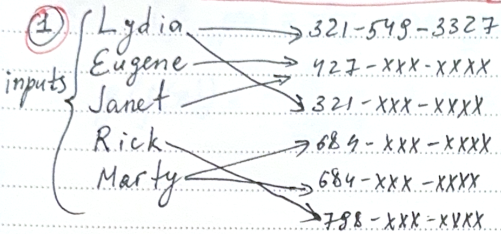
W1 - HW Problems

1)


This relation is not a function. Both Lydia and Marty (inputs) have two (outputs) numbers each.

2) Determine if each of the following equations are functions

a) $y = x^2 + 1$ let $x = 2$

$y = 2^2 + 1$

$y = 5$ This is a function, it has exactly one output.

b) $y^2 = x + 1$

$y^2 = 2 + 1$

$y^2 = 3$

$y = \sqrt{3}$ It can be either negative or positive, so it's not a function.

3, 4) Which functions are surjective (i.e. onto)?

Which functions are injective (i.e.) one-to-one)?

1) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = 3n$

The range of the function is not equal to $\mathbb{Z}$, it's $3\mathbb{Z}$

So it's injective, every output has at most one input.

2) $g : \{1, 2, 3\} \to \{a, b, c\}$ defined by $g = \begin{pmatrix} 1 & 2 & 3 \\ c & a & a \end{pmatrix}$

g isn't surjective. There's no input (in the domain) for $g(x) = b$, which is in the codomain.

g isn't injective.

(5) If $f(x)=\frac{1}{x+2}$ and $g(x)=\frac{1}{x}-2$, is $g=f^{-1}$?

$$g(f(x)) = \frac{1}{\left(\frac{1}{x+2}\right)} - 2 = x+\cancel{2}-\cancel{2} = x$$

$$\boxed{g=f^{-1} \text{ and } f=g^{-1}}$$

(6) Find the inverse of the function $f(x)=2+\sqrt{x-4}$.

$$y = 2+\sqrt{x-4}$$
$$y-2 = \sqrt{x-4}$$
$$(y-2)^2 = x-4$$
$$(y-2)^2+4 = x$$
$$x = (y-2)^2+4$$
$$\boxed{f^{-1}(x) = (x-2)^2+4}$$

(7) Find a formula for the inverse function that gives Fahrenheit temperature as a function of Celsius temperature.

$$C = \frac{5}{9}(F-32) \qquad \boxed{F = \frac{9}{5}C + 32}$$

(8) Find the domain ~~of~~ and range of the following function:

$$g(x) = 2\sqrt{x-4} \qquad g(4) = 2\sqrt{4-4}$$
$$x-4 \geq 0 \qquad g(4) = 2\cdot 0$$
$$x \geq 4 \qquad g(4) = 0$$

Domain $g(x)$: $[4,+\infty)$ Range: ~~$g(x)$~~ $[0,+\infty)$

⑨ Find the domain and range of the following function:

$h(x) = -2x^2 + 4x - 9$

$\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ $\quad a = -2 \quad b = 4$

$\left(-\frac{4}{2\cdot(-2)}, f\left(-\frac{4}{2\cdot(-2)}\right)\right)$

$(1, f(1)) = (1, -2-9)$ $\quad (-1, -11)$

D: $(-\infty, +\infty)$

R: $[-11, +\infty)$

⑩ Find the domain of the following functions:

$f(x) = \frac{x-4}{x^2-2x-15}$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x - 5 = 0$

$x = 5$

$x + 3 = 0$

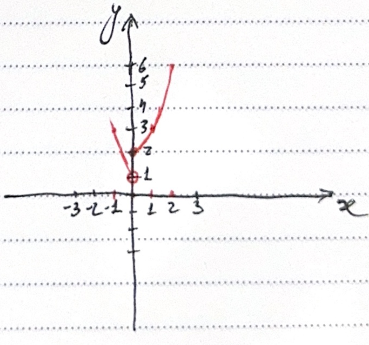
$x = -3$

These are excluded from the domain

D: $(-\infty, -3) \cup (-3, 5) \cup (5, +\infty)$

⑪ Evaluate the following piecewise-defined function for the given values of x, and graph the function:

$$f(x) = \begin{cases} -2x + 1 & -1 \le x < 0 \\ x^2 + 2 & 0 \le x \le 2 \end{cases}$$



$x = -1$

$f(x) = -2x + 1 = -2\cdot(-1) + 1 = 3$

$x = 0$

$f(x) = -2\cdot 0 + 1 = 1$

$x = 0 \rightarrow 0^2 + 2 = 2$

$x = 1 \rightarrow 1^2 + 2 = 3$

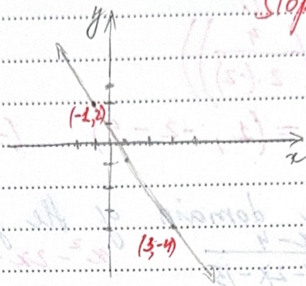
$x = 2 \rightarrow 2^2 + 2 = 6$

(12) Find the slope of the line that passes through the points (-1, 2) and (3, -4). Plot the points and graph the line.

$(x_1, y_1) = (-1, 2)$
$(x_2, y_2) = (3, -4)$

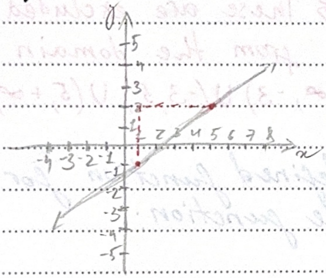
$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-2}{3-(-1)} = \frac{-6}{4} = -\frac{3}{2}$

Slope is $-\frac{3}{2}$



(13) Graph the line passing through the point (1, -1) whose slope is $m = \frac{3}{4}$

$m = \frac{3}{4} = \frac{\text{rise (vertically)}}{\text{run (horizontally)}}$



(14) Given the function $f(t)$ shown in Figure 1.3.1, find the average rate of change on the interval [-1, 2].

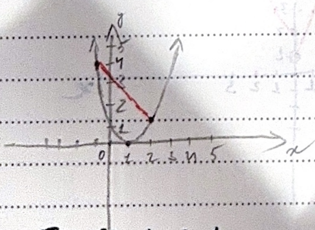


Figure 1.3.1.

$ARC_{[-1,2]} = \frac{f(2) - f(-1)}{2-(-1)} = \frac{1-4}{3} = \frac{-3}{3} = -1.$

(15) Compute the average rate of change of $f(x) = x^2 - \frac{1}{x}$ on the interval $[2,4]$.

$f(2) = 2^2 - \frac{1}{2} = 4 - \frac{1}{2} = \frac{7}{2}$

$f(4) = 4^2 - \frac{1}{4} = 16 - \frac{1}{4} = \frac{63}{4}$

$ARC_{[2,4]} = \frac{f(4) - f(2)}{4-2} = \frac{\frac{63}{4} - \frac{7}{2}}{2} = \frac{\frac{49}{4}}{2} = \frac{49}{8}$

(16) Given $f(t) = t^2 - t$ and $h(x) = 3x + 2$, evaluate $f(h(1))$.

$h(1) = 3 \cdot 1 + 2 = 5$

$f(h(1)) = f(5)$

$f(5) = 5^2 - 5 = 25 - 5 = 20$

$f(h(1)) = 20$

(17) find the domain of $(f \circ g)(x)$ where $f(x) = \frac{5}{x-1}$ and $g(x) = \frac{4}{3x-2}$

$x \neq \frac{2}{3}$ in $g(x)$, also $g(x)$ cannot be 1.

$g(x) = \frac{4}{3x-2} = 1$

$4 = 3x - 2$

$6 = 3x$

$x = 2$

Domain of $(f \circ g)(x)$:

$\left(-\infty, \frac{2}{3}\right) \cup \left(\frac{2}{3}, 2\right) \cup (2, \infty)$

(18) Find and simplify the functions $(g-f)(x)$ and $\left(\frac{g}{f}\right)(x)$, given $f(x) = x - 1$ and $g(x) = x^2 - 1$. Are they the same func?

$(g-f)(x) = g(x) - f(x)$

$(g-f)(x) = x^2 - 1 - (x-1)$

$(g-f)(x) = x^2 - x$

$(g-f)(x) = x(x-1)$

$\left(\frac{g}{f}\right)(x) = \frac{x^2-1}{x-1}$

$\left(\frac{g}{f}\right)(x) = \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}}$

$\left(\frac{g}{f}\right)(x) = x + 1$

$(g-f)(x) \neq \left(\frac{g}{f}\right)(x)$

(19) Write a formula for the graph shown in figure 1.5.12, which is a transformation of the square root function.

$h(x) = f(x - \underset{\text{shift}}{\overset{+1}{\text{horizontal}}}) + \underset{\text{shift}}{\overset{+2}{\text{vertical}}}$

$h(x) = f(x-1)+2$

Graph of a square root function transposed right one unit and up 2.

We use the formula for the square root function, we can write $h(x) = \sqrt{x-1} + 2$

(20) Write a formula for a transformation of the reciprocal function $f(x) = \frac{1}{x}$ that shifts the function's graph one unit to the right and one unit up.

$h(x) = f(x - \overset{1}{\text{hor. shift}}) + \overset{1}{\text{ver. shift}}$ $f(x) = \frac{1}{x}$

$h(x) = \frac{1}{x-1} + 1$

(21) Is the function $f(x) = x^3 + 2x$ even, odd, or neither?

$f(-x) = (-x)^3 + 2\cdot(-x)$

$= -x^3 - 2x$ odd function

(22) Is the function $f(s) = s^4 + 3s^2 + 7$ even, odd, or neither?

$f(-s) = (-s)^4 + 3\cdot(-s)^2 + 7$

$= s^4 + 3s^2 + 7$

$f(-s) = f(s)$

The function is even

(23) Write the point-slope form of equation of a line that passes through the points (5, 1) and (8, 7). Then rewrite it in the slope-intercept form. $y = mx + b$

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $(x_1, y_1) = (5, 1)$ $(x_2, y_2) = (8, 7)$

$m = \frac{7-1}{8-5} = \frac{6}{3} = 2$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - 5)$

$y - 1 = 2x - 10$

$y = 2x - 9$

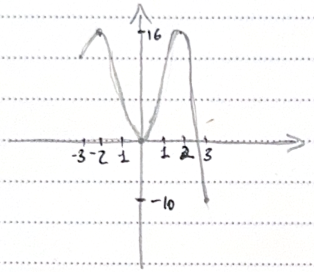
(24) If $f(x)$ is a linear function, and $(3,-2)$ and $(8,1)$ are points on the line, find the slope. Is this function increasing or decreasing?

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $\quad (x_1, y_1) = (3,-2)$ $\quad m = \frac{1+2}{8-3} = \frac{3}{5} > 0$

$(x_2, y_2) = (8, 1)$

The function is increasing because $m > 0$

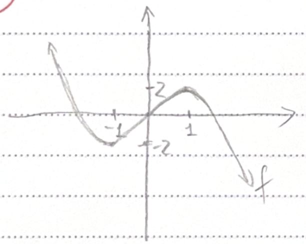
(25) For the function f shown in Figure 3.4.14, find all absolute maxima and minima.



Abs. max. $=16$ $x = 2$, $x = -2$

Abs. min. $= -10$ at $x = 3$

(26) Find all local maxima and minima.



Local maxima at $x = 1$ is 2

Local minima at $x = -1$ is -2.

(27) Given the functions below, identify the functions whose graphs are a pair of parallel lines and a pair of perpendicular lines.

$f(x) = 2x + 3$ $\qquad h(x) = -2x + 2$

$g(x) = \frac{1}{2}x - 4$ $\qquad j(x) = 2x - 6$

$f(x)$ and $j(x)$ are parallel, because they have same slope 2.
$g(x)$ and $h(x)$ are perpendicular, because they have negative and reciprocal slopes $(\frac{1}{2}, -2)$.

(28) $\begin{cases} 2x + y = 7 \\ x - 2y = 6 \end{cases}$ $\Rightarrow y = 7 - 2x$

$\Rightarrow x - 2(7-2x) = 6$

$x - 14 + 4x = 6$

$5x = 20$

$x = 4$

$2x + y = 7$

$2(4) + y = 7$

$8 + y = 7$

$y = -1$

$\begin{cases} 2 \cdot 4 + (-1) = 7 \\ 4 - 2(-1) = 6 \end{cases} \Rightarrow \begin{matrix} 8 - 1 = 7 \\ 4 + 2 = 6 \end{matrix} \Rightarrow \begin{matrix} 7 = 7 \\ 6 = 6 \end{matrix}$

(29) $\begin{cases} 4x + 2y = 4 \\ 6x - y = 8 \end{cases}$ $\Rightarrow 2y = -4x + 4$

$y = -2x + 2$

$6x - y = 8$

$6x - (-2x + 2) = 8$

$6x + 2x - 2 = 8$

$8x = 10$

$x = \frac{5}{4}$

$4\left(\frac{5}{4}\right) + 2y = 4$

$5 + 2y = 4$

$2y = -1$

$y = -\frac{1}{2}$

$\begin{cases} \cancel{4} \cdot \frac{5}{\cancel{4}} + \cancel{2} \cdot \left(-\frac{1}{\cancel{2}}\right) = 4 \\ \overset{3}{\cancel{6}} \cdot \frac{5}{\underset{2}{\cancel{4}}} \cancel{+} - \frac{1}{2} = 8 \end{cases}$

$5 + (-1) = 4$

$5 - 1 = 4$

$4 = 4$

$\frac{15}{2} + \frac{1}{2} = 8$

$\frac{16}{2} = 8$

$8 = 8$

(30) Find the vertex of the quadratic function: $f(x) = 2x^2 - 6x + 7$
Rewrite the quadratic in standard form (vertex form).

$h = -\frac{b}{2a}$

$h = -\frac{-6}{2 \cdot 2} = \frac{6}{4} = \frac{3}{2}$

$k = f(h)$

$k = f\left(\frac{3}{2}\right) = 2 \cdot \left(\frac{3}{2}\right)^2 - 6 \cdot \frac{3}{2} + 7 = \frac{5}{2}$

$f(x) = 2\left(x - \frac{3}{2}\right)^2 + \frac{5}{2}$

(31) Find the domain and range of $f(x) = -5x^2 + 9x - 1$
Quadratic function's domain is all real numbers.

$a = -5$ $\quad h = -\frac{b}{2a}$ $\quad h = -\frac{9}{2 \cdot (-5)} = \frac{9}{10}$

max. value at $f(h)$ $\quad f\left(\frac{9}{10}\right) = -5\left(\frac{9}{10}\right)^2 + 9 \cdot \frac{9}{10} - 1 = \frac{61}{20}$

Range: $\left(-\infty; \frac{61}{20}\right]$

(32) Find the y- and x- intercepts of the quadratic $f(x) = 3x^2 + 5x - 2$

$f(0) = 3 \cdot 0^2 + 5 \cdot 0 - 2 = -2$ y-intercept is at $(0,-2)$

$f(x) = 0$ $3x^2 + 5x - 2 = 0$

$(3x-1)(x+2) = 0$

$3x - 1 = 0$ $x + 2 = 0$

$3x = 1$ $x = -2$ x-intercept ~~is~~ at $(\frac{1}{3}, 0)$ and $(-2, 0)$

$x = \frac{1}{3}$

(33) Solve the inequality, graph the solution set on a number line and show the solution set in interval notation.

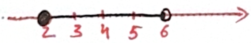
a. $-1 \leq 2x - 5 < 7$

$-1 + 5 \leq 2x - 5 + 5 < 7 + 5$

$4 \leq 2x < 12$

$2 \leq x < 6$

$[2, 6)$

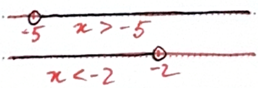


b. $x^2 + 7x + 10 < 0$ $-5 < x < -2$ $(-5, -2)$

$(x+5)(x+2) < 0$

$x + 5 > 0 \cup x + 2 < 0$

$x > -5 \cup x < -2$



c. $-6 < x - 2 < 4$

$-6 + 2 < x - 2 + 2 < 4 + 2$

$-4 < x < 6$

$(-4, 6)$

(34) Solve the inequality and graph the solution set. State the answer in both set builder notation and in interval notation. $10 - (2y + 1) \leq -4(3y + 2) - 3$

$10 - 2y - 1 \leq -12y - 8 - 3$

$-2y + 12y \leq -8 - 3 + 1 - 10$ $\{y \mid y \leq -2\}$

$10y \leq -20$ $(-\infty, -2]$

$y \leq -2$

35 Solve $x(x+3)^2(x-4)<0$

$x=0$ $(x+3)^2=0$ $x-4=0$ $(-\infty,-3)$ $x=-4$

$x=-3$ $x=4$ $(-3,0)$ $x=-2$

We've to test where $f(x)<0$ is correct $(0,4)$ $x=1$

$(4,\infty)$ $x=5$

$-4(-4+3)^2(-4-4)<0$ $-2(-2+3)^2(-2-4)<0$ $1(1+3)^2(1-4)<0$

$-4\cdot1\cdot(-8)<0$ $-2\cdot1\cdot(-6)<0$ $1\cdot16\cdot(-3)<0$

$-4\cdot(-8)<0$ $12\not<0$ $-48<0$ ✓

$32\not<0$

$5(5+3)^2(5-4)<0$

$5\cdot64\cdot1<0$ Solution: $(0,4)$

$320\not<0$ $0<x<4$

36 Solve: $2x^4>3x^3+9x^2$

$2x^4-3x^3-9x^2>0$ $x^2=0$ $2x+3=0$ $x-3=0$

$x^2(2x^2-3x-9)>0$ $x=0$ $2x=-3$ $x=3$

$x^2(2x+3)(x-3)>0$ $x=-\frac{3}{2}$

We'll test where $f(x)>0$ is correct. $(-\infty,-\frac{3}{2})$ ✓ $(-\frac{3}{2},0)$ $(0,3)$ $(3,\infty)$ ✓

1. $x=-2$

$(-2)^2(2\cdot(-2)+3)(-2-3)>0$

$4\cdot(-1)\cdot(-5)>0$

$20>0$ ✓

2. $x=-1$

$(-1)^2\cdot(2\cdot(-1)+3)(-1-3)>0$

$1\cdot1\cdot(-4)>0$

$-4\not>0$

3. $x=1$

$1^2(2\cdot1+3)(1-3)>0$

$1\cdot5\cdot(-2)>0$

$-10\not>0$

4. $x=4$

$4^2(2\cdot4+3)(4-3)>0$

$16\cdot11\cdot1>0$ ✓

$(-\infty,-\frac{3}{2})\cup(3,\infty)$

(37) Given the function $f(x) = -\frac{1}{2}|4x-5|+3$, determine the x-values for which the function values are negative.

$-\frac{1}{2}|4x-5|+3<0$ $\quad$ $4x-5=6$ $\quad$ $4x-5=-6$

$-\frac{1}{2}|4x-5|<-3$ $\quad$ $4x=11$ $\quad$ $4x=-1$

$x=\frac{11}{4}$ $\quad$ $x=-\frac{1}{4}$

$|4x-5|>6$

$x<-\frac{1}{4}$ or $x>2\frac{3}{4}$ $\quad$ $(-\infty,-0.25)\cup(2.75,\infty)$

(38) Solve: $13-2|4x-7|\leq 3$

$-2|4x-7|\leq -10$

$|4x-7|\geq 5$

$4x-7\geq 5$ $\quad$ $4x-7\leq -5$

$4x\geq 12$ $\quad$ $4x\leq 2$

$x\geq 3$ $\quad$ $x\leq\frac{1}{2}$ $\quad$ $(-\infty,\frac{1}{2}]\cup[3,\infty)$