Assignment 3

Due: 2:15 pm, 21 October 2021 (Thursday)

Course: Quantum Mechanics - 1
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Problem 1

Derive an expression for the time-dependent expectation value of an operator $\hat{O}$ by representing the initial ket $|\alpha, 0\rangle$ in a basis of eigenkets of an operator $\hat{A}$ satisfying $[\hat{A}, \hat{H}] = 0$, where $\hat{H}$ is the Hamiltonian operator. Write the resulting expression as a sum of *direct* and *interference* terms: $\sum_{k}[\ldots] + \sum_{k}\sum_{l \neq k}[\ldots]$

Problem 2

For the particle-in-a-box, derive an expression for the time-dependent position expectation value, $\langle \hat{x} \rangle(t)$, for a system in the superposed state $|\alpha\rangle = (|1\rangle + |2\rangle)/\sqrt{2}$. Plot the resulting expression as a function of time.

Problem 3

Suppose a system in some initial ket $|\alpha, 0\rangle$ evolves in time. The shortest time, $t > 0$, when the square of the correlation amplitude, $|\langle \alpha, 0 | \alpha, t \rangle|^2$, reaches 1 is called the revival time. The Hamiltonian operator represented in a basis $|1\rangle, |2\rangle, |3\rangle$ results in the matrix

$$\begin{bmatrix} E_1 & 0 & A \\ 0 & E_2 & 0 \\ A & 0 & E_3 \end{bmatrix}$$

Find the revival time for the initial ket $|\alpha, 0\rangle = |1\rangle$.

Problem 4

A spin-1/2 system in the initial ket $|\alpha, 0\rangle = |+z\rangle$ evolves in a magnetic field given by $\vec{B} = B_0 \hat{z}$.

a) Find expressions for $|\langle \pm x | \alpha, t \rangle|^2$, $|\langle \pm y | \alpha, t \rangle|^2$, and $|\langle \pm z | \alpha, t \rangle|^2$. Plot these as functions of time.

b) Find expressions for $\langle \hat{s}_x \rangle(t)$, $\langle \hat{s}_y \rangle(t)$, and $\langle \hat{s}_z \rangle(t)$. Plot these as functions of time.

Problem 5

Suppose a system in the initial ket given as the linear combination of momentum eigenkets $|\alpha, 0\rangle = |p_0 - \delta p\rangle + 2|p_0\rangle + |p_0 + \delta p\rangle$ evolves in time. What can you say about the normalization of this ket? Derive an expression for the time-evolved wavefunction in the position representation $\psi(x, t) = \langle x | \alpha, t \rangle$. Consult a book on Waves and Oscillations, and identify carrier wave, envelope, group velocity and phase velocity in $\psi(x, t)$.

Problem 6

A spin-1/2 system in the initial ket $|\alpha, 0\rangle = (|+x\rangle + |+y\rangle)/\sqrt{2}$ evolves in a magnetic field given by $\vec{B} = B_0(\hat{x} + \hat{z})/\sqrt{2}$.

a) Find expressions for $|\langle \pm x | \alpha, t \rangle|^2$, $|\langle \pm y | \alpha, t \rangle|^2$, and $|\langle \pm z | \alpha, t \rangle|^2$. Plot these as functions of time.

b) Find expressions for $\langle \hat{s}_x \rangle(t)$, $\langle \hat{s}_y \rangle(t)$, and $\langle \hat{s}_z \rangle(t)$. Plot these as functions of time.