Final Exam 01 (15 marks)

Due: 8:30 am, 29 November 2022 (Tuesday)

Course: Quantum Mechanics - 1
Tata Institute of Fundamental Research Hyderabad, India
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Problem 1 (2 marks)

The eigenvalues and eigenkets of an operator $\hat{A}$ are defined through

$$\hat{A}|a_k\rangle = a_k|a_k\rangle.$$

If $\hat{A}$ commutes with $\hat{B}$, then show that $\langle a_j|\hat{B}|a_k\rangle = b_k\delta_{jk}$.

Problem 2 (2 marks)

Show that two observables A and B, where one is a unitary transformation of the other, have same eigenvalues.

Problem 3 (3 marks)

Prove

$$\langle\beta|\hat{x}|\alpha\rangle = \int dp'\langle\beta|p'\rangle i\hbar\frac{\partial}{\partial p'}\langle p'|\alpha\rangle$$

Problem 4 (4 marks)

Derive the Robertson-Schrödinger uncertainty relation

$$\langle(\Delta A)^2\rangle\langle(\Delta B)^2\rangle \geq \frac{1}{4}|\langle[A, B]\rangle|^2,$$

where A and B are two observables.

Problem 5 (4 marks)

The state of a spin-1/2 particle is describe by the ket

$$|n\rangle = \cos(\beta/2)\,|z+\rangle + e^{i\alpha}\sin(\beta/2)\,|z-\rangle.$$

Determine the values of the angles, α and β, for which $\langle(\Delta S_x)^2\rangle\langle(\Delta S_y)^2\rangle$ is maximum.
