JEE Main 2020 Paper - 4th September 2020 — Shift 1 (Maths)

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1) If $A = \begin{pmatrix} \cos\theta & i\sin\theta \\ i\sin\theta & \cos\theta \end{pmatrix}, \left(\theta = \frac{\pi}{24}\right)$ and $A^5 = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,where $i = \sqrt{-1}$,then which one of the following is not true?
   a) $a^2 - d^2 = 0$
   b) $a^2 - c^2 = 1$
   c) $0 \le a^2 + b^2 \le 1$
   d) $a^2 - b^2 = \frac{1}{2}$

2) Let $\lfloor t \rfloor$ denote the greatest integer $\le t$. Then the equation in $x, \lfloor x \rfloor^2 + 2\lfloor x + 2 \rfloor - 7 = 0$ has:
   a) exactly four integral solutions
   b) infinitely many solutions
   c) no integral solution
   d) exactly two solutions

3) Let α and β be the roots of $x^2 - 3x + p = 0$ and γ and δ be the roots of $x^2 - 6x + q = 0$. If $\alpha, \beta, \gamma, \delta$ form a geometric progression. Then ratio $(2q + p) : (2q - p)$ is:
   a) 33 :31
   b) 9 :7
   c) 3 :1
   d) 5 :3

4) Let $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $(a > b)$ be a given ellipse, the length of whose latus rectum is 10. If its eccentricity is the maximum value of the function, $\phi(t) = \frac{5}{12} + t - t^2$, then $a^2 + b^2$ is equal to
   a) 135
   b) 116
   c) 126
   d) 145

5) A triangle ABC lying in the first quadrant has two vertices as $\mathbf{A}(1, 2)$ and $\mathbf{B}(3, 1)$. If $\angle BAC = 90°$, and ar$(\triangle ABC)$ is $5\sqrt{5}$ s units, then the abscissa of the vertex $\mathbf{C}$ is:
   a) $1 + \sqrt{5}$
   b) $1 + 2\sqrt{5}$
   c) $2\sqrt{5} - 1$
   d) $2 + \sqrt{5}$

6) Let $f(x) = |x - 2|$ and $g(x) = f(f(x)), x \in [0, 4]$. Then $\int_0^3 (g(x) - f(x))\,dx$ is equal to:
   a) $\frac{1}{2}$

b) 0
c) 1
d) $\frac{3}{2}$

7) Given the following two statements :
$(S_1) : (q \vee p) \rightarrow (p \leftrightarrow \sim q)$ is a tautology.
$(S_2) : \sim q \wedge (\sim p \leftrightarrow q)$ is a fallacy.
Then :

a) only (S_1) is correct.
b) both (S_1) and (S_2) are correct.
c) both (S_1) and (S_2) are not correct.
d) only (S_2) is correct.

8) Let $\mathbf{P}(3,3)$ be a point on the hyperbola, $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal to it at $\mathbf{P}$ intersects the x-axis at $(9,0)$ and e is its eccentricity, then the ordered pair $\left(a^2, e^2\right)$ is equal to:

a) $(9,3)$
b) $\left(\frac{9}{2}, 2\right)$
c) $\left(\frac{9}{2}, 3\right)$
d) $\left(\frac{3}{2}, 2\right)$

9) Let $f(x) = \int \frac{\sqrt{x}}{(1+x)^2} dx$. Then $f(3) - f(1)$ is equal to:

a) $-\frac{\pi}{6} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
b) $\frac{\pi}{6} + \frac{1}{2} - \frac{\sqrt{3}}{4}$
c) $-\frac{\pi}{12} + \frac{1}{2} + \frac{\sqrt{3}}{4}$
d) $\frac{\pi}{12} + \frac{1}{2} - \frac{\sqrt{3}}{4}$

10) A survey shows that 63% of the people in a city read newspaper A whereas 76% read newspaper B. If x% of the people read both the newspapers, then a possible value of x can be:

a) 65
b) 37
c) 29
d) 55

11) Let $u = \frac{2z+i}{z-ki}, z = x + iy$ and $k > 0$. If the curve represented by $Re(u) + Im(u) = 1$ intersects the y-axis at the points $\mathbf{P}$ and $\mathbf{Q}$ where $PQ = 5$, then the value of k is:

a) 4
b) $\frac{1}{2}$
c) 2
d) $\frac{3}{2}$

12) Let x_0 be the point of local maxima of $f(x) = \vec{a} \cdot \left(\vec{b} \times \vec{c}\right)$, where $\vec{a} = x\hat{i} + 2\hat{j} + 3\hat{k}, \vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$, and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$. Then the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$ at $x = x_0$ is:

a) -22

b) -4
c) -30
d) 14

13) Two vertical poles AB = 15 m and CD = 10 m are standing apart on a horizontal ground with points **A** and **C** on the ground. If **P** is the point of intersection of BC and AD, then the height of **P** (in m) above the line AC is :

a) $\frac{20}{3}$
b) 5
c) $\frac{10}{3}$
d) 6

14) The mean and variance of 8 observations are 10 and 13.5, respectively. If 6 of these observations are $5, 7, 10, 12, 14, 15$,then the absolute difference of the remaining two observations is :

a) 7
b) 3
c) 5
d) 9

15) The integral $\int \left(\frac{x}{x\sin x+\cos x}\right)^2 dx$ is equal to (where C is a constant of integration):

a) $\tan x - \frac{x\sec x}{x\sin x+\cos x} + C$
b) $\sec x + \frac{x\tan x}{x\sin x+\cos x} + C$
c) $\sec x - \frac{x\tan x}{x\sin x+\cos x} + C$
d) $\tan x + \frac{x\sec x}{x\sin x+\cos x} + C$