Assignment 3

Due: 8:30 am, 11 October 2022 (Tuesday)

Course: Quantum Mechanics - 1
Tata Institute of Fundamental Research Hyderabad, India
Instructor: ramakrishnan@tifrh.res.in

Problem 1

Suppose the Hamiltonian operator is time-dependent, and $\left[\hat{H}(t_1), \hat{H}(t_2)\right] = 0$ for two values of time, t_1 and t_2. Show that the time-evolution operator takes the form

$$\hat{U}(t) = \exp\left[-\frac{i}{\hbar}\int_0^t dt' \hat{H}\left(t'\right)\right].$$

Problem 2

The Hamiltonian operator of a three state system is given by

$$\hat{H} = a\left[|1\rangle\langle 1| + |3\rangle\langle 3|\right] + b|2\rangle\langle 2| + c\left[|1\rangle\langle 3| + |3\rangle\langle 1|\right],$$

where $\{|1\rangle, |2\rangle, |3\rangle\}$ are the three eigenkets. Suppose the state of the system at time $t = 0$ is given by the ket $|\psi(0)\rangle = |1\rangle$. Derive an expression for for the time-dependent ket of the system, $|\psi(t)\rangle$ in the basis of the eigenkets.

Problem 3

Consider a particle-in-a-box confined between $[0, L]$ is in its ground state, $n = 1$. Suppose the box is suddenly expanded to twice its length, so that the new domain is $[0, 2L]$. Derive an expression for the time-evolution of the position expectation value of the particle, $\langle\hat{x}\rangle(t)$.

Problem 4

Suppose the state of a spin-1/2 particle (assume it has a magnetic moment) at time $t = 0$ is given by the ket $|\psi(0)\rangle = |z+\rangle$, and it evolves in a magnetic field $\mathbf{B} = B_0\hat{y}$. Derive an expression for the time-dependent ket of the system, $|\psi(t)\rangle$ represented in the basis $\{|z+\rangle, |z-\rangle\}$.

Problem 5

Suppose the state of a spin-1/2 particle (assume it has a magnetic moment) at time $t = 0$ is given by the ket $|\psi(0)\rangle = |z+\rangle$, and it evolves in a magnetic field $\mathbf{B} = B_0\left(\hat{x} + \hat{z}\right)/\sqrt{2}$. Derive expressions for $\langle \hat{s}_x \rangle(t)$, $\langle \hat{s}_y \rangle(t)$, and $\langle \hat{s}_z \rangle(t)$.
