Mid-Term Examination

30 marks
For Problem 4, you can choose to do option-1 or option-2

Course: Quantum Mechanics - 1
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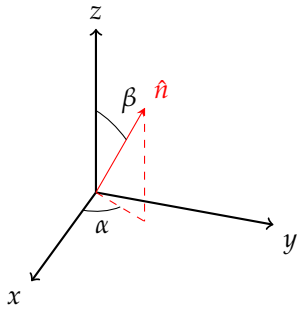
Problem 1 (5 marks)

A particle is confined in a one-dimensional infinite-square well of length L. If the wave function of the particle is given by $\psi(x) = \sin(\pi x/L)\exp(-i\omega t)$; $\quad 0 \leq x \leq L$, calculate the probability of locating the particle in the interval $L/4 \leq x \leq 3L/4$.

Problem 2 (5 marks)

A spin-1/2 particle is entering a Stern–Gerlach magnet polarized along the unit vector, $\hat{n}$, with $\alpha = \pi/4$ and $\beta = 2\pi/3$.



When the particle exits the magnet, its spin-angular momentum has the value $\langle \hat{S}_n \rangle = \hbar/2$. What is the probability that a subsequent measurement on this particle will result in $\langle \hat{S}_x \rangle = \hbar/2$?

Problem 3 (5 marks)

A spin-1/2 system in the initial ket $|\alpha, 0\rangle = |-z\rangle$ evolves in a magnetic field given by $\vec{B} = B_0\hat{y}$. Find the probability of measuring spin up in the z-direction after a time t.

Problem 4, option-1 (15 Marks)

The wave function of a particle in some ket $|\alpha\rangle$ is given by

$$\langle x|\alpha\rangle = \psi(x) \quad = \quad Ax\exp(-x^2/a^2)$$

(a) Normalize the wave function and find the value of x for which the probability density is maximum.
(b) Using $\psi(x)$, find $\langle x\rangle$, $\langle x^2\rangle$, and Δx.
(c) Find the probability to locate the particle between $x = 0$ and $x = a$?
(d) Find $\langle p|\alpha\rangle = \phi(p)$
(e) Using $\phi(p)$, find $\langle p\rangle$, $\langle p^2\rangle$, and Δp.

Problem 4, option-2 (15 Marks)

The wave function of a particle in some ket $|\alpha\rangle$ is given by

$$\langle x|\alpha\rangle = \psi(x) \quad = \quad A\frac{1}{x^2+a^2}$$

(a) Normalize the wave function and find the value of x for which the probability density is maximum.
(b) Using $\psi(x)$, find $\langle x\rangle$, $\langle x^2\rangle$, and Δx.
(c) Find the probability to locate the particle between $x = 0$ and $x = a$?
(d) Find $\langle p|\alpha\rangle = \phi(p)$
(e) Using $\phi(p)$, find $\langle p\rangle$, $\langle p^2\rangle$, and Δp.