

1. Charged particle in electric and magnetic fields [150 points]

- a) Write a Python code for a charged particle moving in electric and magnetic fields in 3D space. Put in an option to either use the Euler or the Euler-Richardson algorithm. The Lorentz force \mathbf{F} for a particle of charge q and velocity $\mathbf{v}(t)$ in the electric field \mathbf{E} and magnetic field \mathbf{B} is given by $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$.
- b) Test the code for a charged particle, e.g., an electron, in a homogeneous electric field. Please use sensible combinations of charge and electric field strengths (at least 3 different combinations of electric field and initial velocity). Compare your numerical solution for the position as a function of time with the analytic solution (similar to that of the falling stone using a constant acceleration). Determine efficient but accurate time steps for the Euler and the Euler-Richardson algorithms by testing if the results change when reducing the time step and checking if the results are the same (within plotting accuracy).
- c) Test the code for a charged particle, e.g., an electron, in a homogeneous magnetic field. Please use sensible combinations of charge and field strengths. Check that the particle moves in a circle in the plane perpendicular to the magnetic field direction (if the initial velocity is non-vanishing). Numerically check that the motion has a constant acceleration.
- d) Turn on the electric and magnetic fields at the same time (at least two different relative orientations). How does the result change with the initial velocity of the particle? Does the trajectory depend on the relative orientation of electric and magnetic fields?
- e) Include a second charged particle and include the electrostatic interaction between the two in the calculations. Run simulations for one repulsive and one attractive case. Discuss the conservation of energy (what do you expect? what do you obtain numerically?)

For all the points above, create sensible plots. At least: position vs. time in 1D, 2D/3D trajectory plots, velocity vs. time, kinetic energy vs. time, plots for different time steps to show convergence and, for the two particle case, plots of distance vs. time and relative trajectory plots. Please explain the plots carefully and connect them to the (physical) expectations.