3-9.3.18

EE24BTECH11009 - Mokshith Kumar Reddy

Question:
Using integration, find the area of the smaller region enclosed by the curve $4x2 + 4y2 = 9$ and the line $2x + 2y = 3$.
Solution:

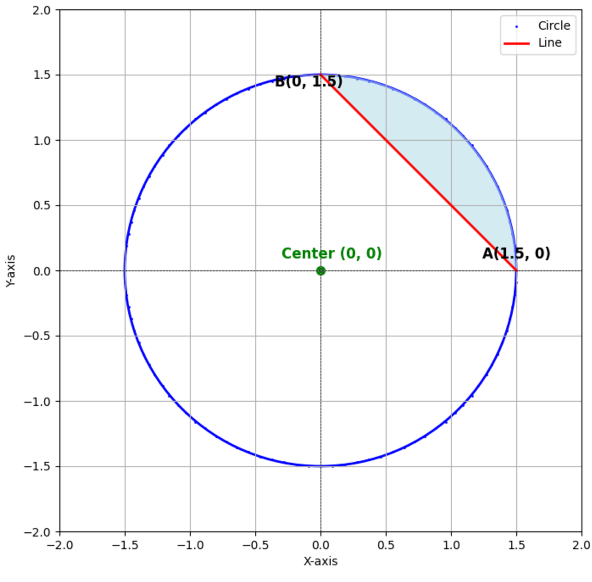

Fig. 0.1

The given circle can be expressed as conics with parameters,

$$V = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}, u = 0, f = -9. \tag{0.1}$$

$$\tag{0.2}$$

The line parameters are:

$$h = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}, m = \begin{pmatrix} 1 \\ -1 \end{pmatrix}. \tag{0.3}$$

The points of intersection of the line

$$L : \quad x = h + \kappa m \quad \kappa \in \mathbb{R} \tag{0.4}$$

with the conic section

$$\mathrm{g}(x) = x^\top V x + 2u^\top x + f = 0 \tag{0.5}$$

are given by

$$x_i = h + \kappa_i m \tag{0.6}$$

where,

$$\kappa_i = \frac{1}{m^\top V m}\left(-m^\top (Vh + u) \pm \sqrt{\left[m^\top (Vh + u)\right]^2 - \mathrm{g}(h)\left(m^\top V m\right)}\right) \tag{0.7}$$

Substituting the parameters in above Equation:

$$k = 0, \frac{3}{2}. \tag{0.8}$$

$$\tag{0.9}$$

yielding the points of intersection as:

$$A = \begin{pmatrix} \frac{3}{2} \\ 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix} \tag{0.10}$$

From ,Fig. 0.1the desired area is:

$$\int_0^{\frac{3}{2}} \frac{\sqrt{9 - 4x^2}}{2} - \int_0^{\frac{3}{2}} \frac{3 - 2x}{2} = \frac{9}{16}\pi - \frac{9}{8} \tag{0.11}$$

$$\tag{0.12}$$

Variable	Description	Value
V	$\lVert n \rVert^2 I - e^2 n n^\top$	$\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$
u	$ce^2 n - \lVert n \rVert^2 F$	0
f	$\lVert n \rVert^2 \lVert F \rVert^2 - c^2 e^2$	-9
h	Point on the line	$\begin{pmatrix} 0 \\ \frac{3}{2} \end{pmatrix}$
m	slope vector of the line	$\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
k_i	varying parameter of the line	$1.5, 0$
A	First points of intersection	$\begin{pmatrix} 1.5 \\ 0 \end{pmatrix}$
B	Second point of intersection	$\begin{pmatrix} 0 \\ 1.5 \end{pmatrix}$

TABLE 0: 3.1