COMP 9024, Homework Solution, 11s2, Class 1

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In each of the following situations, indicate whether $f = O(g)$, or $f = \Omega(g)$, or both (in which case $f = \Theta(g)$).

1. $f(n) = n - 100$ and $g(n) = n - 200$.

   Both are $O(n)$, so $f = \Theta(g)$.

2. $f(n) = n^{1/2}$ and $g(n) = n^{2/3}$.

   For powers of n, just compare the powers. We have that $1/2 < 2/3$, so $f = O(g)$.

3. $f(n) = 100n + \log n$ and $g(n) = n + (\log n)^2$.

   Both are $O(n)$, so $f = \Theta(g)$.

4. $f(n) = n \log n$ and $g(n) = 10n \log 10n$.

   Both are $O(n \log n)$, so $f = \Theta(g)$.

5. $f(n) = \log 2n$ and $g(n) = \log 3n$.

   Both are $O(\log n)$, so $f = \Theta(g)$.

6. $f(n) = 10 \log n$ and $g(n) = \log(n^2)$.

   Both are $O(\log n)$, so $f = \Theta(g)$.

7. $f(n) = n^{1.01}$ and $g(n) = n \log^2 n$.

   If we divide both sides by n, we need to compare $n^{0.01}$ and $\log^2 n$. It takes a really long time, but ultimately the power function wins out, so $f = \Omega(g)$. You can play with `www.wolframalpha.com` to see this.

8. $f(n) = n^2 / \log n$ and $g(n) = n(\log n)^2$.

   If we divide both sides by $n/\log n$, we need to compare n and $(\log n)^3$. The result is $f = \Omega(g)$.

9. $f(n) = n^{0.1}$ and $g(n) = (\log n)^{10}$.

   Once again, $f = \Omega(g)$.

10. $f(n) = (\log n)^{\log n}$ and $g(n) = n/\log n$.

    The function $f(n) = n^{\log\log n}$, so $f = \Omega(g)$.

11. $f(n) = \sqrt{n}$ and $g(n) = (\log n)^3$.

    Once again, $f = \Omega(g)$.

12. $f(n) = n^{1/2}$ and $g(n) = 5^{\log_2 n}$.

    The function $g(n) = n^{\log_2 5} \approx n^{2.32}$, so $f = O(g)$.

13. $f(n) = n2^n$ and $g(n) = 3^n$.

    Here, $f = O(g)$.

14. $f(n) = 2^n$ and $g(n) = 2^{n+1}$.

    Here, $f = \Theta(g)$.

15. $f(n) = n!$ and $g(n) = 2^n$.

    Lookup "factorial" in `www.wikipedia.org`, and you will find that $n! > \sqrt{2\pi n}(\frac{n}{e})^n$. Hence $f = \Omega(g)$.

16. $f(n) = (\log n)^{\log n}$ and $g(n) = 2^{(\log_2 n)^2}$.

    The function $f(n) = n^{\log\log n}$ and the function $g(n) = (2^{\log_2 n})^{\log_2 n} = n^{\log_2 n}$. Hence $f = O(g)$.

17. $f(n) = \sum_{i=1}^{n} i^k$ and $g(n) = n^{k+1}$.

    We have that $f = \Theta(g)$. It is easy to prove that $f = O(g)$, but harder to prove that $f = \Theta(g)$.

These exercises came from the book *Algorithms*, by S. Dasgupta, C. H. Papadimitriou and U.V. Vazirani (freely available on the Web).