

# 2024-April Session-06-04-2024 shift<sup>1</sup>

## 2

EE24BTECH11009-Mokshith

- 16) if three letters can be posted to any one of the 5 different addresses, then the probability that the three are posted to exactly two addresses is:
- $\frac{12}{25}$
  - $\frac{4}{25}$
  - $\frac{6}{25}$
  - $\frac{18}{25}$
- 17) If the locus of the point, whose distances from the point (2, 1) and (1, 3) are in the ratio 5 : 4, is  $ax^2 + by^2 + cxy + dx + ey + 170 = 0$ , then the value of  $a^2 + 2b + 3c + 4d + e$  is equal to:
- 5
  - 27
  - 437
  - 37
- 18) A software company sets up  $m$  number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of  $m$  is equal to:
- 180
  - 125
  - 150
  - 160
- 19) If  $\int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{1}{12} \tan^{-1}(3 \tan x) + \text{constant}$ , then the maximum value of  $a \sin x + b \cos x$ , is:
- $\sqrt{42}$
  - $\sqrt{39}$
  - $\sqrt{41}$
  - $\sqrt{40}$
- 20) Let  $0 \leq r \leq n$ . If  ${}^{n+1}C_{r+1} : {}^nC_r : {}^{n-1}C_{r-1} = 55 : 35 : 21$ , then  $2n + 5r$  is equal to:
- 62
  - 60
  - 55
  - 50
- 21) If the shortest distance between the lines  $\frac{x-1}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$  and  $\frac{x+2}{23} = \frac{y+5}{2} = \frac{z-4}{4}$  is  $\frac{44}{\sqrt{30}}$ , then the largest possible value of  $|\lambda|$  is equal to:

- 22) Let  $[t]$  denote the largest integer less than or equal to  $t$ . If  $\int_0^3 \left( [x^2] + \left[ \frac{x^2}{2} \right] \right) dx = a + b\sqrt{2} - \sqrt{3} - \sqrt{5} + c\sqrt{6} - \sqrt{7}$ , where  $a, b, c \in \mathbb{Z}$ , then  $a + b + c$  is equal to:
- 23) Let  $\alpha, \beta$  be roots of  $x^2 + \sqrt{2}x - 8 = 0$ . If  $U_n = \alpha^n + \beta^n$ , then  $\frac{U_{10} + \sqrt{2}U_9}{2U_8}$  is equal to:
- 24) In a triangle  $ABC$ ,  $BC = 7$ ,  $AC = 8$ ,  $AB = \alpha \in \mathbb{N}$  and  $\cos A = \frac{2}{3}$ . If  $49 \cos(3C) + 42 = \frac{m}{n}$ , where  $\gcd(m, n) = 1$ , then  $m + n$  is equal to:
- 25) The length of the latus rectum and directrices of a hyperbola with eccentricity  $e$  are 9 and  $x = \pm \frac{4}{\sqrt{3}}$ , respectively. Let the line  $y - \sqrt{3}x + \sqrt{3} = 0$  touch this hyperbola at  $(x_0, y_0)$ . If  $m$  is the product of the focal distances of the point  $(x_0, y_0)$ , then  $4e^2 + m$  is equal to:
- 26) If  $S(x) = (1+x) + 2(1+x)^2 + 3(1+x)^3 + \dots + 60(1+x)^{60}$  and  $(60)^2 S(60) = a(b)^b + b$ , where  $a, b \in \mathbb{N}$ , then  $a + b$  is equal to:
- 27) If the system of equations

$$2x + 7y + \lambda z = 3$$

$$3x + 2y + 5z = 4$$

$$x + \mu y + 32z = -1$$

has infinitely many solutions, then  $(\lambda - \mu)$  is equal to:

- 28) Let  $[t]$  denote the greatest integer less than or equal to  $t$ . Let  $f: [0, \infty) \rightarrow \mathbb{R}$  be a function defined by  $f(x) = \left[ \frac{x}{2} + 3 \right] - \left[ \sqrt{x} \right]$ . Let  $S$  be the set of all points in the interval  $[0, 8]$  at which  $f$  is not continuous. Then  $\sum_{a \in S} a$  is equal to:
- 29) If the solution  $y(x)$  of the given differential equation  $(e^y + 1) \cos x dx + e^y \sin x dy = 0$  passes through the point  $\left( \frac{\pi}{2}, 0 \right)$ , then the value of  $e^{y\left(\frac{\pi}{6}\right)}$  is equal to:
- 30) From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable  $X$  denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of  $X$  is  $\frac{m}{n}$ , where  $\gcd(m, n) = 1$ , then  $n - m$  is equal to: