are:

$$h = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}, m = \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \quad (2)$$

The points of intersection of the line

$$L: \quad x = h + \kappa m \quad \kappa \in \mathbb{R} \quad (3)$$

with the conic section

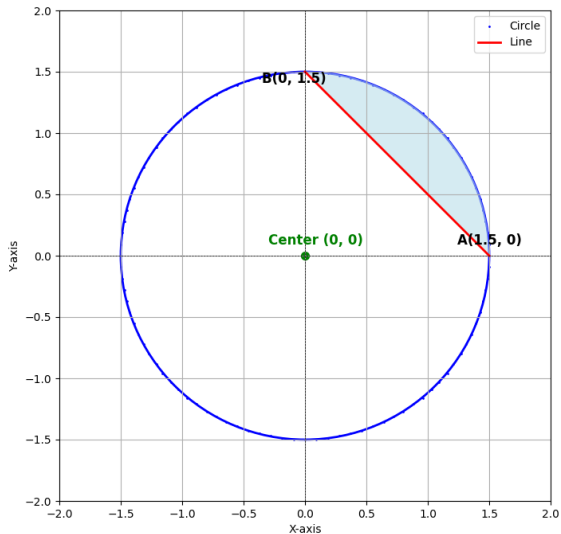
$$g(x) = x^T V x + 2u^T x + f = 0 \quad (4)$$

are given by

$$x_i = h + \kappa_i m \quad (5)$$

where,

Figure



Table

Variable	Description	Value
V	$\ n\ ^2 I - e^2 n n^\top$	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$
u	$c e^2 n - \ n\ ^2 F$	0
f	$\ n\ ^2 \ F\ ^2 - c^2 e^2$	-9
h	Point on the line	$\begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}$
m	slope vector of the line	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
k_i	varying parameter of the line	1.5, 0
A	First points of intersection	$\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$
B	Second point of intersection	$\begin{pmatrix} 0 \\ 1.5 \end{pmatrix}$

Table: Parameters used