

Presentation Template

Mokshith Kumar Reddy
Dept. of Electrical Engg.,
IIT Hyderabad.

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Question:

Using integration, find the area of the smaller region enclosed by the curve $4x^2 + 4y^2 = 9$ and the line $2x + 2y = 3$. 9.3.18

Table for Reference

Variable	Description	Value
V	$\ n\ ^2 I - e^2 n n^\top$	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$
u	$c e^2 n - \ n\ ^2 F$	0
f	$\ n\ ^2 \ F\ ^2 - c^2 e^2$	-9
h	Point on the line	$\begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}$
m	slope vector of the line	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
k_i	varying parameter of the line	1.5, 0
A	First points of intersection	$\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$
B	Second point of intersection	$\begin{pmatrix} 0 \\ 1.5 \end{pmatrix}$

Table: Parameters used

Given Parameters of Conics

The given circle can be expressed as conics with parameters,

$$V = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, u = 0, f = -9. \quad (1)$$

The line parameters are:

$$h = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}, m = \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \quad (2)$$

Formulae for Finding Point of Intersection

The points of intersection of the line

$$L : \quad x = h + \kappa m \quad \kappa \in \mathbb{R} \quad (3)$$

with the conic section

$$g(x) = x^\top Vx + 2u^\top x + f = 0 \quad (4)$$

are given by

$$x_i = h + \kappa_i m \quad (5)$$

where,

$$\begin{aligned} \kappa_i = \frac{1}{m^\top Vm} \left(-m^\top (Vh + u) \right) \\ \pm \sqrt{(m^\top (Vh + u))^2 - g(h) \cdot (m^\top Vm)} \end{aligned} \quad (6)$$

Finding Area of intersection

Substituting the parameters in the above equation yields:

$$k = 0, \frac{3}{2}. \quad (7)$$

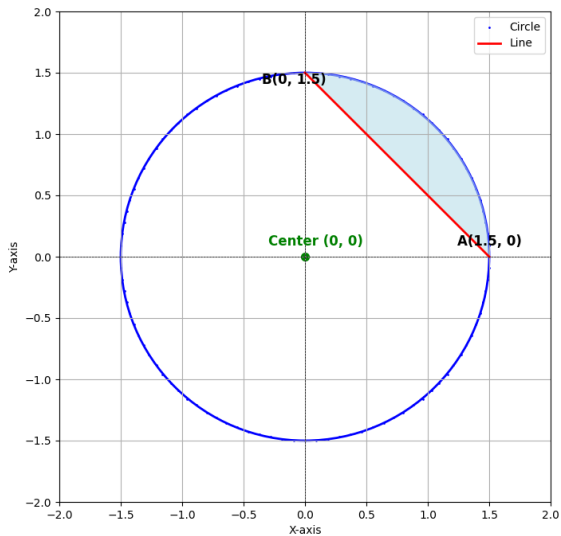
This gives the points of intersection as:

$$A = \left(\frac{3}{2}, 0 \right), B = \left(0, \frac{3}{2} \right). \quad (8)$$

The desired area is:

$$\int_0^{\frac{3}{2}} \frac{\sqrt{9 - 4x^2}}{2} - \int_0^{\frac{3}{2}} \frac{3 - 2x}{2} = \frac{9}{16}\pi - \frac{9}{8}. \quad (9)$$

Figure



Code for Finding Points

```
1 import sympy as sp
2
3 x, y = sp.symbols('x y')
4
5 circle_eq = sp.Eq(4*x**2 + 4*y**2, 9)
6 line_eq = sp.Eq(2*x + 2*y, 3)
7
8 y_expr = sp.solve(line_eq, y)[0]
9
10 circle_substituted = circle_eq.subs(y, y_expr)
11
12 x_solutions = sp.solve(circle_substituted, x)
13
14 intersection_points = []
15 for x_val in x_solutions:
16     y_val = y_expr.subs(x, x_val)
17     intersection_points.append((x_val, y_val))
18
19 print(intersection_points)
20
```

Figure