W1 - HW Problems

①

inputs
$\begin{cases} \text{Lydia} \\ \text{Eugene} \\ \text{Janet} \\ \text{Rick} \\ \text{Marty} \end{cases}$

→ 321-549-3327
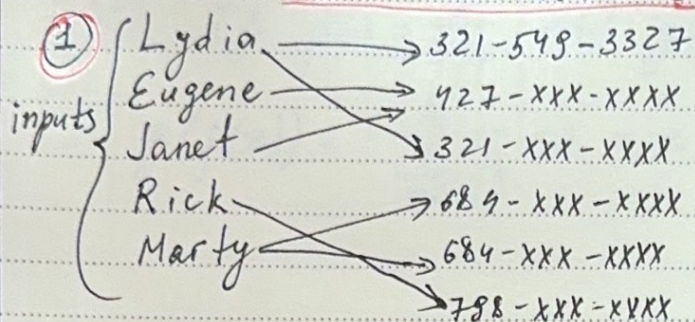→ 427-XXX-XXXX
→ 321-XXX-XXXX
→ 689-XXX-XXXX
→ 684-XXX-XXXX
→ 788-XXX-XXXX

This relation is not a function. Both Lydia and Marty (inputs) have two (outputs) numbers each.

② Determine if each of the following equations are functions

a) $y = x^2 + 1$ let $x = 2$

$y = 2^2 + 1$

$y = 5$ This is a function, it has exactly one output.

b) $y^2 = x + 1$

$y^2 = 2 + 1$

$y^2 = 3$

$y = \sqrt{3}$ It can be either negative or positive, so it's not a function.

③④ Which functions are surjective (i.e. onto)?
Which functions are injective (i.e. one-to-one)?

1) $f: \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$
The range of the function is not equal to $\mathbb{Z}$, it's $3\mathbb{Z}$
So it's injective, every output has at most one input.

2) $g: \{1, 2, 3\} \to \{a, b, c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$
g isn't surjective. There's no input (in the domain) for $g(x) = b$, which is in the codomain.
g isn't injective.

⑤ If $f(x) = \frac{1}{x+2}$ and $g(x) = \frac{1}{x} - 2$, is $g = f^{-1}$?

$$g(f(x)) = \frac{1}{\frac{1}{x+2}} - 2 = x + 2 - 2 = x$$

$$\boxed{g = f^{-1} \text{ and } f = g^{-1}}$$

⑥ Find the inverse of the function $f(x) = 2 + \sqrt{x-4}$.

$$y = 2 + \sqrt{x-4}$$
$$y - 2 = \sqrt{x-4}$$
$$(y-2)^2 = x-4$$
$$(y-2)^2 + 4 = x$$
$$x = (y-2)^2 + 4$$
$$\boxed{f^{-1}(x) = (x-2)^2 + 4}$$

⑦ Find a formula for the inverse function that gives Fahrenheit temperature as a function of Celsius temperature.

$$C = \frac{5}{9}(F - 32) \qquad \boxed{F = \frac{9}{5}C + 32}$$

⑧ Find the domain of and range of the following function.

$g(x) = 2\sqrt{x-4}$ $g(4) = 2\sqrt{4-4}$

$x - 4 \geq 0$ $g(4) = 2 \cdot 0$

$x \geq 4$ $g(4) = 0$

Domain $g(x): [4, +\infty)$ Range: $g(x) \; [0, +\infty)$