Presentation Template

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Question:

Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$. 9.3.18

Table for Reference

Variable	Description	Value
V	$\|n\|^2 I - e^2 n n^\top$	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$
u	$ce^2 n - \|n\|^2 F$	0
f	$\|n\|^2 \|F\|^2 - c^2 e^2$	-9
h	Point on the line	$\begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}$
m	slope vector of the line	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
k_i	varying parameter of the line	$1.5, 0$
A	First points of intersection	$\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$
B	Second point of intersection	$\begin{pmatrix} 0 \\ 1.5 \end{pmatrix}$

Table: Parameters used

Given Parameters of Conics

The given circle can be expressed as conics with parameters,

$$V = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, u = 0, f = -9. \tag{1}$$

The line parameters are:

$$h = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}, m = \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{2}$$

Formulae for Finding Point of Intersection

The points of intersection of the line

$$L : \quad x = h + \kappa m \quad \kappa \in \mathbb{R} \tag{3}$$

with the conic section

$$\mathrm{g}\left(x\right) = x^{\top} V x + 2u^{\top} x + f = 0 \tag{4}$$

are given by

$$x_i = h + \kappa_i m \tag{5}$$

where,

$$\kappa_i = \frac{1}{m^{\top} V m} \left(-m^{\top} (Vh + u)\right) \pm \sqrt{\left(m^{\top} (Vh + u)\right)^2 - g(h) \cdot (m^{\top} V m)} \tag{6}$$

Finding Area of intersection

Substituting the parameters in the above equation yields:

$$k = 0, \frac{3}{2}. \tag{7}$$

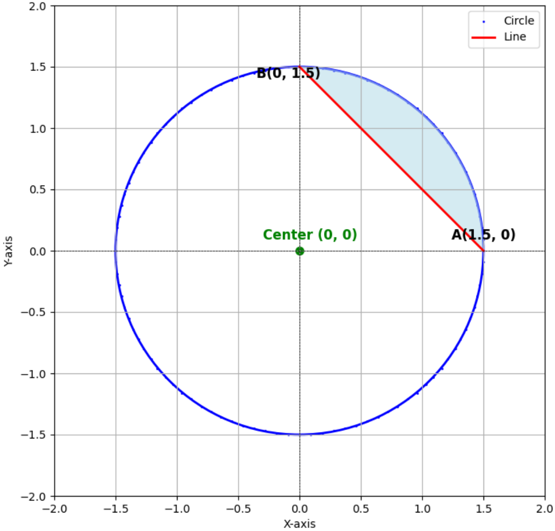
This gives the points of intersection as:

$$A = \begin{pmatrix} \frac{3}{2} \\ 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}. \tag{8}$$

The desired area is:

$$\int_0^{\frac{3}{2}} \frac{\sqrt{9 - 4x^2}}{2} - \int_0^{\frac{3}{2}} \frac{3 - 2x}{2} = \frac{9}{16}\pi - \frac{9}{8}. \tag{9}$$

Figure



Circle
Line
B(0, 1.5)
Center (0, 0)
A(1.5, 0)
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
Y-axis
−2.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
2.0
X-axis

Code for Finding Points

```
import sympy as sp

x, y = sp.symbols('x y')

circle_eq = sp.Eq(4*x**2 + 4*y**2, 9)
line_eq = sp.Eq(2*x + 2*y, 3)

y_expr = sp.solve(line_eq, y)[0]

circle_substituted = circle_eq.subs(y, y_expr)

x_solutions = sp.solve(circle_substituted, x)

intersection_points = []
for x_val in x_solutions:
    y_val = y_expr.subs(x, x_val)
    intersection_points.append((x_val, y_val))

print(intersection_points)
```

Figure