Mid-Term Examination
08 October 2022, 14:00–16:00 pm

30 marks

Course: Quantum Mechanics - 1
Tata Institute of Fundamental Research Hyderabad, India
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Problem 1 (3 marks)

The eigenvalues and eigenkets of a Hermitian operator $\hat{A}$ are given by the equation $\hat{A}|a_k\rangle = a_k|a_k\rangle$. Assume that there is no degeneracy. Determine

$$\prod_k \left(\hat{A} - a_k\right)|n\rangle,$$

where $|n\rangle$ is some arbitrary ket.

Problem 2 (3 marks)

For a spin-1/2 system in the ket $|x+\rangle$, verify the Robertson–Schrödinger uncertainty relation (the generalized uncertainty relation) for the simultaneous measurement of S_x and S_y.

Problem 3 (3 marks)

For a spin-1/2 system, find the matrix representation of the operator

$$\hat{A} = |z+\rangle\langle y+| + |z-\rangle\langle y-|$$

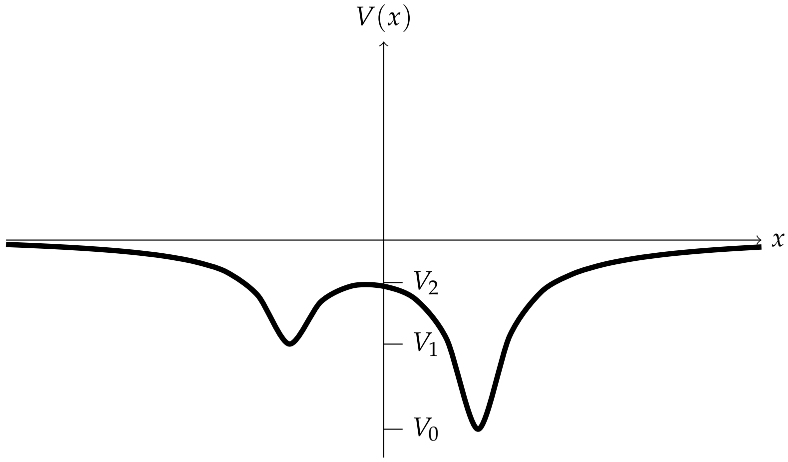
in the z-basis. What is the significance of this matrix?

Problem 4 (3 marks)

A particle confined in a one-dimensional infinite-square well ($0 \leq x \leq L$) is found to be in the ground state, $n = 1$. Calculate the probability of locating the particle in the interval $L/3 \leq x \leq 2L/3$.

Problem 5 (3 marks)

Qualitatively sketch the energy eigenvalue spectrum for a particle confined in a one-dimensional potential shown below.



Problem 6 (5 marks)

A particle is bound by the one-dimensional, negative-Dirac-delta potential, $V(x) = -A\delta(x)$, where A is a positive real number. Find its ground state wavefunction and the corresponding energy.

Problem 7 (5 marks)

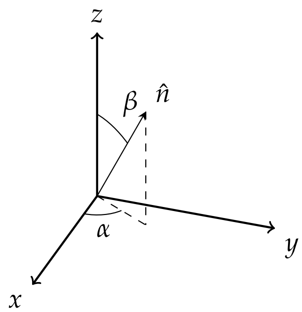
Consider a one-dimensional system bound by an 'analytic' potential $V(x)$ (the term 'analytic' means, $V(x)$ can be expanded as a converging series). For this system, prove the dipole sum rule

$$\sum_j |\langle k|\hat{x}|j\rangle|^2 \left(E_j - E_k\right) = \frac{\hbar^2}{2m},$$

where $\{|k\rangle\}$ are the energy eigenkets satisfying the equation $\hat{H}|k\rangle = E_k|k\rangle$.

Problem 8 (5 marks)

A spin-1/2 particle is entering a Stern–Gerlach magnet polarized along the unit vector, $\hat{n}$, with $\alpha = \pi/4$ and $\beta = 2\pi/3$.



When the particle exits the magnet, its spin-angular momentum has the value $\langle \hat{S}_n \rangle = \hbar/2$. What is the probability that a subsequent measurement of S_x on this particle will give the value $\hbar/2$?