

# 3-9.3.18

EE24BTECH11009 - Mokshith Kumar Reddy

**Question:**

Using integration, find the area of the smaller region enclosed by the curve  $4x^2 + 4y^2 = 9$  and the line  $2x + 2y = 3$ .

**Solution:**

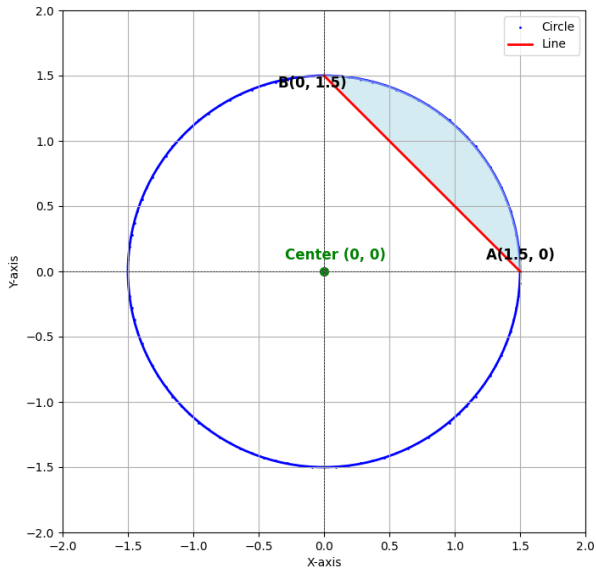


Fig. 0.1

The given circle can be expressed as conics with parameters,

$$V = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, u = 0, f = -9. \quad (0.1)$$

The line parameters are:

$$h = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}, m = \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \quad (0.2)$$

The points of intersection of the line

$$L : \quad x = h + \kappa m \quad \kappa \in \mathbb{R} \quad (0.3)$$

with the conic section

$$g(x) = x^\top Vx + 2u^\top x + f = 0 \quad (0.4)$$

are given by

$$x_i = h + \kappa_i m \quad (0.5)$$

where,

$$\kappa_i = \frac{1}{m^\top Vm} \left( -m^\top (Vh + u) \pm \sqrt{[m^\top (Vh + u)]^2 - g(h)(m^\top Vm)} \right) \quad (0.6)$$

Substituting the parameters in above Equation:

$$k = 0, \frac{3}{2}. \quad (0.7)$$

yielding the points of intersection as:

$$A = \begin{pmatrix} \frac{3}{2} \\ 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \quad (0.8)$$

From ,Fig. 0.1the desired area is:

$$\int_0^{\frac{3}{2}} \frac{\sqrt{9-4x^2}}{2} - \int_0^{\frac{3}{2}} \frac{3-2x}{2} = \frac{9}{16}\pi - \frac{9}{8} \quad (0.9)$$

| Variable | Description                   | Value  |
|----------|-------------------------------|--|
| $V$      | $\ n\ ^2 I - e^2 n n^\top$    | $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$   |
| $u$      | $c e^2 n - \ n\ ^2 F$         | 0  |
| $f$      | $\ n\ ^2 \ F\ ^2 - c^2 e^2$   | -9   |
| $h$      | Point on the line             | $\begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}$ |
| $m$      | slope vector of the line      | $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$          |
| $k_i$    | varying parameter of the line | 1.5, 0   |
| $A$      | First points of intersection  | $\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$         |
| $B$      | Second point of intersection  | $\begin{pmatrix} 0 \\ 1.5 \end{pmatrix}$         |

TABLE 0: 3.1