# *Assignment 1*

*Due: 2:15 pm, 28 September 2021 (Tuesday)*

Course: Quantum Mechanics - 1
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## *Problem 1*

Prove the following identity for the operators $\hat{A}$, $\hat{B}$, $\hat{C}$, and $\hat{D}$

$$\left[\hat{A}\hat{B}, \hat{C}\hat{D}\right] = -\hat{A}\hat{C}\{\hat{D}, \hat{B}\} + \hat{A}\{\hat{C}, \hat{B}\}\hat{D} - \hat{C}\{\hat{D}, \hat{A}\}\hat{B} + \{\hat{C}, \hat{A}\}\hat{D}\hat{B},$$

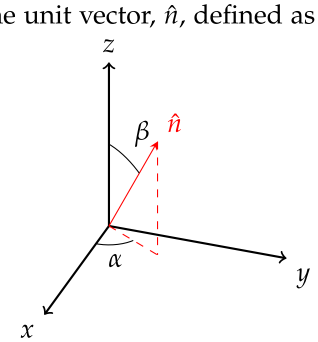
where [ ] and { } denote commutator and anti-commutator, respectively.

## *Problem 2*

The Stern-Gerlach states of a spin-1/2 system, $\{|z+\rangle, |z-\rangle\}$, form a complete set of orthonormal basis kets. In this canonical basis represent the operator $\hat{A} = |z+\rangle\langle z+| + |z-\rangle\langle z-| - |z+\rangle\langle z-| - |z-\rangle\langle z+|$ and calculate its eigenvalues and the corresponding eigenkets as the linear combination of the basis kets.

## *Problem 3*

For a spin-1/2 system, determine the eigenvalues and eigenkets, $\{|n+\rangle, |n-\rangle\}$, of the projection of the spin angular momentum along the unit vector, $\hat{n}$, defined as follows.




Find the conditions when the eigenkets will coincide with $\{|x+\rangle, |x-\rangle\}$, $\{|y+\rangle, |y-\rangle\}$, and $\{|z+\rangle, |z-\rangle\}$.

## *Problem 4*

If an ensemble of spin-1/2 particles in state $|\mathrm{n}+\rangle$ (from Problem 3) is sent through a $z-$polarized Stern-Gerlach setup, what are the probabilities that measurements will yield the values $\hbar/2$ and $-\hbar/2$? What is the mean value, $\langle s_z \rangle$, and the uncertainty, $\Delta s_z$, in the measurements?

## *Problem 5*

Some combination of Stern-Gerlach magnets have created a state given by $|\psi\rangle = (\sqrt{-1/3})|\mathrm{z}+\rangle + (\sqrt{2/3})|\mathrm{z}-\rangle$. Determine $\langle s_z \rangle$, and $\Delta s_z$ for this state. Comment on the minimum size of the ensemble (number of systems), in powers of 10, one has to have in order for the experimental measurements to be within meaningful precision. Hint: Consider statistical fluctuation.

## *Problem 6*

A Hermitian operator, $\hat{A}$, has eigenvalues $\{a_1, a_2, \ldots\}$ for the eigenkets $\{|\phi_1\rangle, |\phi_2\rangle, \ldots\}$. Explain the significance of the operator

$$\prod_{j=1,2,\ldots,j\neq k} \frac{\hat{A} - a_j}{a_k - a_j}$$

## *Problem 7*

A $2 \times 2$ matrix is given by

$$\mathbb{A} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

Suppose this matrix can be diagonalized by a similarity transformation, $\mathbb{U}^\dagger \mathbb{A} \mathbb{U}$, where the transformation matrix is given by

$$\mathbb{U} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}.$$

Find the value of $\theta_0$ (the optimal $\theta$) in terms of $a$, $b$, and $c$ to diagonalize $\mathbb{A}$. Find analytic expressions for the two eigenvalues of $\mathbb{A}$ using $a$, $b$, $c$, and $\theta_0$. The corresponding eigenvectors are simply the two columns of $\mathbb{U}$ with $\theta = \theta_0$. Note this result as it comes handy while solving problems.