JEE Main 2020 Paper - 4th September 2020 — Shift 1 (Maths)

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1) Let $y = y(x)$ be the solution of the differential equation, $xy' - y = x^2(x\cos x + \sin x), x > 0$. If $y(\pi) = \pi$, then $y''\left(\frac{\pi}{2}\right) + y\left(\frac{\pi}{2}\right)$ is equal to
   a) $2 + \frac{\pi}{2} + \frac{\pi^2}{4}$
   b) $2 + \frac{\pi}{2}$
   c) $1 + \frac{\pi}{2}$
   d) $1 + \frac{\pi}{2} + \frac{\pi^2}{4}$
2) The value of $\sum_{r=0}^{20} {}^{50-r}C_6$ is equal to:
   a) ${}^{51}C_7 - {}^{30}C_7$
   b) ${}^{51}C_7 + {}^{30}C_7$
   c) ${}^{50}C_7 - {}^{30}C_7$
   d) ${}^{50}C_6 - {}^{30}C_6$
3) Let $\lfloor t \rfloor$ denote the greatest integer $\leq t$. Then the equation in $x, \lfloor x \rfloor^2 + 2\lfloor x+2 \rfloor - 7 = 0$ has:
   a) exactly four integral solutions
   b) infinitely many solutions
   c) no integral solution
   d) exactly two solutions
4) Let $\mathbf{P}(3,3)$ be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at $\mathbf{P}$ intersects the x-axis at $(9,0)$ and e is its eccentricity, then the ordered pair $\left(a^2, e^2\right)$ is equal to:
   a) $(9,3)$
   b) $\left(\frac{9}{2}, 2\right)$
   c) $\left(\frac{9}{2}, 3\right)$
   d) $\left(\frac{3}{2}, 2\right)$
5) Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ be a given ellipse, the length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to
   a) 135
   b) 116
   c) 126
   d) 145
6) Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$. Then $f(3) - f(1)$ is equal to:
   a) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
   b) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

c) $-\frac{\pi}{12}+\frac{1}{2}+\frac{\sqrt{3}}{4}$
d) $\frac{\pi}{12}+\frac{1}{2}-\frac{\sqrt{3}}{4}$

7) If $1+\left(1-2^2\cdot 1\right)+\left(1-4^2\cdot 3\right)+\left(1-6^2\cdot 5\right)+\cdots+\left(1-20^2\cdot 19\right)=\alpha-220\beta$, then an ordered pair (α,β) is equal to:
a) $(10,97)$
b) $(11,103)$
c) $(11,97)$
d) $(10,103)$

8) The integral $\int\left(\frac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to (where C is a constant of integration):
a) $\tan x-\frac{x\sec x}{x\sin x+\cos x}+C$
b) $\sec x+\frac{x\tan x}{x\sin x+\cos x}+C$
c) $\sec x-\frac{x\tan x}{x\sin x+\cos x}+C$
d) $\tan x+\frac{x\sec x}{x\sin x+\cos x}+C$

9) Let $f(x)=|x-2|$ and $g(x)=f(f(x)), x\in[0,4]$. Then $\int_0^3(g(x)-f(x))\,dx$ is equal to:
a) $\frac{1}{2}$
b) 0
c) 1
d) $\frac{3}{2}$

10) Let x_0 be the point of local maxima of $f(x)=\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$, where $\vec{a}=x\hat{i}+2\hat{j}+3\hat{k}, \vec{b}=-2\hat{i}+x\hat{j}-\hat{k}$, and $\vec{c}=7\hat{i}-2\hat{j}+x\hat{k}$. Then the value of $\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{c}+\vec{c}\cdot\vec{a}$ at $x=x_0$ is:
a) -22
b) -4
c) -30
d) 14

11) A triangle ABC lying in the first quadrant has two vertices as $\mathbf{A}(1,2)$ and $\mathbf{B}(3,1)$. If $\angle BAC=90^\circ$, and ar($\triangle ABC$) is $5\sqrt{5}$ s units, then the abscissa of the vertex $\mathbf{C}$ is:
a) $1+\sqrt{5}$
b) $1+2\sqrt{5}$
c) $2\sqrt{5}-1$
d) $2+\sqrt{5}$

12) Let f be a twice differentiable function on $(1,6)$. If $f(2)=8, f'(2)=5, f'(x)\geq 1$ and $f''(x)\geq 4$, for all $x\in[1,6]$, then:
a) $f(5)+f'(5)\geq 28$
b) $f'(5)+f''(5)\leq 20$
c) $f(5)\leq 10$
d) $f(5)+f'(5)\leq 26$

13) Let α and β be the roots of $x^2-3x+p=0$ and γ and δ be the roots of $x^2-6x+q=0$. If $\alpha,\beta,\gamma,\delta$ form a geometric progression. Then ratio $(2q+p):(2q-p)$ is:
a) 33 :31

b) 9 :7
c) 3 :1
d) 5 :3

14) Let $u = \frac{2z+i}{z-ki}, z = x + iy$ and $k > 0$. If the curve represented by $Re(u) + Im(u) = 1$ intersects the y-axis at the points **P** and **Q** where $PQ = 5$, then the value of k is:
a) 4
b) $\frac{1}{2}$
c) 2
d) $\frac{3}{2}$

15) If $A = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}, \left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,where $i = \sqrt{-1}$,then which one of the following is not true?
a) $a^2 - d^2 = 0$
b) $a^2 - c^2 = 1$
c) $0 \le a^2 + b^2 \le 1$
d) $a^2 - b^2 = \frac{1}{2}$