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Introduction :

Heat is defined as energy in transit. Heat itself cannot be seen but its effect can be felt and measured as a property called temperature. Accordingly there are two theories of each, any theory should be able to explain the facts given below:

1. Whenever there is a exchange of heat, heat is consumed i.e. heat lost by the hot body is equal to heat gained by the cold body. The heat flow takes place from higher temperature to lower temperature.
2. Substance expand on heating.
3. In order to change a state of a body from solid to liquid or liquid to gas without rise in temperature, certain amount of heat is required.

Heat transfer may be defined as transmission of energy from one region to other as the result of temperature gradient. In other words, the science which deals with determination of rates of energy transfer is called as heat transfer.

In heat transfer the driving potential is temperature.

The study of heat transfer is done for following purpose:

a. To estimate the rate of flow of energy as heat through the boundary of the system for both steady and unsteady condition.

b. to determine the temperature field under steady and transient conditions.

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Applications of heat transfer

1. Mechanical engineering - boilers, heat exchangers, IC engines
2. Metallurgical engineering - furnace, heat treatment of components
3. Electrical engineering - cooling system for electric motors, generators, transformers
4. Aerospace engineering - design of aircraft systems and components
5. Nuclear engineering - removal of heat generated by nuclear fission using liquid metal coolants.

* Basic / Fundamental laws governing heat transfer:

1. First law of thermodynamics:
   It gives conservation of energy.
2. Second law of thermodynamics:
   It gives direction of heat flow
3. Equation of continuity:
   It gives conservation of mass
4. Rate equations governing the three modes of heat transfer:
   a) Conduction - Fourier law of conduction
   b) Convection - Newton's law of cooling
   c) Radiation - Stephen's Boltzman law
5. Equation of flow:
   It gives fluid flow parameters.
6. Emperical relations for fluid properties such as specific heat, thermal conductivity, viscosity, etc.

Modes of heat transfer

a. Conduction:

It is a transfer of heat from one part of substance to another part of same substance by physical contact through lattice vibrations and transport of free electrons.

Eg: When a rod is heated at one end, after some time heat can be felt at other end.

b. Convection:

It is a transfer of heat within a fluid by mixing of one portion of fluid with other. There are two types of convection

1. Free or natural convection:

It occurs when the fluid circulates by virtue of natural difference in the density of hot and cold fluids. The denser portion move downward due to gravity as compared to lesser denser fluid.

2. Forced convection:

If the fluid motion is caused by an external agencies such as pump, fan, blower the heat transfer is said to be forced convection. The fluid is made to flow along the hot surface due to the pressure difference generated by the device

Eg: when boiling water, the heat passes from the burner into the pot, heating the water

Using a fan on a hot summer day. When

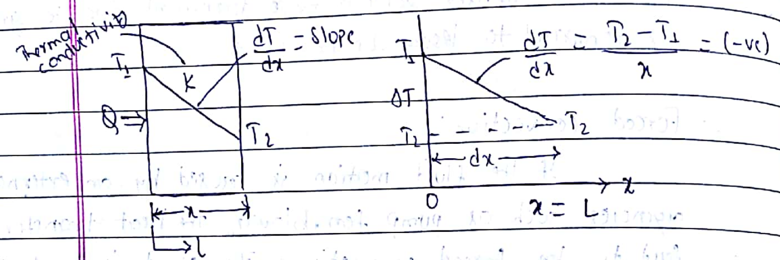
c. Radiation:

It is a transfer of heat through space or matter by means other than conduction or convection i.e. electromagnetic waves or quantum.

Eg: Heat from sun warming your face
heat from fire.

* Fourier law of conduction:

It is an emperical law based on experiments of biots but formulated by French mathematician, Physist Fourier in 1822. It states that the rate of heat by conduction in a given direction is proportional to area measured normal to the direction of heat flow and temperature gradient in that direction.



$$Q \propto A \frac{dT}{dx}$$

$$Q = KA \frac{dT}{dx}$$

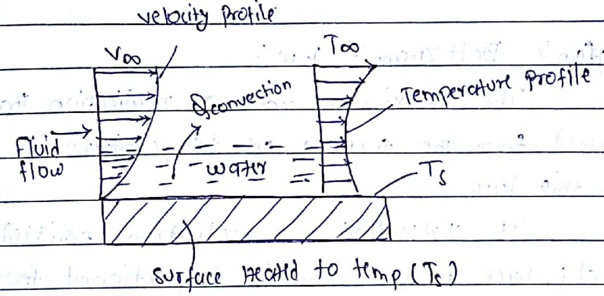
$$\text{or, } Q = -K A \frac{(T_2 - T_1)}{L}$$

$$\text{or } \boxed{Q = KA \frac{(T_1 - T_2)}{L}}$$

Q = heat flow rate in x - direction in watts
A = Area normal to direction of heat flow in m^2
K = proportionality constant called as thermal conductivity W/m°C
$\frac{dT}{dx}$ = temperature gradient in °C/m.

Thermal conductivity depends on
i) microstructure
ii) density
iii) Moisture content
iv) Pressure and temperature difference

Newton's law of cooling



It states that heat transfer by convection between surface & fluid is directly proportional to surface area of heat transfer and temperature difference between them

i.e. $Q \propto A (T_s - T_\infty)$

or, $Q \propto A \Delta T$

or, $\boxed{Q = hA\Delta T}$

where,

Q = heat transfer rate in watts
A = surface area in m^2
ΔT = Temp. diff. in °C
h = convective heat transfer coefficient in $W/m^2 °C$

Heat transfer coefficient 'h' is ability of fluid to carry the heat from the surface. It is a function of
i) Fluid properties such as density, viscosity, thermal conductivity, specific heat.
ii) Type of flow i.e. laminar / turbulent

iii) Orientation & geometry of surface

* Heat flux (q)

It is defined as the rate of heat transfer per unit area normal to direction of heat transfer.

i.e. $q = \frac{Q}{A}$ W/m^2

* Stefan's Boltzmann's law:

The maximum rate of radiation that can be emitted from the surface at temperature 'T' is given by this law.

It states that the radiation emitted by ideal (black body) body is proportional to surface area and fourth power of absolute temperature.

i.e. $Q \propto AT^4$

or, $Q = \sigma AT^4$

σ = Stefan's Boltzmann constant

$= 5.67 \times 10^{-8}\ W/m^2\,{}^{\circ}K^4$

* Initial & Boundary conditions

Initial conditions:

It describes the temperature at the surface at initial moment of time and it is applicable only in unsteady condition or transient conditions. It can be expressed as

At t_0 - $T_i = T(x, y, z)$

Boundary condition

The boundary condition specify the thermal condition at boundary surfaces of the region such as temperature and heat flow.

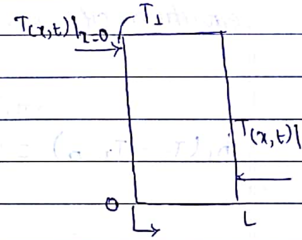
The mathematical expression of thermal condition at the boundaries are called as boundary conditions.

There are three types of boundary conditions:

1. Prescribed temperature boundary condition or Dirichlet boundary condition
2. Prescribed heat flux boundary condition or Neumann boundary condition
3. Convection boundary condition

a. Prescribed temperature BC

Here temperature at boundaries are specified and constant. Eg: Temperature at a surface is constant if that surface is in contact with melting ice or boiling liquid



$T(x,t)|_{x=0} = T(0,t)|_{x=0} = T_1$

$T(x,t)|_{x=L} = T(L,t)|_{x=L} = T_2$

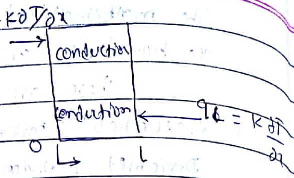
b. Prescribed heat flux BC

Here, heat flux at the boundaries are assumed. Eg: If the surface is heated by electric heater, the heat flux entering the surface is known.

Consider a plate of thickness 'L' as shown in figure. The heat supplied into the plate at a rate of q_0 watt/m^2 through the boundary surface at $x = 0$ and another heat supply into the plate at a rate of q_1 watt/m^2 through the boundary surface at $x = L$. The mathematical expression can be written as:

At $x=0$, $q_0 = -k\frac{\partial T}{\partial x}$

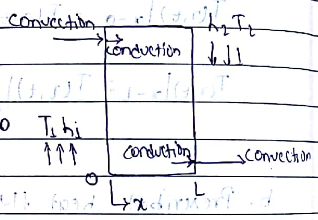
At $x=L$, $q_L = k\frac{\partial T}{\partial x}$



3 Convection BC:

In most practical situation, heat transfer at boundary surface is by convection with known heat transfer coefficient 'h' into an ambient fluid at a prescribed temperature.

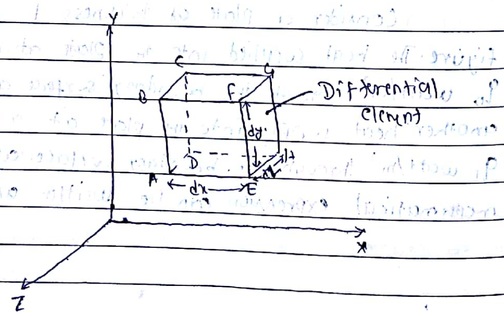
Consider a plate of thickness 'L', a fluid at temperature T_1 with heat transfer coefficient 'h_1' flows over a surface of plate at $x=0$. The mathematical expression or boundary condition is obtained by energy balance equation at surface $x=0$



$$h_1(T_1 - T_{x=0}) = -k\frac{\partial T}{\partial x}\Big|_{x=0}$$

$$h_2(T_{x=L} - T_2) = -k\frac{\partial T}{\partial x}\Big|_{x=L}$$

Imp * 3D - Heat conduction equation in cartesian co-ordinates:



In heat transfer analysis one of the objective is to determine the temperature distribution within the body at any given instance. We calculate heat transfer rate, at any given point in a given direction by applying Fouriers law. General technique to obtain the temperature distribution over the entire body is to consider a differential controlled volume within the body and apply the law of conservation of energy to this differential control volume which results in differential equation called as 3D - Heat conduction equation. for which the solution can be obtained by appropriate initial and boundary conditions which gives temperature field at any point within the body.

Consider a small differential volume having sides dx, dy & dz as shown in fig. It has six surfaces and each surface is assumed to be isothermal and rigid body. The various energy term involved are:

1. Energy conducted into the element.
2. Energy conducted out of the element.
3. Internal heat generated due to chemical or nuclear reaction.
4. Net heat conducted into the element or increase in internal energy or energy stored

$E_{in} \rightarrow Q_x + Q_y + Q_z$

$E_{out} \rightarrow Q_{(x+dx)} + Q_{y+dy} + Q_{z+dz}$

$E_{gen} \rightarrow \dot{q}\, dx\,dy\,dz$

$E_{stor} \rightarrow m C_p \frac{\partial T}{\partial t}$

$= \rho\, dx\,dy\,dz\, C_p \frac{\partial T}{\partial t}$

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Energy balance equation :

$$E_{in} - E_{out} + E_{gen} = E_{stored} \quad \text{--- (1)}$$

where, $E_{in} - E_{out} = (Q_x + Q_y + Q_z) - (Q_{x+dx} + Q_{y+dy} + Q_{z+dz})$

$$Q_{x+dx} = Q_x + \frac{\partial (Q_x)}{\partial x} dx \quad \text{- from Taylor series}$$

$$Q_x - Q_{x+dx} = \cancel{Q_x} - \left(\cancel{Q_x} + \frac{\partial (Q_x)}{\partial x} dx\right)$$

$$= -\frac{\partial Q_x}{\partial x} dx$$

from Fourier's law of conduction

$$Q = -KA\frac{\partial T}{\partial x}$$

$$\therefore Q_x - Q_{x+dx} = -\frac{\partial}{\partial x}\left(-K_x A_x \frac{\partial T}{\partial x}\right) dx$$

$$= K_x \frac{\partial^2 T}{\partial x^2} dx\,dy\,dz \quad (\because A_x = dy\,dz)$$

Similarly,

$$Q_y - Q_{y+dy} = K_y \frac{\partial^2 T}{\partial y^2} dx\,dy\,dz$$

$$Q_z - Q_{z+dz} = K_z \frac{\partial^2 T}{\partial z^2} dx\,dy\,dz$$

$$\therefore E_{in} - E_{out} = K_x \frac{\partial^2 T}{\partial x^2} dx\,dy\,dz + K_y \frac{\partial^2 T}{\partial y^2} dx\,dy\,dz + K_z \frac{\partial^2 T}{\partial z^2} dx\,dy\,dz$$

Substituting in eqn (1)

$$\left[K_x \frac{\partial^2 T}{\partial x^2} dx\,dy\,dz + K_y \frac{\partial^2 T}{\partial y^2} dx\,dy\,dz + K_z \frac{\partial^2 T}{\partial z^2} dx\,dy\,dz\right] + \dot{q}\,dx\,dy\,dz$$

$$= \rho\,dx\,dy\,dz\,C_p \frac{\partial T}{\partial t}$$

$$\therefore \boxed{K_x \frac{\partial^2 T}{\partial x^2} + K_y \frac{\partial^2 T}{\partial y^2} + K_z \frac{\partial^2 T}{\partial z^2} + \dot{q} = \rho C_p \frac{\partial T}{\partial t}}$$

3D- heat conduction equation.

For isotropic material

$$K = K_x = K_y = K_z$$

$$\therefore \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\dot{q}}{K} = \frac{\rho C_p}{K}\frac{\partial T}{\partial t}$$

Thermal diffusivity, $\alpha = \frac{K}{\rho C_p}$

$$\therefore \boxed{\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}\right] + \frac{\dot{q}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

Special cases:

1. Steady state (temperature at any point doesnot change with time)

$$\text{i.e } \frac{1}{\alpha}\frac{\partial T}{\partial t} = 0$$

$$\therefore \boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = 0}$$

This eqn is called as Poisson's equation

2. No internal heat generation [Diffusion equation]

$$\frac{\dot{q}}{K} = 0$$

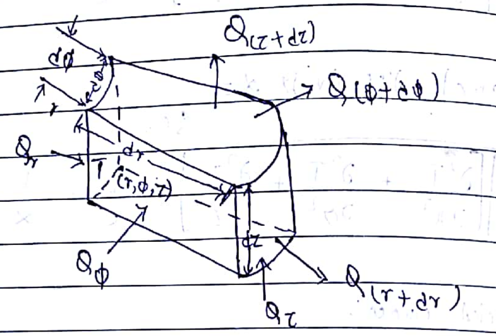
$$\therefore \boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

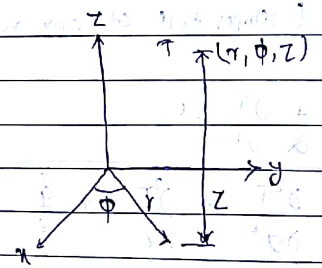
3. Steady state and no internal heat generation (Laplace Equation)

$\frac{\dot{q}}{K} = 0$ & $\frac{1}{\alpha}\frac{\partial T}{\partial t} = 0$

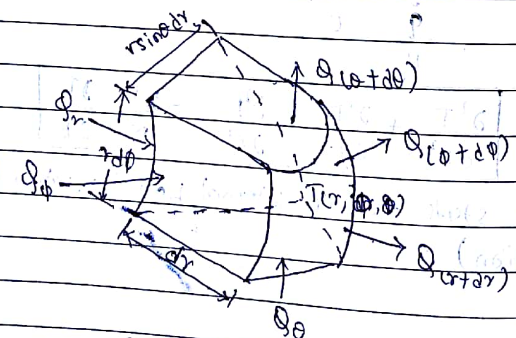
$$\therefore \boxed{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0.}$$

Cylindrical co-ordinates :-





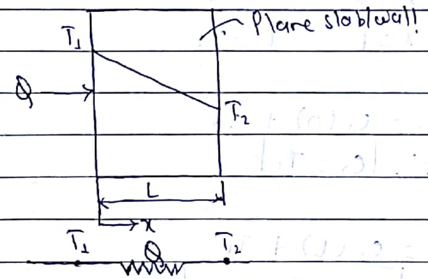
$$\boxed{\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$



Spherical co-ordinates.

$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial T}{\partial \phi^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

* One Dimensional Heat conduction equation for Plane slab:



$$R = \left(L/KA\right)$$

1. Temperature distribution (T).
2. Heat flow rate (Q)

Assumption:

1. 1D
2. Steady state
3. NO heat generation
4. Isotropic & Homogenous
5. Uniform thermal conductivity

$$\frac{\partial^2 T}{\partial x^2} + \cancel{\frac{\partial^2 T}{\partial y^2}}^{0} + \cancel{\frac{\partial^2 T}{\partial z^2}}^{0} + \cancel{\frac{\dot{q}}{K}}^{0} = \frac{1}{\alpha}\cancel{\frac{\partial T}{\partial t}}^{0}$$

$$\therefore \frac{\partial^2 T}{\partial x^2} = 0$$

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Integrating

$$\frac{\partial T}{\partial x} = C_1$$

Again Integrating

$$T = C_1 x + C_2 \quad -(1)$$

Boundary condition

At $x = 0 \quad T = T_1$

$x = l \quad T = T_2$

$\therefore T_1 = C_1(0) + C_2$

$\therefore \boxed{C_2 = T_1}$

$T_2 = C_1(L) + T_1$

$\therefore \boxed{C_1 = \frac{T_2 - T_1}{L}}$

Substituting in (1)

$$\boxed{T = \left(\frac{T_2 - T_1}{L}\right)x + T_1}$$ Linear in nature

(2) Heat flow rate (Q)

Fourier Law

$$Q = -KA\frac{dT}{dx}$$

$$Q\,dx = -KA\,dT$$

Integrating

$$Q\int_{x=0}^{x=L} dx = -KA\int_{T=T_1}^{T=T_2} dT$$

Or, $QL = -KA(T_2 - T_1)$

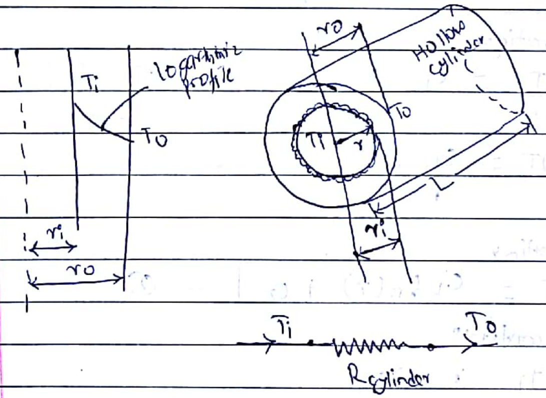
or, $$\boxed{Q = \frac{KA(T_1 - T_2)}{L}}$$

$$Q = \frac{T_1 - T_2}{L/KA}$$

$$\therefore Q = \frac{\Delta T}{R_{cond}} \quad \left(\because R_{cond} = \frac{L}{KA}\right) \ (°C/W)$$

conductive Resistance

One Dimensional heat conduction equation for hollow cylinder



Consider a Hollow cylinder of thermal conductivity 'K' inside radius 'r_i', outside radius 'r_o' and length 'L'. The inside and outside surfaces are kept constant at temperature T_i and T_o respectively.

Assumption

1. Steady state
2. One-dimensional
3. No heat generation
4. Uniform thermal conductivity
5. Heat transfer is only in radial direction

3D heat conduction equation for cylinder

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\cancel{\frac{\partial^2 T}{\partial \theta^2}} + \cancel{\frac{\partial^2 T}{\partial z^2}} + \cancel{\frac{\dot{q}}{K}} = \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

$$\frac{\partial^2 T}{\partial^2 r} + \frac{1}{r}\frac{\partial T}{\partial r} = 0$$

or $\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{r\,\partial T}{\partial r}\right) = 0$

$\because \frac{1}{r} \neq 0$

$\therefore \boxed{\frac{\partial}{\partial r}\left(\frac{r\,\partial T}{\partial r}\right) = 0}$ — (1)

Integrating

$\frac{r\,dT}{\partial r} = c_1$

or, $dT = \frac{c_1\,\partial r}{r}$

Integrating

$\boxed{T = c_1 \ln(r) + c_2}$ — (2)

Boundary condition

At $T = T_i$ $r = r_i$

$T = T_o$ $r = r_o$

$\therefore T_i = c_1 \ln(r_i) + c_2$ — A

$T_o = c_1 \ln(r_o) + c_2$ — B

Subtracting (A) − (B)

$T_i - T_o = c_1 \ln(r_i) - c_1 \ln(r_o)$

or, $T_i - T_o = c_1 \ln\left(\frac{r_i}{r_o}\right)$

$\therefore \boxed{c_1 = \frac{T_i - T_o}{\ln\left(\frac{r_i}{r_o}\right)}}$ — (3)

Substituting in (A)

$T_i = \frac{T_i - T_o}{\ln\left(\frac{r_i}{r_o}\right)} \cdot \ln(r_i) + c_2$

$\therefore \boxed{c_2 = T_i - \frac{T_i - T_o}{\ln(r_i/r_o)} \ln(r_i)}$ (4)

Substituting c_1 & c_2 in eqn (2)

$T = \frac{T_i - T_o}{\ln(r_i/r_o)} \ln(r) + T_i - \frac{T_i - T_o}{\ln(r_i/r_o)} \ln(r_i)$

$= \frac{T_i - T_o}{\ln(r_i/r_o)} \left[\ln(r) - \ln(r_i)\right] + T_i$

or, $T - T_i = \frac{T_i - T_o}{\ln(r_i/r_o)} \ln(r/r_i)$

$\therefore \boxed{\frac{T - T_i}{T_i - T_o} = \frac{\ln(r/r_i)}{\ln(r_i/r_o)}}$ page no. 53 DHB

Logarithmic in nature

This is the temperature distribution equation.

ii) Heat flow

$Q = -KA\frac{dT}{dr}$

or, $Q = -K\,2\pi r L \cdot \frac{dT}{dr}$ $(\because A = 2\pi r l)$

or, $Q\frac{dr}{r} = -K\,2\pi L\,dT$

Integrating

$Q \ln r \Big|_{r_i}^{r_o} = -K\,2\pi L \cdot T \Big|_{T_i}^{T_o}$

or, $Q \ln\left(\frac{r_o}{r_i}\right) = -2\pi K L\,(T_o - T_i)$

or, $Q = \frac{2\pi K L\,[T_i - T_o]}{\ln(r_o/r_i)}$

$= \frac{T_i - T_o}{\frac{\ln(r_o/r_i)}{2\pi K L}}$

$\therefore \boxed{Q = \frac{\Delta T}{R_{cylinder}}}$ $R_{cylinder} = \frac{\ln(r_o/r_i)}{2\pi K L}$

Heat flow equation

One dimensional heat conduction equation for sphere

One dimensional heat conduction eqn for sphere.

3D Heat conduction eqn

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)} + \cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}} + \cancel{\frac{\dot{q}}{K}}$$

$$= \cancel{\frac{1}{\alpha}\frac{\partial T}{\partial t}} \to 0$$

$$\therefore \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0$$

$$\because \frac{1}{r^2} \neq 0$$

$$\therefore \frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = 0$$

Integrating

$$r^2\frac{\partial T}{\partial r} = C_1$$

Or, $\partial T = \frac{C_1}{r^2}\partial r$

$$\therefore \boxed{T = \frac{-C_1}{r} + C_2} \quad -(1)$$

Boundary condn:

$T = T_i \quad r = r_i \Rightarrow T_i = \frac{-C_1}{r_i} + C_2 \quad$ — A

$T = T_o \quad r = r_o \Rightarrow T_o = \frac{-C_1}{r_o} + C_2 \quad$ —(B)

$$T_i - T_o = \frac{-C_1}{r_i} + \frac{C_1}{r_o}$$

$$T_i - T_o = C_1\left(\frac{1}{r_o} - \frac{1}{r_i}\right)$$

$$\therefore C_1 = \frac{T_i - T_o}{\left(\frac{1}{r_o} - \frac{1}{r_i}\right)}$$

From (A)

$$T_i = -\frac{T_i - T_o}{\left(\frac{1}{r_o} - \frac{1}{r_i}\right)} \times \frac{1}{r_i} + C_2$$

$$\therefore C_2 = T_i + \frac{T_i - T_o}{\left(\frac{1}{r_o} - \frac{1}{r_i}\right)} \times \frac{1}{r_i}$$

Substitute C_1 & C_2 in eqn (1)

$$T = -\frac{T_i - T_o}{\left(\frac{1}{r_o} - \frac{1}{r_i}\right)} \times \frac{1}{r} + T_i + \frac{T_i - T_o}{\left(\frac{1}{r_o} - \frac{1}{r_i}\right)} \times \frac{1}{r_i}$$

$$= \frac{T_i - T_o}{\left(\frac{1}{r_o} - \frac{1}{r_i}\right)}\left[\frac{1}{r_i} - \frac{1}{r}\right] + T_i$$

or, $$\boxed{\frac{T - T_i}{T_i - T_o} = \frac{\left(1/r_i - 1/r\right)}{\left(\frac{1}{r_o} - \frac{1}{r_i}\right)}}$$ Hyperbolic in nature

Temperature distribution

2) Heat flow

$$Q = -KA\frac{dT}{dr} \qquad A = 4\pi r^2$$

$$= -K\,4\pi r^2\frac{dT}{dr}$$

$$Q\frac{dr}{r^2} = -4\pi K\,dT$$

or, $$Q\int_{r_i}^{r_o}\frac{dr}{r^2} = -4\pi K\int_{T_i}^{T_o} dT$$

or, $$-Q\left.\frac{1}{r}\right|_{r_i}^{r_o} = -4\pi K\left.T\right|_{T_i}^{T_o}$$

$$\text{or, } -Q\left(\frac{1}{r_0}-\frac{1}{r_i}\right) = -4\pi K\left[T_0 - T_i\right]$$

$$\text{or, } Q\left[\frac{r_i - r_0}{r_0 \cdot r_i}\right] = 4\pi K\left[T_0 - T_i\right]$$

$$\text{or, } Q\left[r_i - r_0\right] = 4\pi K r_i r_0 \left[T_0 - T_i\right]$$

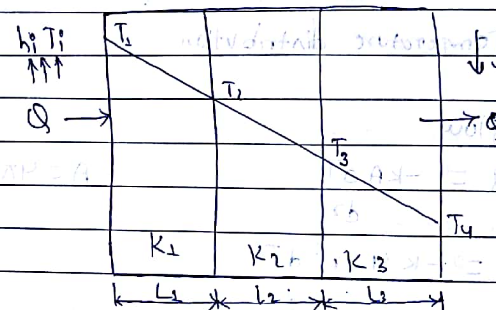
$$\therefore Q = \frac{4\pi K r_i r_0 \left[T_0 - T_i\right]}{r_i - r_0}$$

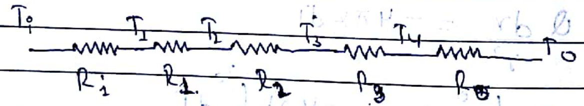
$$= \frac{4\pi K r_i r_0 \left[T_i - T_0\right]}{r_0 - r_i}$$

$$= \frac{\left[T_i - T_0\right]}{\frac{(r_0 - r_i)}{4\pi K r_i r_0}}$$

$$\therefore \boxed{Q = \frac{\Delta T}{R_{sphere}}} \qquad R_{sphere} = \frac{r_0 - r_i}{4\pi K r_i r_0}$$

Composite wall :-





Consider a composite wall made up of three layer with thickness L_1, L_2 & L_3 and thermal conductivity K_1, K_2 & K_3.

The wall is exposed to convective heat transfer at both the ends with heat transfer coefficient h_i & h_o. The resistance is given by:

$R_i = \frac{1}{h_i A}$ $\qquad R_3 = \frac{L_3}{K_3 A}$

$R_1 = \frac{L_1}{K_1 A}$ $\qquad R_o = \frac{1}{h_o A}$

$R_2 = \frac{L_2}{K_2 A}$

Now,

$$Q = h_i A (T_i - T_1) \quad -(1)$$

$$\Rightarrow T_i - T_1 = Q\left(\frac{1}{h_i A}\right)$$

$$Q = -K_1 A \frac{(T_2 - T_1)}{L_1} \quad -(2)$$

$$\Rightarrow -(T_2 - T_1) = Q\left(\frac{L_1}{K_1 A}\right)$$

$$Q = -K_2 A \frac{(T_3 - T_2)}{L_2} \quad -(3)$$

$$\Rightarrow -(T_3 - T_2) = Q\left(\frac{L_2}{K_2 A}\right) \quad -(4)$$

$$Q = -K_3 A \frac{(T_4 - T_3)}{L_3} \quad -(4)$$

$$\Rightarrow -(T_4 - T_3) = Q\left(\frac{L_3}{K_3 A}\right) \quad -$$

$$Q = h_o A (T_4 - T_0) \quad -(5)$$

$$\Rightarrow (T_4 - T_0) = Q\left(\frac{1}{h_o A}\right)$$

Adding.

$$T_i - \cancel{T_1} - \cancel{T_2} + \cancel{T_1} - \cancel{T_3} + \cancel{T_2} - \cancel{T_4} + \cancel{T_3} + \cancel{T_4} - T_0 =$$

$$Q\left[\frac{1}{h_i A} + \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A} + \frac{L_3}{K_3 A} + \frac{1}{h_o A}\right]$$

Or, $T_i - T_o = Q\left[R_i + R_1 + R_2 + R_3 + R_o\right]$

Or, $Q = \dfrac{T_i - T_o}{R_i + R_1 + R_2 + R_3 + R_o}$

$\therefore \boxed{Q = \dfrac{\Delta T}{\sum R_{th}}}$

Q. A component slab is made up of three layers A, B & C with thickness 25cm, 10cm and 15cm respectively. The thermal conductivity of A & B are 1.7 W/m°C and 9.5 W/m°C respectively. The outside surface is exposed to air at 20°C with the convection heat transfer coefficient 15 $W/m^2{}^\circ C$ and the inside surface is exposed to gases at 1200°C with convection heat transfer coefficient 28 $W/m^2{}^\circ C$. The inside surface is at 1080°C. Take Area = $1m^2$ and determine the thermal conductivity of layer C.

Soln:

$L_1 = 25cm = 0.25m$

$L_2 = 10cm = 0.1m$

$L_3 = 15cm = 0.15m$

$K_1 = 1.7\ W/m^\circ C$

$K_2 = 9.5\ W/m^\circ C$

$T_o = 20^\circ C$

$h_o = 15\ W/m^2{}^\circ C$

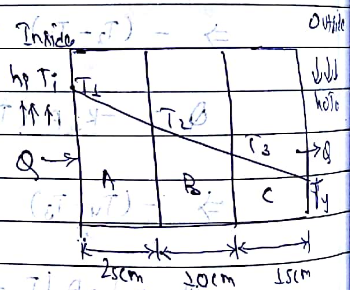
$T_i = 1200^\circ C$

$h_i = 28\ W/m^2{}^\circ C$

$T_1 = 1080^\circ C$

$A = 1m^2$

$K_3 = ?$



We have,

$T_i - T_1 = Q\left(\dfrac{1}{h_i A}\right)$

Or, $1200 - 1080 = Q \dfrac{1}{28 \times 1}$

$\therefore Q = 3360\ W$

Again,

$R_i = \dfrac{1}{h_i A} = \dfrac{1}{28 \times 1} = 0.0357\ ^\circ C/W$

$R_1 = \dfrac{L_1}{K_1 A} = \dfrac{0.25}{1.7 \times 1} = 0.147\ ^\circ C/W$

$R_2 = \dfrac{L_2}{K_2 A} = \dfrac{0.1}{9.5 \times 1} = 0.01052\ ^\circ C/W$

$R_3 = \dfrac{L_3}{K_3 A} = \dfrac{0.15}{K_3 \times 1}$

$R_o = \dfrac{1}{h_o A} = \dfrac{1}{15 \times 1} = 0.0666\ ^\circ C/W$

Also,

$Q = \dfrac{\Delta T}{R_{th}} = \dfrac{\Delta T}{R_i + R_1 + R_2 + R_3 + R_o}$

Or, $3360 = \dfrac{1200 - 20}{0.259 + \frac{0.15}{K_3}}$

Or, $2.847 = \dfrac{1}{0.259 + 0.15/K_3} = \dfrac{K_3}{0.259K_3 + 0.15}$

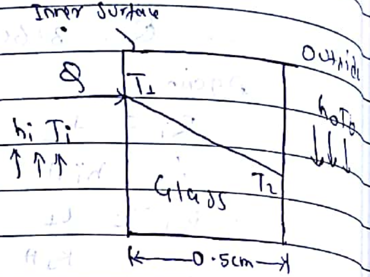
Or, $0.737K_3 + 0.427 = K_3$

Or, $0.263K_3 = 0.427$

$\therefore \boxed{K_3 = 1.63\ W/m^\circ C}$

Q. A large window glass 0.5cm thick and 0.78 W/m°C thermal conductivity is exposed to warm air at 25°C over its inner surface with convection heat transfer coefficient 15 $W/m^2{}^\circ C$. The outside air is at −15°C with convection heat transfer coefficient 60 $w/m^2{}^\circ c$. Determine the heat transfer rate and temperature at inner and outer surface of the glass. $A = 1m^2$

Soln:



$L = 0.5cm = 0.005m$

$h_i = 15\ w/m^2{}^\circ c$

$T_i = 25^\circ c$

$h_o = 50\ w/m^2{}^\circ c$

$T_o = -15^\circ c$

$K = 0.78\ w/m^\circ c$

Now,

$R_i = \frac{1}{h_i A} = \frac{1}{15\times1} = 0.0667\ ^\circ c/w$

$R = \frac{L}{KA} = \frac{0.005}{0.78\times1} = 6.41\times10^{-3}\ ^\circ c/w$

$R_o = \frac{1}{h_o A} = \frac{1}{50\times1} = 0.02\ ^\circ c/w$

$Q = \frac{T_i - T_o}{R_i + R + R_o}$

$= \frac{25-(-15)}{0.0667 + 6.41\times10^{-3} + 0.02}$

$\therefore \boxed{Q = 429.59w}$

$Q = h_i A (T_i - T_1)$

or, $429.59 = 15\times1 \times (25 - T_1)$

or, $25 - T_1 = 28.63$

$\therefore \boxed{T_1 = -3.639\ ^\circ c}$

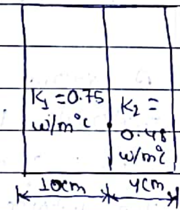
$Q = h_o A (T_2 - T_o)$

or, $429.59 = 50\times1\ (T_2 + 15)$

$\therefore \boxed{T_2 = -6.4082\ ^\circ C}$

Q. Exterior wall of a house may be approximated by 10cm layer of brick with thermal conductivity $K = 0.75\ w/m^\circ c$ followed by 4cm layer of plaster with $K = 0.48\ w/m^\circ c$. What thickness of loosely packed wool insulation with thermal conductivity, $K = 0.065\ w/m^\circ c$ should be added to reduce the heat loss or gain through the wall by 80%. Take $A = 1m^2$

Soln: Case 1:



$L_1 = 10cm = 0.1m$

$K_1 = 0.75\ w/m^\circ c$

$\therefore L_2 = 4cm = 0.04m$

$K_2 = 0.48\ w/m^\circ c$

Now,

$Q_1 = \frac{\Delta T}{R_1 + R_2}$

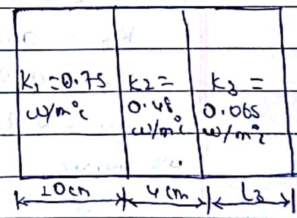
$R_1 = \frac{L_1}{K_1 A} = \frac{0.1}{0.75\times1} = 0.133\ ^\circ c/w$

$R_2 = \frac{L_2}{K_2 A} = \frac{0.04}{0.48\times1} = 0.083\ ^\circ c/w$

$\therefore Q_1 = \frac{\Delta T}{0.133 + 0.083}$

$\therefore Q_1 = 4.629\Delta T$

Case 2:



$Q_2 = (1 - 0.8)\times Q_1$

$Q_2 = 0.2 Q_1$

$Q_2 = \frac{\Delta T}{R_1 + R_2 + R_3}$

or, $0.2 Q_1 = \frac{\Delta T}{0.133 + 0.083 + R_3}$

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$Q = uA\Delta T$

Or, $\dfrac{0.2 \times 4.629\,\cancel{\Delta T}}{0.216 + R_3} = \cancel{\Delta T}$

or, $R_3 = 0.864$

or, $\dfrac{L_3}{K_3 A} = 0.864$

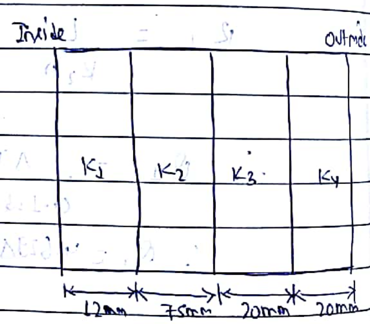
or, $\dfrac{L_3}{0.065} = 0.864$

$\therefore L_3 = 0.0562$ m

$\therefore \boxed{L_3 = 5.62\,cm}$

Q. A exterior wall of a building in constructed of 4 materials. Layer 1 12 mm thick, layer 2 75mm thick, layer 3 20mm thick, Layer 4 20mm thick. The inside and outside temperature are 20°C and −10°C. Determine the heat flux and overall heat transfer coefficient $K_1 = 0.176$ w/m°c $k_2 = 0.115$ w/m°c, $K_3 = 0.036$ w/m°c and $K_4 = 0.215$ w/m°c

Soln:



$L_1 = 12mm = 0.012m$

$L_2 = 75mm = 0.075\,m$

$L_3 = 20mm = 0.020\,m$

$L_4 = 20mm = 0.02\,m$

$T_i = 20°c$

$T_o = -10°c$

$K_1 = 0.176$ w/m°c

$K_2 = 0.115$ w/m°c

$K_3 = 0.036$ w/m°c

$K_4 = 0.215$ w/m°c

we have,

$$Q = \frac{\Delta T}{\Sigma R_{th}} = \frac{T_i - T_o}{R_1 + R_2 + R_3 + R_4}$$

$R_1 = \dfrac{L_1}{K_1 A} = \dfrac{0.012}{0.176} = 0.0682$ °C/w

$R_2 = \dfrac{L_2}{K_2 A} = \dfrac{0.075}{0.115} = 0.652$ °C/w

$R_3 = \dfrac{L_3}{K_3 A} = \dfrac{0.020}{0.036} = 0.555$ °C/w

$R_4 = \dfrac{L_4}{K_4 A} = \dfrac{0.02}{0.215} = 0.093$ °C/w

$\therefore Q = \dfrac{20 - (-10)}{0.0682 + 0.652 + 0.555 + 0.093}$

$\therefore Q = 21.926$ w

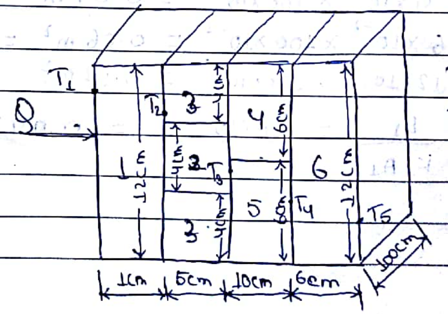
we have,

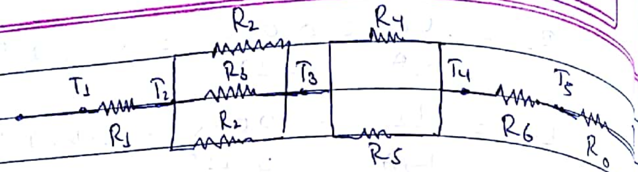
$Q = UA\Delta T$

or, $21.926 = U \times 1 \times [20 - (-10)]$

$\therefore$ Overall heat transfer coff., $U = 0.73$ w/m²°c

Q Find the heat flow rate through the composite wall as shown in figure. The thermal conductivities of various material are: $K_1 = K_6 = 2$ w/m°c, $K_2 = 8$ w/m°c, $K_3 = 20$ w/m°c, $K_4 = 16$ w/m°c, $K_5 = 35$ W/m°c, the left surface of the wall is maintained at a temperature of 300°c and the right surface is exposed to convection environment at 50°c and $h = 20$ W/m²°c. Determine the temperature at section 2 and 4 and temperature drop at section 6.





Soln:

$K_1 = K_6 = 2\ w/m°c$

$K_2 = 8\ w/m°c$

$K_3 = 20\ w/m°c$

$K_4 = 15\ w/m°c$

$K_5 = 35\ w/m°c$

$T_1 = 300°c$

$T_0 = 50°c$

$h_0 = 20\ w/m^2 ?$

$Q = ?$

$T_2 = ?$ $\quad T_4 - T_5 = ?$

$T_3 = ?$

$$Q = \frac{\Delta T}{\Sigma R_{th}}$$

$$\Sigma R_{th} = R_1 + \underbrace{\frac{1}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_2}}}_{R_{equ1}} + \underbrace{\frac{1}{\frac{1}{R_4} + \frac{1}{R_5}}}_{R_{equ2}} + R_6 + R_0$$

$A_1 = 12\times10^{-2} \times 100\times10^{-2} = 0.12\ m^2 = A_6$

$A_2 = 4\times10^{-2} \times 100\times10^{-2} = 0.04\ m^2 = A_3$

$A_4 = 6\times10^{-2} \times 100\times10^{-2} = 0.06\ m^2 = A_5$

$A_0 = 12\times10^{-2} \times 100\times10^{-2} = 0.12\ m^2$

$$R_1 = \frac{L_1}{K_1A_1} = \frac{1\times10^{-2}}{2\times0.12} = 0.042\ °C/w$$

$$R_2 = \frac{L_2}{K_2A_2} = \frac{5\times10^{-2}}{8\times0.04} = 0.156\ °C/w$$

$$R_3 = \frac{L_3}{K_3A_3} = \frac{5\times10^{-2}}{20\times0.04} = 0.0625\ °C/w$$

$$R_{equ1} = \frac{1}{\frac{1}{R_3} + \frac{1}{R_3} + \frac{1}{R_2}} = \frac{1}{\frac{1}{0.0625} + \frac{1}{0.0625} + \frac{1}{0.156}}$$

$\therefore R_{equ1} = 0.0260\ °C/w$

$$R_4 = \frac{L_4}{K_4A_4} = \frac{10\times10^{-2}}{15\times0.06} = 0.111\ °C/w$$

$$R_5 = \frac{L_5}{K_5A_5} = \frac{10\times10^{-2}}{35\times0.06} = 0.0476\ °C/w$$

$$R_{equ2} = \frac{1}{1/R_4 + 1/R_5} = \frac{1}{1/0.111 + 1/0.0476}$$

$\therefore R_{equ2} = 0.033$

$$R_6 = \frac{L_6}{K_6A_6} = \frac{6\times10^{-2}}{2\times0.12} = 0.25\ °C/w$$

$$R_0 = \frac{1}{h_0A_0} = \frac{1}{20\times0.12} = 0.416\ °C/w$$

$\Sigma R_{th} = R_1 + R_{equ1} + R_{equ2} + R_6 + R_0$

$= 0.042 + 0.026 + 0.033 + 0.25 + 0.416$

$\therefore \Sigma R_{th} = 0.767\ °C/w$

$$Q = \frac{\Delta T}{\Sigma R_{th}} = \frac{300 - 50}{0.767}$$

$\therefore \boxed{Q = 325.94\ w}$

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$$Q = \frac{T_1 - T_2}{R_1}$$

or, $325.94 = \frac{300 - T_2}{0.042}$

$\therefore \boxed{T_2 = 286.31\ ^\circ C}$

$$Q = \frac{T_2 - T_3}{R_{eq.1}}$$

or, $325.94 = \frac{286.31 - T_3}{0.0260}$

$\therefore \boxed{T_3 = 277.835\ ^\circ C}$

Again,

$$Q = \frac{T_4 - T_5}{R_6}$$

or, $325.95 = \frac{T_4 - T_5}{0.25}$

$\therefore \boxed{T_4 - T_5 = 81.485\ ^\circ C}$

Q. A steel pipe with 50mm outside diameter is covered with 6.4mm asbestos insulation ($K = 0.166\ w/m^\circ c$) followed by a 25mm layer of fibre glass insulation ($K = 0.0485\ w/m^\circ c$). The pipe wall temperature is $393^\circ C$ and the outside insulation temperature is $311^\circ C$. Calculate the interface temperature between asbestos and fibre glass. L = 1m.

soln:

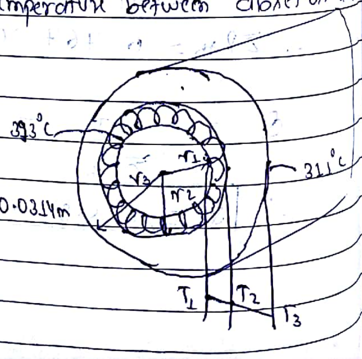
D = 50mm

$\Rightarrow r_1 = 25mm = 0.025m$

$r_2 = 0.025 + (6.4 \times 10^{-3}) = 0.0314m$

$r_3 = r_2 + (25 \times 10^{-3})$

$= 0.0564\ m$



$K_1 = 0.166\ w/m^\circ c$ $\quad T_1 = 393^\circ C$

$K_2 = 0.0485\ w/m^\circ c$ $\quad T_3 = 311^\circ C$

$Q = ?\quad T_2 = ?$

$$Q = \frac{\Delta T}{R_{cylinder}}$$

$$R_{cylinder} = \frac{\ln(r_o/r_i)}{2\pi K L}$$

$$R_{cylinder} = R_1 + R_2$$

$$= \frac{\ln(r_2/r_1)}{2\pi K_1 L} + \frac{\ln(r_3/r_2)}{2\pi K_2 L}$$

$$= \frac{\ln(0.0314/0.025)}{2\pi \times 0.166} + \frac{\ln(0.0564/0.0314)}{2\pi \times 0.0485}$$

$\therefore R_{cylinder} = 0.2185 + 1.922 = 2.14\ ^\circ C/w$

$$Q = \frac{T_1 - T_3}{R_{cylinder}}$$

$$= \frac{393 - 311}{2.14}$$

$\therefore \boxed{Q = 38.32\ w/m}$

Also,

$$Q = \frac{T_1 - T_2}{R_1}$$

or $38.32 = \frac{393 - T_2}{0.2185}$

$\therefore \boxed{T_2 = 384.62\ ^\circ C}$

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Q. A 240mm steam main, 210m long is covered with 50mm of high temperature insulation material ($k = 0.092\ W/m^\circ C$) of 40mm of low temperature insulation material ($k = 0.062\ W/m^\circ C$). The inner & outer surface temperature are $390^\circ C$ & $40^\circ C$. calculate the following:

i) the total heat loss per hour

ii) the heat lost per m^2 of pipe surface (pipe inner surface)

iii) the total heat lost / m^2 of outer surface

iv) the temperature between two layer of insulation.

Soln:

$D = 240mm$

$\therefore r_1 = 120mm = 0.12m$

$L = 210m$

$r_2 = r_1 + 50mm = 0.12 + 50\times10^{-3}$

$= 0.17m$

$r_3 = r_2 + 40\times10^{-3} = 0.21m$

$k_1 = 0.092\ W/m^\circ C$

$k_2 = 0.062\ W/m^\circ C$

$T_1 = 390^\circ C$ $\quad Q = ?$

$T_3 = 40^\circ C$ $\quad T_2 = ?$

Now,

$$Q = \frac{\Delta T}{R_{cylinder}}$$

$$R_{cylinder} = R_1 + R_2$$

$$= \frac{\ln(r_2/r_1)}{2\pi k_1 l} + \frac{\ln(r_3/r_2)}{2\pi k_2 L}$$

$$= \frac{\ln(0.17/0.12)}{2\pi\times0.092\times210} + \frac{\ln(0.21/0.17)}{2\pi\times0.062\times210}$$

$$= 2.869\times10^{-3} + 2.583\times10^{-3}$$

$$\therefore R_{cylinder} = 5.452\times10^{-3}\ ^\circ C/W$$

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$$Q = \frac{T_1 - T_3}{R_{cylinder}} = \frac{390 - 40}{5.452\times10^{-3}}$$

$\therefore Q = 64196.62\ W$

$\therefore Q = 64.196\ kW$

i) total heat lost per hour

$Q = 64.196\times3600$

$\therefore \boxed{Q = 231106.6\ kJ/hr}$

ii) heat lost per m^2 of inner surface

$$Q = \frac{64.196}{A_{inner\ surface}} = \frac{64.196}{2\pi\times r_1\times L}$$

$$= \frac{64.196}{2\pi\times0.12\times210}$$

$= 0.40544\ kW/m^2$

$\therefore \boxed{Q_i = 405.44\ W/m^2}$

iii) heat lost / m^2 of outer surfaces

$$Q = \frac{64196.62}{2\pi\times0.21\times210} \qquad A_o = 2\pi r_3 L$$

$\therefore \boxed{Q_o = 231.686\ W/m^2}$

iv) temperature between two layer of insulation

$$Q = \frac{T_1 - T_2}{R_1}$$

or, $$64196.62 = \frac{390 - T_2}{2.869\times10^{-3}}$$

$\therefore \boxed{T_2 = 205.82\ ^\circ C}$

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Q A steam pipe with internal & external diameters 18cm & 21cm is covered with 2 layers of insulation which is 30mm with thermal conductivities 0.18 w/m°C & 0.09w/m°C. The difference in temperature between inside and outside surface is 250°c. Calculate the quantity of heat loss per m length of the pipe if its thermal conductivity is 60 w/m°c. What is the % error if the calculation is carried out considering the pipe as plane wall.

Soln:

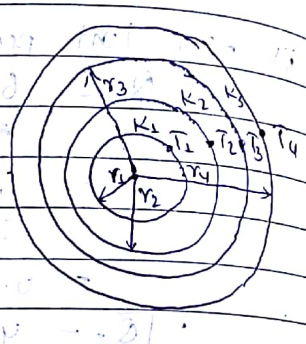
$D_1 = 18cm = 0.18m$

=> $r_1 = 0.09m$

$D_2 = 0.21m \quad r_2 = 0.105m$

$r_3 = 0.105 + 0.03 = 0.135m$

$r_4 = 0.135 + 0.03 = 0.165m$



$K_1 = 60 \, w/m°c$

$K_2 = 0.18 \, w/m°c$

$K_3 = 0.09 \, w/m°c$

$\Delta T = T_1 - T_4 = 250°c$

$Q = ?$

% error = ?

Case 1:

Heat loss in composite cylinder

Total thermal resistance

$$\Sigma R_t = R_1 + R_2 + R_3$$

$$= \frac{\ln(r_2/r_1)}{2\pi k_1 L} + \frac{\ln(r_3/r_2)}{2\pi k_2 L} + \frac{\ln(r_4/r_3)}{2\pi k_3 L}$$

$$= \frac{\ln(0.105/0.1)}{2\pi \times 60 \times 1} + \frac{\ln(0.135/0.105)}{2\pi \times 0.18 \times 1} + \frac{\ln(0.165/0.135)}{2\pi \times 0.09 \times 1}$$

$$= 4 \times 089 \times 10^{-4} + 0.222 + 0.355$$

$$\therefore \Sigma R_t = 0.578 \, °c/w$$

Heat lost/m length

$$Q = \frac{\Delta T}{\Sigma R_t} = \frac{250}{0.578} = 432.58 \, w$$

Case 2:

When composite cylinder is considered as a composite plane wall.

Q A 3m inner dia spherical tank made of 2cm thick stainless steel ($K = 15\ W/m^{\circ}C$) is used to store ice water at $0^{\circ}C$. The tank is located in a large room maintained at $22^{\circ}C$. The outer surface of tank is black and heat is convected and radiated on the outer surface of the tank. The convection heat transfer coefficient at the inner and outer surface of tank are $80\ W/m^2{}^{\circ}C$ and $10\ W/m^2{}^{\circ}C$ resp. The radiation heat transfer coefficient is $5.34\ W/m^2{}^{\circ}C$. Determine rate of heat transfer to iced water.

Soln:

$D_1 = 3\ m$

$\rightarrow r_1 = 1.5\ m$

$r_2 = 1.5 + 0.02 = 1.52\ m$

$K = 15\ W/m^{\circ}C$

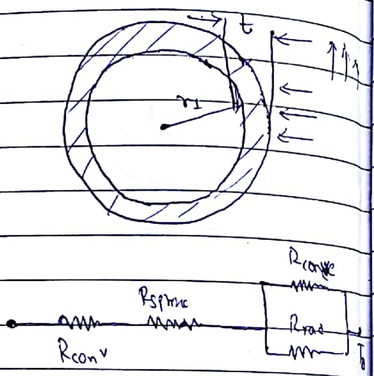
$T_i = 22^{\circ}C$

$T_o = 0^{\circ}C$

$h_i = 80\ W/m^2{}^{\circ}C$

$h_o = 10\ W/m^2{}^{\circ}C$

$h_r = 5.34\ W/m^2{}^{\circ}C$



$A_i = 4\pi r_1^2 = 4\pi \times 1.5^2 = 28.27\ m^2$

$A_o = 4\pi r_2^2 = 4\pi \times 1.52^2 = 29.03\ m^2$

$$Q = \frac{\Delta T}{\Sigma R_{th}} = \frac{\Delta T}{(R_{conv_1} + R_{sphere} + R_{eq})}$$

$$R_{conv_1} = \frac{1}{h_i A_i} = \frac{1}{80 \times 28.27} = 4.42 \times 10^{-4}\ {}^{\circ}C/W$$

$$R_{sphere} = \frac{r_2 - r_1}{4\pi K r_1 r_2} = \frac{1.52 - 1.5}{4\pi \times 15 \times 1.5 \times 1.52}$$

$$= 4.653 \times 10^{-5}\ {}^{\circ}C/W$$

$$R_{conv_2} = \frac{1}{h_o A_o} = \frac{1}{10 \times 29.03} = 3.44 \times 10^{-3}\ {}^{\circ}C/W$$

$$R_{rad} = \frac{1}{h_r A_o} = \frac{1}{5.34 \times 29.03} = 6.45 \times 10^{-3}\ {}^{\circ}C/W$$

$$\frac{1}{R_{eq}} = \frac{1}{R_{conv_2}} + \frac{1}{R_{rad}} = \frac{1}{3.44 \times 10^{-3}} + \frac{1}{6.45 \times 10^{-3}}$$

$\therefore R_{eq} = 2.245 \times 10^{-3}\ {}^{\circ}C/W$

$$Q = \frac{22 - 0}{4.42 \times 10^{-4} + 4.653 \times 10^{-5} + 2.245 \times 10^{-3}}$$

$\therefore \boxed{Q = 8.05\ kW}$

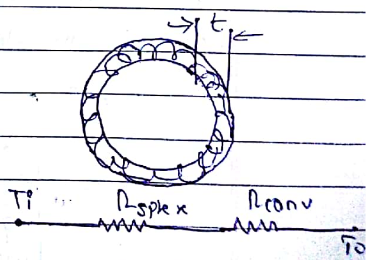
Q A spherical thin wall metallic container is used to store liquid nitrogen at $77^{\circ}K$. The container has a diameter of 0.5m and is covered with an evacuated reflective insulation of silica powder ($K = 0.0017\ W/m^{\circ}K$). The insulation is 25mm thick and its outer surface is exposed to ambient air at $300^{\circ}K$. The convective coeff. is known to be $20\ W/m^2{}^{\circ}K$. The latent heat of vaporization of liquid nitrogen is $2 \times 10^5\ J/kg$. Determine the rate of heat transfer to liquid nitrogen. What is the rate of liquid boil off. ($\dot{m}$)

Soln:

$D_1 = 0.5m \Rightarrow r_1 = 0.25m$

$r_2 = 0.25 + 0.025$

$= 0.275m$



$K = 0.0017\ W/m^{\circ}K$

$T_i = 77^{\circ}K$

$T_o = 300^{\circ}K$

$h_o = 20\ W/m^2{}^{\circ}K$

$h_{fg} = 2 \times 10^5\ J/kg$

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$A_i = 4\pi r_1^2 = 4\pi \times 0.25^2 = 0.785\ m^2$

$A_o = 4\pi r_2^2 = 4\pi \times 0.275^2 = 0.9503\ m^2$

$$Q = \frac{\Delta T}{\Sigma R_{th} (R_{sphere} + R_{conv})}$$

$$R_{sphere} = \frac{r_2 - r_1}{4\pi K r_1 r_2} = \frac{0.275 - 0.25}{4\pi \times 0.0017 \times 0.25 \times 0.275}$$

$$= 17.02\ ^\circ K/W$$

$$R_{conv} = \frac{1}{h_o A_o} = \frac{1}{20 \times 0.9503} = 0.052\ ^\circ K/W$$

$$Q = \frac{300 - 77}{17.02 + 0.052}$$

$\therefore \boxed{Q = 13.06\ W}$

Also,

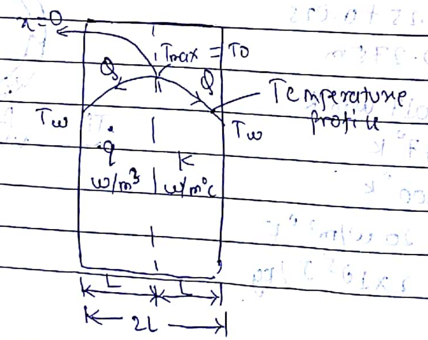
$Q = \dot{m} h_{fg}$

or, $13.06 = \dot{m} \times 2 \times 10^5$

$\therefore \boxed{\dot{m} = 6.53 \times 10^{-5}\ kg/s}$

* One dimensional steady state heat ~~convection~~ conduction equation with heat generation

Case 1 : Both sides same temperature



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Consider a plane slab of thickness 2L, as shown in figure. both the sides of slab are maintained at uniform temp. T_w. It is clear that maxm temp. will occur at centre line.

→ Since heat flow is from centre to outward.

Assumption

1) One dimensional
2) steady state heat conduction
3) Heat generation $\dot{q}$
4) Material of slab is homogeneous & isotropic

$$\frac{d^2T}{dx^2} + \cancel{\frac{d^2T}{dy^2}} + \cancel{\frac{d^2T}{dz^2}} + \frac{\dot{q}}{K} = \cancel{\frac{1}{\alpha}\frac{dT}{dt}}$$

$$\boxed{\frac{d^2T}{dx^2} + \frac{\dot{q}}{K} = 0}$$

$$\frac{d^2T}{dx^2} = \frac{-\dot{q}}{K}$$

Integrating

$$\frac{dT}{dx} = -\frac{\dot{q}}{K}x + C_1 \rightarrow (a)$$

$$\boxed{T(x) = -\frac{\dot{q}}{2K}x^2 + C_1 x + C_2}$$

Boundary conditions

At $x = 0$ $\frac{dT}{dx} = 0 \rightarrow C_1 = 0$

$x = \pm L$ $T = T_w$

$$T_w = -\frac{\dot{q}L^2}{2K} + C_2$$

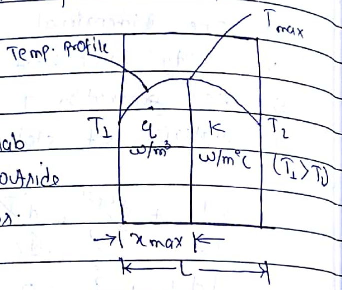
$$C_2 = T_w + \frac{\dot{q}L^2}{2K}$$

$$T(x) = -\frac{\dot{q}x^2}{2K} + T_w + \frac{\dot{q}L^2}{2K}$$

$$\therefore \boxed{T(x) = T_w + \frac{\dot{q}}{2K}(L^2 - x^2)}$$

Case 2: Both side different temperature.

Let $T_1 > T_2$, T_{max} must occur within the slab, since heat is generated in the slab and flowing from inside to outside both to left and right faces.



$$T(x) = -\frac{\dot{q}x^2}{2K} + C_1 x + C_2$$

Boundary condition

At $x = 0$ $T = T_1$

$x = L$ $T = T_2$

$\therefore T_1 = 0 + 0 + C_2$

$\therefore \boxed{C_2 = T_1}$

$$T_1 = -\frac{\dot{q}L^2}{2K} + C_1 L + C_2$$

or, $$T_2 = -\frac{\dot{q}L^2}{2K} + C_1 L + T_1$$

or, $$\boxed{C_1 = \frac{T_2 - T_1}{L} + \frac{\dot{q}L}{2K}}$$

$$\therefore T(x) = -\frac{\dot{q}x^2}{2K} + \left[\frac{T_2 - T_1}{L} + \frac{\dot{q}L}{2K}\right]x + T_1$$

$$= -\frac{\dot{q}x^2}{2K} + \frac{\dot{q}Lx}{2K} + \left(\frac{T_2 - T_1}{L}\right)x + T_1$$

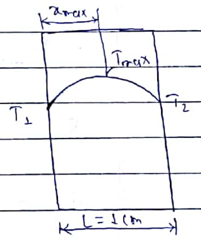
$$\therefore \boxed{T(x) = T_1 + \left[\frac{\dot{q}(L-x)}{2K} + \frac{T_2 - T_1}{L}\right]x}$$

Q. Heat is generated uniformly in a stainless steel plate $K = 20$ W/m°C. The thickness of the plate is 1 cm and rate of heat generation is 500 MW/m³. The two sides of the plate are maintained at 100 and 200°C respectively. Determine i) the temperature at the centre of plate

ii) position & value of max^m temperature

iii) heat transfer rate at left & right faces. Take $A = 1\,m^2$

Soln:



$K = 20$ W/m°C

$\dot{q} = 500$ MW/m³ $= 500 \times 10^6$ W/m³

$T_1 = 200°C$

$T_2 = 100°C$

$L = 1\,cm = 1 \times 10^{-2}\,m$

we have,

$$T(x) = -\frac{\dot{q}x^2}{2K} + C_1 x + C_2$$

Boundary condition

i) At $x = 0$ $T = T_1 = 200°C$

$\therefore C_2 = T_1 = 200°C$

ii) At $x = L$ $T = T_2 = 100°C$

$$\therefore T_2 = -\frac{\dot{q}L^2}{2K} + C_1 L + C_2$$

or, $$100 = -\frac{500 \times 10^6 \times (10^{-2})^2}{2 \times 20} + C_1 \times 10^{-2} + 200$$

$\therefore C_1 = 1.15 \times 10^5$

$\therefore T(x) = \dfrac{-500\times10^6 (x)^2}{2\times20} + 1.15\times10^5 x + 200$ — (1)

i) at $x = 0.005$

$T_{max} = \dfrac{-500\times10^6 \times (0.005)^2}{2\times20} + 1.15\times10^5 \times 0.005 + 200$

$\therefore \boxed{T_{max} = 462.5^\circ C}$ → at centre

ii) Differentiating eqn (1) wrt x & equating to zero

$\dfrac{dT}{dx} = -\dfrac{\dot{q}\,2x}{2K} + C_1 = 0$

or, $-\dfrac{500\times10^6 \times x}{20} + 1.15\times10^5 = 0$

$\therefore \boxed{x = 4.6\times10^{-3}\ m}$ → position

substituting x in eqn (1)

$T_{max} = \dfrac{-500\times10^6 \times (4.6\times10^{-3})^2}{2\times20} + 1.15\times10^5 \times 4.6\times10^{-3} + 200$

$\therefore \boxed{T_{max} = 464.5^\circ C}$ → value of max^m temp.

iii) $Q = -KA\dfrac{dT}{dx}$

$\dfrac{dT}{dx} = -\dfrac{\dot{q}x}{2K} + C_1$

a) at left face $x = 0$

$\therefore \dfrac{dT}{dx} = C_1 = 1.15\times10^5$

$\therefore Q = -20\times1\times1.15\times10^5$

$\therefore \boxed{Q_{left} = -2.3\times10^6\ W/m^2}$

b) at right face $x = L = 1cm = 0.01m$

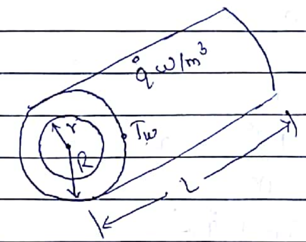
$\therefore \dfrac{dT}{dx} = -\dfrac{500\times10^6 \times 0.01}{20} + 1.15\times10^5$

$= -135000$

$\therefore Q = -20\times-135000\times1$

$\therefore \boxed{Q_{right} = 2.7\times10^6\ W/m^2}$

* One dimensional heat conduction equation for cylinder with heat generation



Assumptions:-

i) Steady state with heat generation

ii) 1D heat conduction in r-direction

iii) homogeneous & isotropic material with constant 'K'

iv) Uniform heat generation

$\dfrac{d^2T}{dr^2} + \dfrac{1}{r}\dfrac{dT}{dr} + \cancel{\dfrac{1}{r^2}\dfrac{d^2T}{d\phi^2}}^{0} + \cancel{\dfrac{d^2T}{dz^2}}^{0} + \dfrac{\dot{q}}{K} = \cancel{\dfrac{1}{\alpha}\dfrac{dT}{dt}}^{0}$

or, $\dfrac{d^2T}{dr^2} + \dfrac{1}{r}\dfrac{dT}{dr} + \dfrac{\dot{q}}{K} = 0$

Multiply by r

$r\dfrac{d^2T}{dr^2} + \dfrac{dT}{dr} + \dfrac{\dot{q}r}{K} = 0$

or, $\dfrac{d}{dr}\left(r\dfrac{dT}{dr}\right) = -\dfrac{\dot{q}r}{K}$ — (1)

Integrating

$r\dfrac{dT}{dr} = -\dfrac{\dot{q}r^2}{2K} + C_1$

or, $\dfrac{dT}{dr} = -\dfrac{\dot{q}r}{2K} + \dfrac{C_1}{r}$

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Again, Integrating

$$\boxed{T = -\frac{\dot{q} r^2}{4K} + C_1 \ln(K) + C_2} \quad \text{--- (2)}$$

Boundary condition

i) $r = 0 \quad \frac{dT}{dr} = 0$

$\therefore C_1 = 0$

ii) $r = R \quad T = T_\omega$

$\therefore T_\omega = -\frac{\dot{q} R^2}{4K} + C_2$

$\therefore C_2 = T_\omega + \frac{\dot{q} R^2}{4K}$

Eqn (2) becomes

$$T = -\frac{\dot{q} r^2}{4K} + T_\omega + \frac{\dot{q} R^2}{4K}$$

$$\therefore \boxed{T(x) = T_\omega + \frac{\dot{q}}{4K}(R^2 - r^2)}$$

* One dimensional heat conduction equation for sphere with heat generation

↳ Assumption same as cylinder

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \cancel{\frac{1}{r^2 \sin\theta}\frac{d}{d\theta}\left(\sin\theta \frac{dT}{d\theta}\right)} + \cancel{\frac{1}{r^2 \sin^2\theta}\frac{dT}{d\phi^2}} + \frac{\dot{q}}{K} = \cancel{\frac{1}{\alpha}\frac{dT}{dt}}$$

Or, $$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{\dot{q}}{K} = 0$$

Multiply by r^2

$$\frac{d}{dr}\left(r^2 \frac{dT}{dr}\right) + \frac{\dot{q} r^2}{K} = 0$$

Integrating

$$r^2 \frac{dT}{dr} + \frac{\dot{q} r^3}{3K} = C_1$$

$\div r^2$

$$\frac{dT}{dr} = -\frac{\dot{q} r}{3K} + \frac{C_1}{r^2}$$

Again, Integrating

$$\boxed{T = -\frac{\dot{q} r^2}{6K} - \frac{1}{r} C_1 + C_2} \quad \text{--- (1)}$$

Boundary condition

i) $r = 0 \quad \frac{dT}{dr} = 0$

$\therefore C_1 = 0$

ii) $r = R \quad T = T_\omega$

$\therefore T_\omega = -\frac{\dot{q} R^2}{6K} + C_2$

$\therefore C_2 = T_\omega + \frac{\dot{q} R^2}{6K}$

Eqn (1) becomes

$$T(x) = -\frac{\dot{q} r^2}{6K} + T_\omega + \frac{\dot{q} R^2}{6K}$$

$$\therefore \boxed{T(x) = T_\omega + \frac{\dot{q}}{6K}(R^2 - r^2)}$$

Unit - II

Fins

Heat transfer through extended surfaces:

Fins are used to reduce the thermal resistance at the surface and thereby increase the heat transfer rate from the surface to adjacent fluid. In other words, wherever the available surface is inadequate to heat transfer the required heat with available temperature drop and the convective heat transfer coefficient fins are introduced. This helps in reducing the thermal resistance which increases heat transfer rate.

From Newton's law of cooling 'Q' is given by:

$$Q = hA_c\Delta T \approx hA_c(T_s - T_\infty)$$

$A_c \rightarrow$ Cross section area

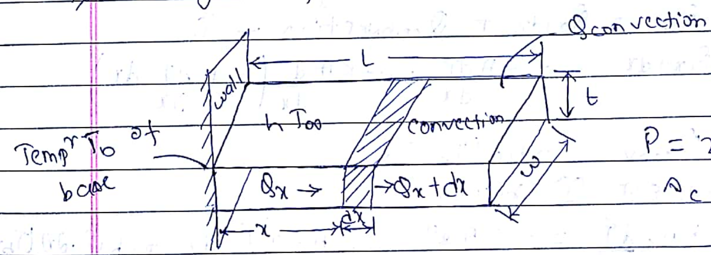
$T_s \rightarrow$ surface tempr. of fin

$T_\infty \rightarrow$ Ambient tempr. of surroundings

For proper design of fin the temperature distribution along the fins is necessary.

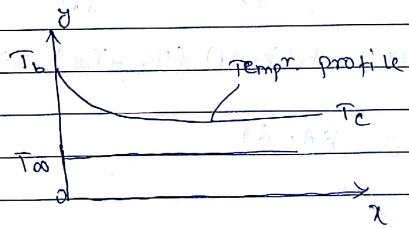
Fins of uniform cross section

i) Rectangular fin



$P = 2(w + t)$

$A_c = w \times t$



Consider a rectangular fin protruding from wall as shown in fig:

Assumption:

i) steady state heat conduction with no heat generation

ii) 1D heat conduction

iii) homogenous and isotrope fin material

let,

L = length of the fin

w = width of the fin

t = thickness

P = perimeter

A_c = cross section area

T_b = temperature at base of fin

T_∞ = Ambient or surrounding temp.

K = thermal conductivity

h = heat transfer coefficient

Consider an elemental section of thickness dx at a distance x from the base of the fin, the energy balance equation can be written as:

Energy conducted into fin = Energy leaving through conduction + Energy by convection

$$Q_x = Q_{x+dx} + Q_{convection} \quad \text{—(1)}$$

$$Q_{x+dx} = -KA\frac{dT}{dx} + -KA\frac{d}{dx}\left(T + \frac{\partial T}{\partial x}dx\right)$$

$$Q_{conv} = hA_c(T_b - T_\infty)$$

Now, Eqn (1) becomes

$$-KA_c\cancel{\frac{dT}{dx}} = -KA\cancel{\frac{dT}{dx}} - KA_c\frac{d^2T}{dx^2}dx + h(P.dx)(T_b-T_\infty)$$

or, $$KA_c\frac{d^2T}{dx^2}dx - h(P.dx)(T_b-T_\infty) = 0$$

Dividing by $KA_c dx$

$$\frac{d^2T}{dx^2} - \frac{h(Pdx)}{A_cKdx}(T_b-T_\infty) = 0$$

or, $$\frac{d^2T}{dx^2} - \frac{hP}{KA_c}(T_b-T_\infty) = 0$$

or, $$\frac{d^2T}{dx^2} - m^2(T_b-T_\infty) = 0 \quad \text{—(2)}$$

$$\left(\because m = \sqrt{\frac{hP}{KA_c}}\right)$$

Further simplified by transforming the dependent variable by defining the eqn's

$$\theta(x) : (T(x) - T_\infty)$$

$$\theta = T_b - T_\infty$$

Diff. w.r.to x

$$\frac{d\theta}{dx} = \frac{dT}{dx} \quad \left(T_\infty \text{ is const. so } \frac{dT_\infty}{dx} = 0\right)$$

or, $$\frac{d^2\theta}{dx^2} = \frac{d^2T}{dx^2}$$

Substituting in eqn (2), we get

$$\boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$$

Governing eqn for fin with uniform cross-section.

The general solution for above eqn is given by calculus theory in two equivalent forms:

$$\theta(x) : C_1e^{-mx} + C_2e^{mx} \quad \text{—(a)}$$

$$\theta(x) = A\cosh(mx) + B\sinh(mx) \quad \text{—(b)}$$

Above two equation (a) & (b) describe the tempr. distribution in a fin along its length.

Case I: Infinitely long fin

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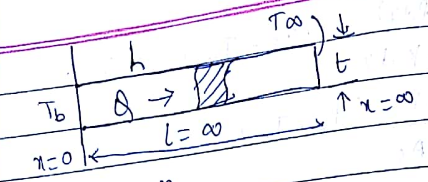

Governing eqⁿ

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

$$\theta(x) = C_1 e^{-mx} + C_2 e^{mx} \quad (1)$$

$$\theta(x) = T(x) - T_\infty) - (2)$$

Boundary conditions

At $x = 0$ $\quad T = T_b$

$x = \infty$ $\quad T = T_\infty$

substituting BC's in eqⁿ (2)

$$(T_b - T_\infty) = \theta$$

$$(T_\infty - T_\infty) = 0 = \theta(\infty)$$

at $x = 0$, eqⁿ (1)

$$\theta(0) = C_1 e^{-m0} + C_2 e^{m \times 0}$$

$$\therefore C_1 + C_2 = \theta$$

$$0 = C_1 e^{-\infty} + C_2 e^{\infty}$$

$$\therefore C_2 = 0$$

$$C_1 = \theta$$

$$\theta(x) : \theta e^{-mx}$$

$$\frac{\theta(x)}{\theta} = e^{-mx}$$

$$\boxed{\frac{T(x) - T_\infty}{T_b - T_\infty} = e^{-mx}}$$

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