Monte-Carlo examples in R

Serial vs parallel code Meelis Utt

Setup

Let's source the setup (function, analytical solution, number of iterations).

```
##
## > options(scipen = 1000)
##
## > n <- 10000000
##
## > header <- c("n", "computational", "analytical", "error",
## + "walltime", "type")
##
## > f <- function(x) {
## + return(x^2 + x^4 + sin(x) + cos(x) + x^25)
## + }
##
## > analytical <- integrate(f, 0, 1)$value</pre>
```

Serial implementations

start <- Sys.time()</pre>

source("Setup.R",echo = T)

Let's start with a simple implementation of Monte-Carlo method.

```
MCser1 <- function(n){</pre>
  start <- Sys.time()</pre>
  i \leftarrow runif(n,0,1)
  EX <- mean(f(i))</pre>
  end <- Sys.time()</pre>
  time <- difftime(end,start)</pre>
  error <- (EX - analytical) %>% abs
  return(c(n,EX,analytical,error,time,"MCser1"))
}
data.table(t(MCser1(n))) %>% setNames(header)
##
                    computational
                                           analytical
## 1: 10000000 1.87312157698611 1.87296355073463 0.000158026251479537
              walltime
## 1: 2.1086106300354 MCser1
Let's try a bit more vectorized solution, using the *apply function.
MCser2 <- function(n,ncols=1000){</pre>
```

```
dt <- matrix(runif(n,0,1),ncol = ncols)</pre>
  EX <- sapply(1:ncols,function(i,dt){</pre>
      EX <- dt[,i] %>% f %>% mean
    },dt) %>% mean
  end <- Sys.time()</pre>
  time <- difftime(end,start)</pre>
  error <- (EX - analytical) %>% abs
  return(c(n,EX,analytical,error,time,"MCser2"))
}
data.table(t(MCser2(n))) %>% setNames(header)
##
                   computational
                                         analytical
                                                                      error
## 1: 10000000 1.87300128353283 1.87296355073463 0.0000377327982024056
##
              walltime
                          type
## 1: 1.7653968334198 MCser2
Let's try an approach using data.table.
MCser3 <- function(n){</pre>
  start <- Sys.time()</pre>
  dt <- data.table(unif = runif(n,0,1))</pre>
  EX <- dt[,.(EX = mean(f(unif)))] %>% unlist %>% unname
  end <- Sys.time()</pre>
  time <- difftime(end,start)</pre>
  error <- (EX - analytical) %>% abs
  return(c(n,EX,analytical,error,time,"MCser3"))
data.table(t(MCser3(n))) %>% setNames(header)
              n
                   computational
                                         analytical
## 1: 10000000 1.87298681647747 1.87296355073463 0.000023265742841172
               walltime
                           type
## 1: 1.66933250427246 MCser3
Let's try divide-and-conquer approach with data.table.
MCser4 <- function(n,ncols=1000){</pre>
  start <- Sys.time()</pre>
  dt <- matrix(runif(n,0,1),ncol=ncols) %>% data.table
  EX <- dt[,lapply(.SD,f)][,lapply(.SD,mean)][,.(EX = sum(.SD)/ncols)] %>% unlist %>% unname
  end <- Sys.time()</pre>
  time <- difftime(end,start)</pre>
  error <- (EX - analytical) %>% abs
  return(c(n,EX,analytical,error,time,"MCser4"))
}
data.table(t(MCser4(n))) %>% setNames(header)
                   computational
                                         analytical
## 1: 10000000 1.87284836166252 1.87296355073463 0.000115189072112054
               walltime
                           type
## 1: 1.81064772605896 MCser4
```

Parallel implementations

Let's try different parallel implementations. First let's start with package parallel.

```
MCpar1 <- function(n){</pre>
  start <- Sys.time()</pre>
  # Calculate the number of cores
  no_cores <- detectCores()</pre>
  # Initiate cluster
  cl <- makeCluster(no_cores)</pre>
  intermean <- parSapply(cl, rep(n/no_cores,no_cores),function(ni,f){</pre>
      EX <- mean(f(runif(ni,0,1)))</pre>
    },f
  on.exit(stopCluster(cl))
  EX <- mean(intermean)</pre>
  end <- Sys.time()</pre>
  time <- difftime(end,start)</pre>
  error <- (EX - analytical) %>% abs
  return(c(n,EX,analytical,error,time,"MCpar1"))
data.table(t(MCpar1(n))) %>% setNames(header)
                                          analytical
              n
                    computational
## 1: 10000000 1.87309074553834 1.87296355073463 0.000127194803714481
               walltime
                            type
## 1: 1.22565364837646 MCpar1
Now let's try approach analogical to MCser2.
MCpar2 <- function(n){</pre>
  start <- Sys.time()</pre>
  # Calculate the number of cores
  no cores <- detectCores()</pre>
  cl <- makeCluster(no_cores)</pre>
  dt <- matrix(runif(n,0,1),ncol = no_cores)</pre>
  intermean <- parSapply(cl, 1:no_cores,function(i,f,dt){</pre>
      EX <- mean(f(dt[,i]))</pre>
    },f,dt
  on.exit(stopCluster(cl))
  EX <- mean(intermean)</pre>
  end <- Sys.time()</pre>
  error <- (EX - analytical) %>% abs
  time <- difftime(end,start)</pre>
  return(c(n,EX,analytical,error,time,"MCpar2"))
data.table(t(MCpar2(n))) %>% setNames(header)
                    computational
                                          analytical
              n
## 1: 10000000 1.87326255732499 1.87296355073463 0.000299006590361861
               walltime
## 1: 3.25085091590881 MCpar2
This approach was not very good. But let's have one more try at analogical solution to MCser2.
MCpar2_2 <- function(n,ncols=1000){</pre>
  start <- Sys.time()</pre>
  # Calculate the number of cores
  no_cores <- detectCores()</pre>
```

```
cl <- makeCluster(no_cores)</pre>
  dt <- matrix(runif(n,0,1),ncol = ncols)</pre>
  intermean <- parSapply(cl, 1:ncols,function(i,f,dt){</pre>
      EX <- mean(f(dt[,i]))</pre>
    },f,dt
  on.exit(stopCluster(cl))
  EX <- mean(intermean)</pre>
  end <- Sys.time()</pre>
  error <- (EX - analytical) %>% abs
  time <- difftime(end,start)</pre>
  return(c(n,EX,analytical,error,time,"MCpar2_2"))
data.table(t(MCpar2_2(n))) %>% setNames(header)
##
                   computational
                                         analytical
                                                                      error
## 1: 10000000 1.87314601528394 1.87296355073463 0.000182464549312922
              walltime
                            type
## 1: 3.1351101398468 MCpar2_2
This solution was bit better, but still worse than the previous examples.
Let's try the package foreach now.
MCpar3 <- function(n){</pre>
  start <- Sys.time()</pre>
  # Calculate the number of cores
  no_cores <- detectCores()</pre>
  # Initiate cluster
  cl<-makeCluster(no cores)</pre>
  # registerDoParallel(cl)
  EX <- foreach(ni = rep(n/no_cores,no_cores),.combine=mean,.export="f") %dopar%
    mean(f(runif(ni,0,1)))
  on.exit(stopCluster(cl))
  # stopImplicitCluster()
  end <- Sys.time()</pre>
  time <- difftime(end,start)</pre>
  error <- (EX - analytical) %>% abs
  return(c(n,EX,analytical,error,time,"MCpar3"))
data.table(t(MCpar3(n))) %>% setNames(header)
## Warning: executing %dopar% sequentially: no parallel backend registered
##
                   computational
                                         analytical
                                                                      error
## 1: 10000000 1.87333968656737 1.87296355073463 0.000376135832744318
               walltime
```

Benchmarking

1: 1.82036471366882 MCpar3

Now, let's visualize the walltimes of implemented solutions.

```
iterations <- c(2.5,5,7.5,10)*10**(5:7)
## Warning in c(2.5, 5, 7.5, 10) * 10^(5:7): longer object length is not a multiple
## of shorter object length</pre>
```

```
funs <- c(</pre>
  MCser1, MCser2, MCser3, MCser4,
  MCpar1, MCpar3 #MCpar2, MCpar2_2,
)
data <- sapply(funs,function(f,iterations){</pre>
  sapply(iterations,function(n){
    f(n)
  })
}, iterations) %>%
  matrix(ncol=6,byrow=T) %>%
  data.table %>%
  setNames(header) %>%
  mutate_at(header[-grep(x=header,pattern="type")],as.numeric)
data
##
              n computational analytical
                                                   error
                                                            walltime
                                                                       type
##
         250000
                     1.873130
                                 1.872964 0.00016643353
                                                         0.03385305 MCser1
    1:
##
    2:
        5000000
                     1.873030
                                 1.872964 0.00006595650
                                                          0.69568539 MCser1
##
    3: 75000000
                     1.872907
                                 1.872964 0.00005654916 11.68683290 MCser1
##
   4:
       1000000
                     1.872530
                                 1.872964 0.00043352101
                                                         0.14016461 MCser1
##
   5:
         250000
                     1.871317
                                 1.872964 0.00164682859
                                                         0.20487976 MCser2
##
    6:
       5000000
                     1.872773
                                 1.872964 0.00019064751
                                                         0.77347493 MCser2
##
                     1.872913
                                 1.872964 0.00005082639 11.00078845 MCser2
   7: 75000000
##
    8:
       1000000
                     1.873018
                                 1.872964 0.00005454452
                                                         0.25182199 MCser2
##
  9:
         250000
                     1.875004
                                 1.872964 0.00204025997
                                                          0.03408027 MCser3
## 10:
        5000000
                     1.873260
                                 1.872964 0.00029644140
                                                         0.65575528 MCser3
## 11: 75000000
                     1.873049
                                 1.872964 0.00008524746 11.90426970 MCser3
## 12:
       1000000
                     1.872933
                                 1.872964 0.00003028198 0.13201976 MCser3
## 13:
         250000
                     1.874320
                                 1.872964 0.00135683934
                                                         0.23717046 MCser4
## 14:
        5000000
                     1.873407
                                 1.872964 0.00044341028
                                                         0.82303739 MCser4
## 15: 75000000
                     1.873138
                                 1.872964 0.00017413568 11.54206014 MCser4
## 16:
       1000000
                     1.873115
                                 1.872964 0.00015100499
                                                         0.34342837 MCser4
## 17:
         250000
                     1.871166
                                 1.872964 0.00179786906
                                                          0.44379544 MCpar1
## 18:
        5000000
                     1.873437
                                 1.872964 0.00047358039
                                                         0.81169033 MCpar1
## 19: 75000000
                                 1.872964 0.00014894196
                     1.872815
                                                          6.08725572 MCpar1
## 20:
       1000000
                                                          0.59535599 MCpar1
                     1.873463
                                 1.872964 0.00049947203
## 21:
         250000
                     1.870885
                                 1.872964 0.00207891484
                                                          0.46692729 MCpar3
## 22: 5000000
                     1.873080
                                 1.872964 0.00011612905
                                                         1.09262633 MCpar3
## 23: 75000000
                     1.873205
                                 1.872964 0.00024189381 10.45459032 MCpar3
                                 1.872964 0.00012044145
## 24: 1000000
                     1.872843
                                                         0.57980013 MCpar3
              n computational analytical
                                                  error
                                                            walltime
                                                                       type
ggplot(data=data,aes(x=n,y=walltime,group=type,color=type)) +
  geom_point() +
  geom line() +
  labs(
       title="Serial vs parallel implementations",
       x="Nr of iterations",
       y="Walltime (sec)"
       ) +
  theme(
        plot.title = element_text(hjust = 0.5),
        plot.subtitle = element_text(hjust = 0.5)
```

```
) +
guides(color=guide_legend(title="Function"))
```

Serial vs parallel implementations

