2021-XE-'14-26'

EE24BTECH11009-Mokshith

4) Let $f(x)$ be a non-negative continuous function of real variable x. If the area under the curve $y = f(x)$ from $x = 0$ to $x = a$ is $\frac{a^2}{2} + \frac{\pi}{2}\sin a + \frac{\pi}{2}\cos a - \frac{\pi}{2}$, then the value of $f\left(\frac{\pi}{2}\right)$ is _____(round off to one decimal place).
5) If the numerical approximation of the value of the integral $\int_0^4 2^{\alpha x} dx$ using the Trapezoidal rule with two sub intervals is 9, then the value of the real constant α is _____(round off to one decimal place).
6) Let the transformation $y(x) = e^x v(x)$ reduce the ordinary differential equation

$$x\frac{d^2y}{dx^2} + 2(1-x)\frac{dy}{dx} + (x-2)y = 0; \quad x > 0$$

to

$$\alpha x\frac{d^2v}{dx^2} + \beta\frac{dv}{dx} + \gamma v = 0,$$

where α, β, γ are real constants. Then, the arithmetic mean of α, β, γ is _____(round off to three decimal places).
7) A person, who speaks the truth 3 out of 4 times, throws a fair dice with six faces and informs that the outcome is 5. The probability that the outcome is really 5 _____(round off to three decimal places).
8) Let $f(x,y) = x^4 + y^4 - 2x^2 + 4xy - 2y^2 + \alpha$ be a real valued function. Then, which one of the following statements is TRUE for all α?
   a) $(0,0)$ is not a stationary point of f
   b) f has a local maxima at $(0,0)$
   c) f has a local minima at $(0,0)$
   d) f has a saddle point at $(0,0)$
9) Let $u(x,y) = \left(x^2 - y^2\right)v(x,y)$ be such that both $u(x,y)$ and $v(x,y)$ satisfy the Laplace equation in a domain Ω of the xy-plane. Then, which one of the following is TRUE in Ω?
   a) $x\frac{\partial v}{\partial x} - y\frac{\partial v}{\partial y} = 0$
   b) $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 0$
   c) $x\frac{\partial v}{\partial y} - y\frac{\partial v}{\partial x} = 0$
   d) $x\frac{\partial v}{\partial y} + y\frac{\partial v}{\partial x} = 0$
10) Let I denote the identity matrix of order 7, and A be a 7×7 real matrix having characteristic polynomial $C_A(\lambda) = \lambda^2(\lambda-1)^\alpha(\lambda+2)^\beta$, where α and β are positive integers. If A is diagonalizable and $rank(A) = rank(A+2I)$, then $rank(A-I)$ is _____(in integer).
11) Let C_1 be the line segment from $(0,1)$ to $\left(\frac{4}{5}, \frac{3}{5}\right)$, and let C_2 be the arc of the circle

$x^2 + y^2 = 1$ from $(0, 1)$ to $\left(\frac{4}{5}, \frac{3}{5}\right)$. If

$$\alpha = \int_{C_1} \left(\frac{2x}{y} i + \frac{1 - x^2}{y^2} j\right) \cdot dr \text{ and } \beta = \int_{C_2} \left(\frac{2x}{y} i + \frac{1 - x^2}{y^2} j\right) \cdot dr,$$

where $r = xi + yj$, then the value of $\alpha^2 + \beta^2$ is _____(round off to two decimal places).

Fluid Mechanics (XE-B)

1) The general relationship between shear stress, τ, and the velocity gradient (du/dy) for a fluid is given by $\tau = k\,(du/dy)^n$, where k is a constant with appropriate units. The fluid is Newtonian if
   a) $n > 1$
   b) $n < 1$
   c) $n = 1$
   d) $n = 0$
2) Which one of the following options is TRUE?
   a) Pathlines and streaklines are the same in an unsteady flow, and streamlines are tangential to the local fluid velocity at a point.
   b) Streamlines are perpendicular to the local fluid velocity at a point, and streamlines and streaklines are the same in a steady flow.
   c) Pathlines and streaklines are the same in an unsteady flow, and streamlines and streaklines are the same in a steady flow.
   d) Streamlines are tangential to the local fluid velocity at a point, and streamlines and streaklines are the same in a steady flow.
3) If $P_m = 1.2Pa$ and $P_{out} = 1.0Pa$ are the average pressures at inlet and outlet respectively for a fully-developed flow inside a channel having a height of $50cm$, then the absolute value of average shear stress $(inPa)$ acting on the walls of the channel of length 5 m is
   a) 0.005
   b) 0.02
   c) 0.01
   d) 0.05
4) Consider the fully-developed flow of a Newtonian fluid (density ρ; viscosity μ) through a smooth pipe of diameter D and length L. The average velocity of the flow is V. If the length of the pipe is doubled, keeping V, D, ρ, μ constant, the friction factor
   a) increases by two times
   b) remains the same
   c) decreases by two times
   d) increases by four times
5) The absolute value of pressure difference between the inside and outside of a spherical soap bubble of radius, R, and surface tension, γ, is:
   a) $\frac{2\gamma}{R}$

b) $\frac{\gamma}{R}$
c) $\frac{\gamma}{2R}$
d) $\frac{4\gamma}{R}$