W4 HW Problems. ~~Bo~~Boolean Logic.

① a) Quixote Media had the largest annual revenue False
b) Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue. Both of them are TRUE
c) Acme Computer had the largest net profit or Quixote Media had the largest net profit. One of them is true. So statement is TRUE.
d) If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue. At least one of them is true. So, statement is TRUE
e) Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue. TRUE

② Let p and q be the propositions
p: It is below freezing
q: It is snowing.
a) It is below freezing and snowing $p \wedge q$
b) It is below freezing but not snowing $p \wedge \neg q$
c) It isn't below freezing and it is not snowing $\neg p \wedge \neg q$
d) It is either snowing or below freezing (or both) $q \vee p$
e) If it is below freezing, it is also snowing. $p \rightarrow q$
f) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing $(p \vee q) \wedge (p \rightarrow \neg q)$
g) That it is below freezing is necessary and sufficient for it to be snowing. $p \leftrightarrow q$

③ How many rows appear in a truth table for each of these compound propositions.

⑪ Construct a truth table for each of these compound propositions

a) $p \wedge \neg p$ b) $p \vee \neg p$

p	$\neg p$	$p \wedge \neg p$	$p \vee \neg p$
T	F	F	T
F	T	F	T

c) $(p \vee \neg q) \to q$

p	q	$\neg q$	$p \vee \neg q$	$(p \vee \neg q) \to q$
T	T	F	T	T
T	F	T	T	F
F	T	F	F	T
F	F	T	T	F

d) $(p \vee q) \to (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$(p \vee q) \to (p \wedge q)$
T	T	T	T	T
T	F	T	F	F
F	T	T	F	F
F	F	F	F	T

e) $(p \to q) \leftrightarrow (\neg q \to \neg p)$

p	q	$p \to q$	$\neg q \to \neg p$	$(p \to q) \leftrightarrow (\neg q \to \neg p)$
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

f) $(p \to q) \to (q \to p)$

p	q	$p \to q$	$q \to p$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

⑤ The question was: „Does everyone want coffee?" The first and second professors answer: „I do not know". We can understand that they want coffee but they're not sure about others. Finally the third says: „No, not everyone wants". We understand only the third professor does not want coffee.

⑥ Use De Morgan's laws to find the negation of each of the following statements.

a) Jan is rich(r) and happy(h) $\neg r \vee \neg h$

b) Carlos will bicycle(p) or run tomorrow(q) $\neg p \wedge \neg q$

c) Mei walks(p) or takes(q) the bus to class $\neg p \wedge \neg q$

d) Ibrahim is smart(p) and hard working(q) $\neg p \vee \neg q$

⑦ a) $(p \wedge q) \rightarrow p$ b) $p \rightarrow (p \vee q)$ c) $\neg p \rightarrow (p \rightarrow q)$

p	q	$(p \wedge q) \rightarrow p$	$p \rightarrow (p \vee q)$	$\neg p$	$\rightarrow$	$(p \rightarrow q)$	$\equiv$
T	T	T	T	F	$\rightarrow$	T	$\equiv$ T
T	F	T	T	F	$\rightarrow$	F	$\equiv$ T
F	T	T	T	T	$\rightarrow$	T	$\equiv$ T
F	F	T	T	T	$\rightarrow$	T	$\equiv$ T

d) $(p \wedge q) \rightarrow (p \rightarrow q)$

$p \wedge q$	$p \rightarrow q$	$(p \wedge q) \rightarrow (p \rightarrow q)$
T	T	T
F	F	T
F	T	T
F	T	T

e) $\neg(p \rightarrow q) \rightarrow p$

$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow p$
F	T
T	T
F	T
F	T

f) $\neg(p \rightarrow q) \rightarrow \neg q$

$\neg(p \rightarrow q)$	$\neg q$	$\neg(p \rightarrow q) \rightarrow \neg q$
F	F	T
T	T	T
F	F	T
F	T	T

(10) $P(x)$ = „x can speak Russian"
$Q(x)$ = „x knows the computer language C++"
The domain for quantifiers consists of all students at your school.

a) There is a student at your school who can speak Russian and who knows C++. $\exists x\,(P(x) \land Q(x))$

b) There is a student at your school who can speak Russian but who doesn't know C++. $\exists x\,(P(x) \land \neg Q(x))$

c) Every student at your school either can speak Russian or knows C++. $\forall x\,(P(x) \lor Q(x))$

d) No student at your school can speak Russian or knows C++. $\forall x\,\neg(P(x) \lor Q(x))$

(11) Determine the truth value of each of these statements if the domain for all variables consists of all integers

a) $\forall n\,(n^2 \geq 0)$ True
c) $\forall n\,(n^2 \geq n)$ True
b) $\exists n\,(n^2 = 2)$ False
d) $\exists n\,(n^2 < 0)$ False

(12) $P(x) = \{0, 1, 2, 3, 4\}$. Write out each of these propositions using disjunctions, conjunctions, and negations

a) $\exists x\, P(x)$ $P(0) \lor P(1) \lor P(2) \lor P(3) \lor P(4)$

b) $\forall x\, P(x)$ $P(0) \land P(1) \land P(2) \land P(3) \land P(4)$

c) $\exists x \neg P(x)$ $\neg P(0) \lor \neg P(1) \lor \neg P(2) \lor \neg P(3) \lor \neg P(4)$

d) $\forall x \neg P(x)$ $\neg P(0) \land \neg P(1) \land \neg P(2) \land \neg P(3) \land \neg P(4)$

e) $\neg \exists x\, P(x)$ $\neg(P(0) \lor P(1) \lor P(2) \lor P(3) \lor P(4))$

f) $\neg \forall x\, P(x)$ $\neg(P(0) \land P(1) \land P(2) \land P(3) \land P(4))$

(13) Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.

a) Someone in your class can speak Hindi $\exists x P(x)$

b) Everyone in your class is friendly $\forall x F(x)$

c) There is a person in your class who was not born in California $\exists x \neg B(x)$

d) A student in your class has been in a movie $\exists x M(x)$

e) No student in your class has taken a course in logic programming. $\forall x \neg L(x)$

(14) Let $M(x,y)$ be „x has sent y an e-mail message" and $T(x,y)$ be „x has telephoned y," where the domain consists of all students in your class. Use quantifiers to express each of these statements. ~~(Assume~~

a) Chou has never sent an e-mail message to Koko. $\neg M(Chou, Koko)$

b) Arlene has never sent an e-mail message to or telephoned Sarah. $\neg M(A,S) \land \neg T(A,S)$

c) Jose has never received an e-mail message from Deborah. $\neg M(Deborah, Jose)$

d) Every student in your class has sent an e-mail message to Ken. $\forall x\, M(x, Ken)$

e) No one in your class has telephoned Nina. $\forall x \neg T(x, Nina)$

f) Everyone in your class has either telephoned Avi or sent him an e-mail message. $\forall x (T(x, Avi) \lor M(x, Avi))$

g) There is a student in your class who has sent everyone else in your class an e-mail message. $\exists x \forall y (y \neq x \rightarrow M(x,y))$

h) There is someone in your class who has either sent an e-mail message or telephoned everyone else in your class. $\exists x \forall y (y \neq x \rightarrow (M(x,y) \lor T(x,y)))$

(i) There are two different students in your class who have sent each other e-mail messages. $\exists x \exists y (x \neq y \land M(x,y) \land M(y,x))$

j) There's a student who has sent himself or herself an e-mail message. $\exists x\, M(x,x)$

k) There's a student in your class who has not received an e-mail message from anyone else in the class and who has not been called by any other student in the class.

$\exists x \forall y (x \neq y \to (\neg M(y,x) \land \neg T(y,x)))$

l) Every student in the class has either received an e-mail message or received a telephone call from another student in the class. $\forall x \exists y (x \neq y \land (M(y,x) \lor T(y,x)))$

m) There are ~~two~~ at least two students in your class such that one student has sent the other e-mail and the second student has telephoned. $\exists x \exists y (x \neq y \land M(x,y) \land T(y,x))$

n) There are two different students in your class who between them have sent an e-mail message to or telephoned everyone else in the class.

$\exists x \exists y (x \neq y \land \forall z ((z \neq x \land z \neq y) \to ((M(x,z) \lor M(y,z)) \lor (T(x,z) \lor T(y,z)))))$