Assignment 1

Due: 8:30 am, 13 September 2022 (Tuesday)

Course: Quantum Mechanics - 1
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Problem 1

A beam of silver atoms is created by heating a vapor in an oven to 1000°C, and selecting atoms with a velocity close to the mean of the thermal distribution. The beam moves through a one-meter long magnetic field with a vertical gradient 10 T/m, and impinges a screen one meter downstream of the end of the magnet. Assuming the silver atom has spin 1/2 with a magnetic moment of one Bohr magneton, find the separation distance in millimeters of the two states on the screen.

Problem 2

A 2×2 matrix is given by

$$\mathbb{A} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$

Suppose this matrix can be diagonalized by a similarity transformation, $\mathbb{U}^\dagger \mathbb{A} \mathbb{U}$, where the transformation matrix is given by

$$\mathbb{U} = \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}.$$

Find the value of θ_0 (the optimal θ) in terms of a, b, and c to diagonalize $\mathbb{A}$. Find analytic expressions for the two eigenvalues of $\mathbb{A}$ using a, b, c, and θ_0. The corresponding eigenvectors are the two columns of $\mathbb{U}$ with $\theta = \theta_0$. Note this result as it comes handy while solving problems.

Problem 3

Some combination of Stern-Gerlach magnets have created a state, ψ, represented by the ket $|\psi\rangle = (\sqrt{-1/3})|z+\rangle + (\sqrt{2/3})|z-\rangle$. Determine $\langle s_z \rangle$, and Δs_z for this state. Comment on the minimum size of the ensemble (number of systems), in powers of 10, one has to have in order for the experimental measurements to be within meaningful precision. Hint: Consider statistical fluctuation.

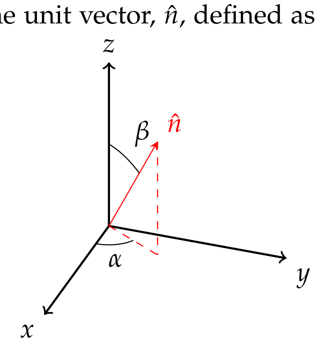
Problem 4

A Hermitian operator, $\hat{A}$, has eigenvalues $\{a_1, a_2, \ldots\}$ for the eigenkets $\{|\phi_1\rangle, |\phi_2\rangle, \ldots\}$. Explain the significance of the operator

$$\prod_{j=1,2,\ldots,j\neq k} \frac{\hat{A} - a_j}{a_k - a_j}$$

Problem 5

For a spin-1/2 system, determine the eigenvalues and eigenkets, $\{|\mathrm{n}+\rangle, |\mathrm{n}-\rangle\}$, of the projection of the spin angular momentum along the unit vector, $\hat{n}$, defined as follows.



1. Find the conditions in terms of the angles, when the eigenkets will coincide with $\{|\mathrm{x}+\rangle, |\mathrm{x}-\rangle\}$, $\{|\mathrm{y}+\rangle, |\mathrm{y}-\rangle\}$, and $\{|\mathrm{z}+\rangle, |\mathrm{z}-\rangle\}$. You may begin by finding the eigenvectors (using the analytic expressions from Problem 2) of the $\hat{S}_n$ operator in the z-basis.

2. If an ensemble of spin-1/2 particles in state $|\mathrm{n}+\rangle$ is sent through a $z-$polarized Stern-Gerlach setup, what are the probabilities that measurements will yield the values $\hbar/2$ and $-\hbar/2$? What is the mean value, $\langle s_z \rangle$, and the uncertainty, Δs_z, in this measurement?