

解答・解説(数と式②)

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1. (1) $2x(xy-3)$ (2) $a(3-4b^2)$ (3) $x^2(a+b-c)$ (4) $2xy(x^2y-2x+y^2)$
2. (1) $(x-y)(a+1)$ (2) $(b+c)(a-1)$ (3) $(x-2y)(3-x)$ (4) $a(b-2c)(4a-1)$
3. (1) $(x-8)^2$ (2) $(2x-5)^2$ (3) $9(a+b)^2$ (4) $(x+y+3)^2$
4. (1) $(a+7)(a-7)$ (2) $(3x+4y)(3x-4y)$ (3) $(x+y+z)(x-y-z)$ (4) $(1-a)(2x+y)(2x-y)$
5. (1) $(x-1)(x-9)$ (2) $2(a-3b)(a-4b)$ (3) $(x-y-4)(x-y-5)$ (4) $2(2a-3)(a+2)$
6. (1) $(2x-3)(x+1)$ (2) $(3x-4)(2x-1)$ (3) $(3x-1)(x-2)$ (4) $(3x-1)(2x+3)$
7. (1) $(x+2y-3)(x-y+1)$ (2) $(x-3y+4)(2x+y-1)$
 (3) $(3x-y+2)(x+2y+1)$ (4) $(x-2y-4)(2x+y-5)$
8. (1) $(x^2+7)(x^2-6)$ (2) $(2x^2-3)(x^2+2)$
 (3) $(x^2+4x+8)(x^2-4x+8)$ (4) $(x^2+2x-2)(x^2-2x-2)$
9. (1) $(x-1)^3$ (2) $(2a+b)^3$
 (3) $(2x+1)(4x^2-2x+1)$ (4) $(4a-3b)(16a^2+12ab+9b^2)$
10. (1) $(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$ (2) $(a+b-c)(a^2+2ab+b^2+ac+bc+c^2)$
 (3) $(x+1)(x^2-x+1)(x-2)(x^2+2x+4)$ (4) $(a+b)^2(a^2-ab+b^2)$
11. (2), (4), (6)
12. (1) 0.375 (2) 1.571428 (3) 0.43 (4) 0.255
13. (1) $\frac{4}{9}$ (2) $\frac{122}{99}$ (3) $\frac{521}{999}$ (4) $\frac{232}{99}$
14. 例) $a=1+\sqrt{2}$, $b=1-\sqrt{2}$
15. (1) 4 (2) 2 (3) 4 (4) $\sqrt{3}-1$ (5) 1 (6) 1
16. (1) 0 (2) 3 (3) -1 (4) 0
17. (1) $|x-3| = \begin{cases} -x+3 & (x<3) \\ x-3 & (x\geq 3) \end{cases}$ (2) $-2x-1$
18. (1) $2\sqrt{6}$ (2) 8 (3) $2-\sqrt{2}$ (4) ± 5
19. (1) $3\sqrt{2}-2\sqrt{3}$ (2) $5-2\sqrt{6}$ (3) 2 (4) $-1-2\sqrt{2}$
 (5) $3\sqrt{6}-\sqrt{2}$
20. (1) $3\sqrt{2}$ (2) $\frac{2\sqrt{3}}{3}$ (3) $3+\sqrt{2}$ (4) $3-2\sqrt{2}$
21. (1) 10 (2) 1 (3) 98 (4) 970
22. (1) 3 (2) ± 1 (3) $2\sqrt{5}$
23. (1) $a=3$, $b=2+\sqrt{3}-3=\sqrt{3}-1$ (2) $\frac{11\sqrt{3}+7}{6}$
24. (1) $\frac{19+11\sqrt{2}-5\sqrt{3}-2\sqrt{6}}{17}$ (2) $\frac{6-10\sqrt{3}-4\sqrt{5}+3\sqrt{15}}{11}$
25. (1) $\sqrt{7}-2$ (2) $\sqrt{3}+\sqrt{2}$
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解説

1. (1) $2x^2y - 6x = 2x(xy - 3)$
 (3) $(a+b)x^2 - cx^2 = x^2\{(a+b) - c\}$
 $= x^2(a+b-c)$
- (2) $3a - 4ab^2 = a(3 - 4b^2)$
 (4) $2x^3y^2 - 4x^2y + 2xy^3 = 2xy(x^2y - 2x + y^2)$
2. (1) $a(x-y) + x - y = a(x-y) + (x-y)$
 $= (x-y)(a+1)$
 (3) $3x - 6y - x(x-2y) = 3(x-2y) - x(x-2y)$
 $= (x-2y)(3-x)$
- (2) $a(b+c) - b - c = a(b+c) - (b+c)$
 $= (b+c)(a-1)$
 (4) $4a^2(b-2c) - ab + 2ac = 4a^2(b-2c) - a(b-2c)$
 $= (b-2c)(4a^2 - a)$
 $= a(b-2c)(4a-1)$
3. (1) $x^2 - 16x + 64 = (x-8)^2$
 (3) $9a^2 + 18ab + 9b^2 = 9(a^2 + 2ab + b^2)$
 $= 9(a+b)^2$
- (2) $4x^2 - 20x + 25 = (2x-5)^2$
 (4) $(x+y)^2 - 6(x+y) + 9 = \{(x+y) + 3\}^2$
 $= (x+y+3)^2$
4. (1) $a^2 - 49 = (a+7)(a-7)$
 (3) $x^2 - (y+z)^2 = \{x + (y+z)\}\{x - (y+z)\}$
 $= (x+y+z)(x-y-z)$
- (2) $9x^2 - 16y^2 = (3x+4y)(3x-4y)$
 (4) $4(1-a)x^2 + (a-1)y^2 = 4(1-a)x^2 - (1-a)y^2$
 $= (1-a)(4x^2 - y^2)$
 $= (1-a)(2x+y)(2x-y)$
5. (1) $x^2 - 10x + 9 = (x-1)(x-9)$
 (2) $2a^2 - 14ab + 24b^2 = 2(a^2 - 7ab + 12b^2)$
 $= 2(a-3b)(a-4b)$
 (3) $(x-y)^2 - 9(x-y) + 20 = \{(x-y) - 4\}\{(x-y) - 5\}$
 $= (x-y-4)(x-y-5)$
 (4) $(2a-1)^2 + 3(2a-1) - 10 = \{(2a-1) - 2\}\{(2a-1) + 5\}$
 $= (2a-3)(2a+4)$
 $= 2(2a-3)(a+2)$
6. (1) $2x^2 - x - 3 = (2x-3)(x+1)$
 (3) $3x^2 - 7x + 2 = (3x-1)(x-2)$
- (2) $6x^2 - 11x + 4 = (3x-4)(2x-1)$
 (4) $6x^2 + 7x - 3 = (3x-1)(2x+3)$
7. (1) $x^2 + xy - 2y^2 - 2x + 5y - 3 = x^2 + (y-2)x - 2y^2 + 5y - 3$
 $= x^2 + (y-2)x - (2y-3)(y-1)$
 $= (x+2y-3)(x-y+1)$
 (2) $2x^2 - 5xy - 3y^2 + 7x + 7y - 4 = 2x^2 - (5y-7)x - 3y^2 + 7y - 4$
 $= 2x^2 - (5y-7)x - (3y-4)(y-1)$
 $= (x-3y+4)(2x+y-1)$
 (3) $3x^2 + 5xy - 2y^2 + 5x + 3y + 2 = 3x^2 + 5(y+1)x - 2y^2 + 3y + 2$
 $= 3x^2 + 5(y+1)x - (y-2)(2y+1)$
 $= (3x-y+2)(x+2y+1)$
 (4) $2x^2 - 3xy - 2y^2 - 13x + 6y + 20 = 2x^2 - (3y+13)x - 2y^2 + 6y + 20$
 $= 2x^2 - (3y+13)x - 2(y+2)(y-5)$
 $= (x-2y-4)(2x+y-5)$

8. (1) $x^4 + x^2 - 42 = (x^2 + 7)(x^2 - 6)$ (2) $2x^4 + x^2 - 6 = (2x^2 - 3)(x^2 + 2)$
 (3) $x^4 + 64 = (x^4 + 16x^2 + 64) - 16x^2$ (4) $x^4 - 8x^2 + 4 = (x^4 - 4x^2 + 4) - 4x^2$
 $= (x^2 + 8)^2 - 16x^2$ $= (x^2 - 2)^2 - 4x^2$
 $= \{(x + 8) + 4x\}\{(x + 8) - 4x\}$ $= \{(x^2 - 2) + 2x\}\{(x^2 - 2) - 2x\}$
 $= (x^2 + 4x + 8)(x^2 - 4x + 8)$ $= (x^2 + 2x - 2)(x^2 - 2x - 2)$
9. (1) $x^3 - 3x^2 + 3x - 1 = (x - 1)^3$ (2) $8a^2 + 12a^2b + 6ab^2 + b^3 = (2a + b)^3$
 (3) $8x^3 + 1 = (2x + 1)(4x^2 - 2x + 1)$ (4) $64a^3 - 27b^3 = (4a - 3b)(16a^2 + 12ab + 9b^2)$
10. (1) $x^6 - y^6 = (x^3 + y^3)(x^3 - y^3)$
 $= (x + y)(x - y)(x^2 + xy + y^2)(x^2 - xy + y^2)$
 (2) $(a + b)^3 - c^3 = \{(a + b) - c\}\{(a + b)^2 + (a + b)c + c^2\}$
 $= (a + b - c)(a^2 + 2ab + b^2 + ac + bc + c^2)$
 (3) $x^6 - 8x^3 - 9 = (x^3 + 1)(x^3 - 8)$
 $= (x + 1)(x^2 - x + 1)(x - 2)(x^2 + 2x + 4)$
 (4) $a^4 + a^3b + ab^3 + b^4 = a^3(a + b) + b^3(a + b)$
 $= (a + b)(a^3 + b^3)$
 $= (a + b)(a + b)(a^2 - ab + b^2)$
 $= (a + b)^2(a^2 - ab + b^2)$
11. (1) $\sqrt{2}, \pi, -1 + \sqrt{3}$ は無理数である。0.5 は分数に直すことができるので有理数である。
12. (1) $\frac{3}{8} = 0.375$ (2) $\frac{11}{7} = 1.\dot{5}7142\dot{8}$
 (3) $\frac{43}{99} = 0.4\dot{3}$ (4) $\frac{255}{999} = 0.\dot{2}5\dot{5}$
13. (1) $0.\dot{4} = \frac{4}{9}$ (2) $1.\dot{2}\dot{3} = \frac{122}{99}$
 (3) $0.\dot{5}2\dot{1} = \frac{521}{999}$ (4) $2.\dot{2}\dot{4} = \frac{232}{99}$
14. (1) 例) $a = 1 + \sqrt{2}, b = 1 - \sqrt{2}$
15. (1) $|4| = 4$ (2) $|-2| = 2$
 (3) $|(-2)^2| = 4$ (4) $|1 - \sqrt{3}| = \sqrt{3} - 1$
 (5) $|-2| - |2| = 2 - 3 = 1$ (6) $|\sqrt{2} - 2| + |\sqrt{2} - 1| = 2 - \sqrt{2} + \sqrt{2} - 1 = 1$
16. (1) $|a + 2| - |a| = |-1 + 2| - |-1|$ (2) $|a^2| + |2a| = |(-1)^2| + |2 \times (-1)|$
 $= 1 - 1$ $= 1 + 2$
 $= 0$ $= 3$
 (3) $|-a| - |a + 3| = |-(-1)| - |-1 + 3|$ (4) $|2a| - 2|a| = |2 \times (-1)| - 2|-1|$
 $= 1 - 2$ $= 2 - 2$
 $= -1$ $= 0$
17. (1) $|x - 3| = \begin{cases} -x + 3 & (x < 3) \\ x - 3 & (x \geq 3) \end{cases}$
 (2) $x < -2$ のとき、 $x + 2 < 0, x - 1 < 0$ より、
 $|x + 2| + |x - 1| = -x - 2 + (-x + 1)$
 $= -2x - 1$

18. (1) $\sqrt{24} = 2\sqrt{6}$ (2) $\sqrt{64} = 8$
 (3) $\sqrt{(2-\sqrt{2})^2} = |2-\sqrt{2}|$ (4) ± 5
 $= 2-\sqrt{2}$
19. (1) $3\sqrt{2} + \sqrt{2} = 4\sqrt{2}$ (2) $\sqrt{18} - \sqrt{27} + \sqrt{3} = 3\sqrt{2} - 3\sqrt{3} + \sqrt{3}$
 $= 3\sqrt{2} - 2\sqrt{3}$
 (3) $(\sqrt{3} - \sqrt{2})^2 = 3 - 2\sqrt{6} + 2$ (4) $(\sqrt{5} - \sqrt{3})(\sqrt{5} + \sqrt{3}) = 5 - 3$
 $= 5 - 2\sqrt{6}$ $= 2$
 (5) $(\sqrt{2} - 3)(\sqrt{2} + 1) = 2 - 2\sqrt{2} - 3$ (6) $(\sqrt{6} - \sqrt{2})(\sqrt{3} + 4) = 3\sqrt{2} + 4\sqrt{6} - \sqrt{6} - 4\sqrt{2}$
 $= -1 - 2\sqrt{2}$ $= 3\sqrt{6} - \sqrt{2}$
20. (1) $\frac{6}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{2}}{2}$ (2) $\frac{4}{\sqrt{12}} = \frac{4}{2\sqrt{3}}$
 $= 3\sqrt{2}$ $= \frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$
 $= \frac{2\sqrt{3}}{3}$
 (3) $\frac{1}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{3+\sqrt{2}}{3-2}$ (4) $\frac{\sqrt{2}-1}{\sqrt{2}+1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1} = \frac{2-2\sqrt{2}+1}{2-1}$
 $= 3+\sqrt{2}$ $= 3-2\sqrt{2}$
21. (1) $x+y = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$
 $= \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}} + \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$
 $= \frac{(\sqrt{3}-\sqrt{2})^2}{3-2} + \frac{(\sqrt{3}+\sqrt{2})^2}{3-2}$
 $= (3-2\sqrt{6}+2) + (3+2\sqrt{6}+2)$
 $= 10$
 (2) $xy = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$
 $= 1$
 (3) $x^2 + y^2 = (x+y)^2 - 2xy$
 $= 10^2 - 2 \times 1$
 $= 98$
 (4) $x^3 + y^3 = (x+y)^3 - 3xy(x+y)$
 $= 10^3 - 3 \times 1 \times 10$
 $= 970$
22. $a \times \frac{1}{a} = 1$ に着目する。
 (1) $a^2 + \frac{1}{a^2} = \left(a + \frac{1}{a}\right)^2 - 2$ (2) $\left(a - \frac{1}{a}\right)^2 = \left(a + \frac{1}{a}\right)^2 - 4$
 $= (\sqrt{5})^2 - 2$ $= (\sqrt{5})^2 - 4$
 $= 3$ $= 1$
 (3) $a^3 + \frac{1}{a^3} = \left(a + \frac{1}{a}\right)^3 - 3\left(a + \frac{1}{a}\right)$ \therefore であるから、 $a - \frac{1}{a} = \pm 1$
 $= (\sqrt{5})^3 - 3\sqrt{5}$
 $= 2\sqrt{5}$

23. (1) $\frac{\sqrt{3}+1}{\frac{\sqrt{3}-1}{a} + \frac{1}{b}} \times \frac{\sqrt{3}+1}{\frac{\sqrt{3}+1}{a} + \frac{1}{b}} = \frac{3+2\sqrt{3}+1}{3-1} = 2+\sqrt{3}$ である。 $1 < \sqrt{3} < 2$ より、 $a=3, b=2+\sqrt{3}-3=\sqrt{3}-1$
- (2) $\frac{\frac{\sqrt{3}-1}{a} + \frac{1}{b}}{\frac{1}{b} + \frac{1}{a}} = \frac{\frac{\sqrt{3}-1}{a} + \frac{1}{b}}{\frac{1}{ab}}$
 $= \frac{3^2 + (\sqrt{3}-1)^2}{3 \times (\sqrt{3}-1)}$
 $= \frac{9 + (3 - 2\sqrt{3} + 1)}{3(\sqrt{3}-1)}$
 $= \frac{13 - 2\sqrt{3}}{3(\sqrt{3}-1)} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$
 $= \frac{(13 - 2\sqrt{3})(\sqrt{3}+1)}{3(3-1)}$
 $= \frac{13\sqrt{3} + 13 - 6 - 2\sqrt{3}}{6}$
 $= \frac{11\sqrt{3} + 7}{6}$
24. (1) $\frac{1}{\sqrt{2} + \sqrt{3} + \sqrt{4}} = \frac{1}{(2 + \sqrt{2}) + \sqrt{3}} \times \frac{(2 + \sqrt{2}) + \sqrt{3}}{(2 + \sqrt{2}) - \sqrt{3}}$
 $= \frac{2 + \sqrt{2} - \sqrt{3}}{4 - 2\sqrt{2} + 2 - 3}$
 $= \frac{3 + \sqrt{2} - \sqrt{3}}{5 - 2\sqrt{2}} \times \frac{5 + 2\sqrt{2}}{5 + 2\sqrt{2}}$
 $= \frac{15 + 5\sqrt{2} - 5\sqrt{3} + 6\sqrt{2} + 4 - 2\sqrt{6}}{25 - 8}$
 $= \frac{19 + 11\sqrt{2} - 5\sqrt{3} - 2\sqrt{6}}{17}$
- (2) $\frac{\sqrt{3} - \sqrt{4} + \sqrt{5}}{\sqrt{3} - \sqrt{4} - \sqrt{5}} = \frac{\sqrt{3} - 2 + \sqrt{5}}{(\sqrt{3} - 2) - \sqrt{5}} \times \frac{\sqrt{3} - 2 + \sqrt{5}}{(\sqrt{3} - 2) + \sqrt{5}}$
 $= \frac{(\sqrt{3} - 2 + \sqrt{5})^2}{3 - 4\sqrt{3} + 4 - 5}$
 $= \frac{3 + 4 + 5 - 4\sqrt{3} - 4\sqrt{5} + 2\sqrt{15}}{2 - 4\sqrt{3}}$
 $= \frac{12 - 4\sqrt{3} - 4\sqrt{5} + 2\sqrt{15}}{2(1 - 2\sqrt{3})}$
 $= \frac{6 - 2\sqrt{3} - 2\sqrt{5} + \sqrt{15}}{1 - 2\sqrt{3}} \times \frac{1 + 2\sqrt{3}}{1 + 2\sqrt{3}}$
 $= \frac{6 - 2\sqrt{3} - 2\sqrt{5} + \sqrt{15} + 12\sqrt{3} - 12 - 4\sqrt{15} + 6\sqrt{5}}{1 - 12}$
 $= \frac{-6 + 10\sqrt{3} + 4\sqrt{5} - 3\sqrt{15}}{-11}$
 $= \frac{6 - 10\sqrt{3} - 4\sqrt{5} + 3\sqrt{15}}{11}$
25. (1) $\sqrt{11 - 2\sqrt{28}} = \sqrt{(\sqrt{7} - \sqrt{4})^2}$
 $= \sqrt{7} - \sqrt{4}$
 $= \sqrt{7} - 2$
- (2) $\sqrt{5 + \sqrt{24}} = \sqrt{5 + 2\sqrt{6}}$
 $= \sqrt{(\sqrt{3} + \sqrt{2})^2}$
 $= \sqrt{3} + \sqrt{2}$