3-9.3.18

EE24BTECH11009 - Mokshith Kumar Reddy

Question:
Using integration, find the area of the smaller region enclosed by the curve $4x2 + 4y2 = 9$ and the line $2x + 2y = 3$.
Solution:

Variable	Description
V, u, f	Parameters of Circle
h, m, k_i	Parameters of Line
A, B	points of intersection

TABLE 0: 3.1

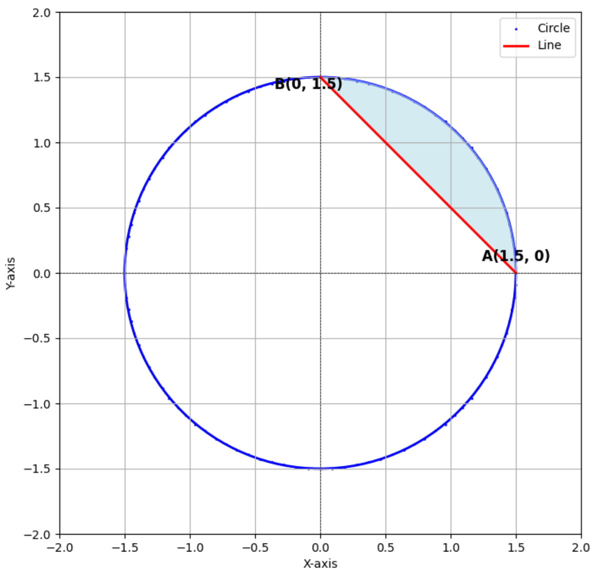

Fig. 0.1

The given circle can be expressed as conics with parameters,

$$V = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, u = 0, f = -9. \tag{0.1}$$

$$\tag{0.2}$$

The line parameters are:

$$h = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}, m = \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{0.3}$$

$$\kappa_i = \frac{1}{\mathbf{m}^\top \mathbf{V} \mathbf{m}} \left(-\mathbf{m}^\top \left(\mathbf{V} \mathbf{h} + \mathbf{u} \right) \pm \sqrt{\left[\mathbf{m}^\top \left(\mathbf{V} \mathbf{h} + \mathbf{u} \right) \right]^2 - g \left(\mathbf{h} \right) \left(\mathbf{m}^\top \mathbf{V} \mathbf{m} \right)} \right) \tag{0.4}$$

Substituting the parameters in above Equation:

$$k = 0, \frac{3}{2}. \tag{0.5}$$

$$\tag{0.6}$$

yielding the points of intersection as:

$$A = \begin{pmatrix} \frac{3}{2} \\ 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \tag{0.7}$$

From ,Fig. 0.1the desired area is:

$$\int_0^{\frac{3}{2}} \frac{\sqrt{9 - 4x^2}}{2} - \int_0^{\frac{3}{2}} \frac{3 - 2x}{2} = \frac{9}{16}\pi - \frac{9}{8} \tag{0.8}$$

$$\tag{0.9}$$