The potential of a large dust grain in a collisionless plasma

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Abstract

1 Introduction

2 Radial motion theory (ABR)

The ABR model is a radial motion theory derived by Allen, Boyd and Reynolds. It describes the equilibrium surface potential acquired by a dust grain immersed in an infinite and stationary plasma [1].

Consider a spherical dust grain, of arbitrary radius a, immersed in this infinite plasma. Far from the surface we assume that the electron and ion densities are equal, denoted n_e and n_i respectively; this is known as quasi-neutrality, which is mathematically written as $Zn_i \approx n_e$. As electrons are faster than ions, it can be shown that such a dust grain will become negatively charged [2], thus ions will experience an attractive force due to the potential on the dust surface, ϕ_a. We assume that ions at infinity have no kinetic energy, hence, they move radially towards the dust grain. Therefore, it is appropriate to say that an ion at a distance r from the dust center has radial speed v_i. Using energy conservation, one can show the following,

$$\frac{1}{2}m_i v_i^2 = -Ze\phi(r), \tag{1}$$

where m_i is the ion mass, Z is the relative ion charge, e is the electron charge and $\phi(r)$ is the potential at r, which vanishes as $r \to \infty$ [1].

Equation (1) then leads to an expression for the ion current, which is entirely dependant on the radial distance from the dust grain, given by

$$I_i = \frac{4\sqrt{2}\ n_i \pi r^2 Z^{\frac{3}{2}} e^{\frac{3}{2}} \phi_a^{\frac{1}{2}}}{m_i^{\frac{1}{2}}}. \tag{2}$$

As the potential is negative, few electrons reach the dust grain, hence, the electron density obeys a Boltzmann distribution:

$$n_e(r) = n_0 \exp\left(\frac{e\phi_a}{k_B T_e}\right), \tag{3}$$

where n_0 is the electron density at infinity, k_B is the Boltzmann constant and T_e is the electron temperature. We further assume that only inbound electrons contribute to the electron current at the surface of the dust grain, given as

$$I_i = I_e = 4\pi a^2 n_0 e \sqrt{\frac{k_B T_e}{2\pi m_e}} \exp\left(\frac{e\phi_a}{k_B T_e}\right). \tag{4}$$

where m_e is the electron mass [1].

It is useful to apply the following normalisations, noting that Φ is the opposite sign for simplicity:

$$\Phi = -\frac{e\phi}{k_B T_e},\ \rho = \frac{r}{\lambda_D},\ \alpha = \frac{a}{\lambda_D},\ J = \frac{I_i}{4\pi\lambda_D^2 n_0 e\sqrt{\frac{2k_B T_e}{m_i}}}, \tag{5}$$

where λ_D is the electron Debye length, which is a characteristic length over which quasi-neutrality breaks down,

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_e}{n_0 e^2}}. \tag{6}$$

Poisson's law allows for the formation of a differential equation which relates the spatial variation of the potential to the difference in electron and ion densities,

$$\frac{d}{d\rho}\left(\rho^2 \frac{d\Phi}{d\rho}\right) = JZ^{-\frac{1}{2}}\Phi^{-\frac{1}{2}} - \rho^2 \exp{(-\Phi)}. \tag{7}$$

This equation may be solved using the boundary conditions;

$$\rho \approx J^{\frac{1}{2}} Z^{-\frac{1}{4}} \Phi^{-\frac{1}{4}} \exp\left(\frac{\Phi}{2}\right), \tag{8}$$

$$\left.\frac{d\Phi}{d\rho}\right|_{\rho_b} = \frac{2\rho_b Z^{\frac{1}{2}} J^{-1} \Phi_b^{\frac{3}{2}}}{\Phi_b - \frac{1}{2}} exp(-\Phi_b), \tag{9}$$

$$\frac{J}{\Gamma} = \frac{4Z^{\frac{1}{2}}\Phi_b^{\frac{3}{2}}(2\Phi_b - 3)(2\Phi_b + 1)}{(2\Phi_b - 1)^3}, \tag{10}$$

$$\frac{J}{\alpha^2} = \frac{\mu}{\sqrt{4\pi}} \exp{(-\Phi_a)}, \tag{11}$$

which are formed by assuming that there exists a certain distance ρ_b, past which, quasi-neutrality applies. The potential at ρ_b is given by Φ_b and Γ is a number much greater than unity [1]. In order to find a value for the surface potential we must solve (7), this may be achieved using a 4th order Runge-Kutta. We choose $\Gamma = 10000$ and find the roots of (10) allowing us to determine the necessary boundary conditions using (8) and (9). Hence, solving the differential equation numerically yields the following graph of normalised dust potential as a function of normalised dust radius.

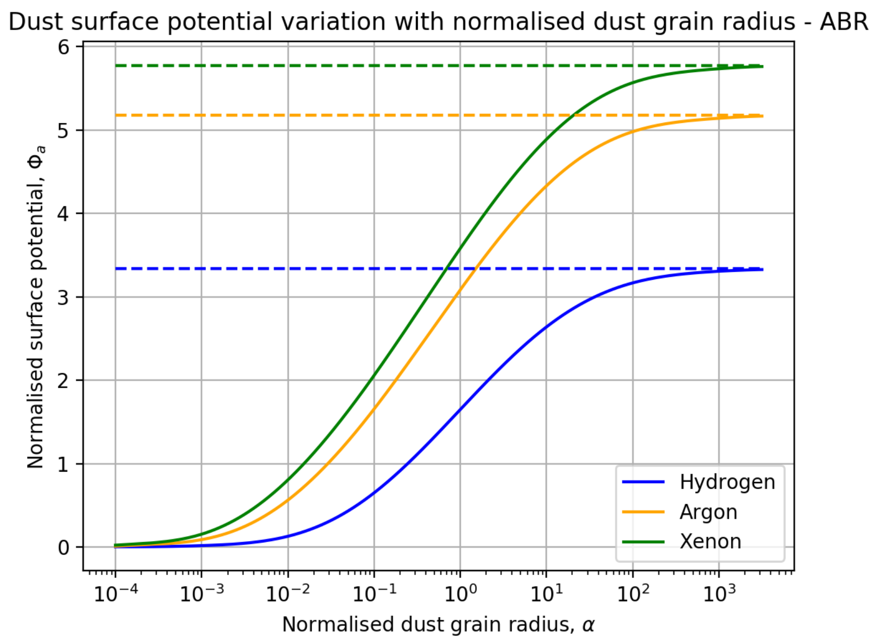

Figure 1: ABR predictions for Φ_a as a function of α for a dust grain in singly ionised Hydrogen, Argon and Xenon plasmas ($Z = 1$) [1] [2].

Thomas [2] discusses that in the limit of $\alpha \to \infty$ the ABR potential approaches the cold planar wall limit, given as the following

$$\lim_{\alpha \to \infty} \Phi_a = \frac{1}{2} \ln(2\pi) - \frac{1}{2} - \ln(\mu), \tag{12}$$

where the $-\frac{1}{2}$ is due to the potential drop across a cold ion pre-sheath, $\Theta = 0$, as discussed by Stangeby [3]. Furthermore, one can clearly see that in the limit of $\alpha \to 0$ the ABR prediction tends to zero also, which makes physical sense [1].

3 Modified orbital motion limited (MOML)

4 SCEPTIC numerical fit

5 Comparison of MOML and ABR with SCEPTIC data

6 Flowing sheath approximation

7 Conclusion

8 References and Acknowledgements

[1] K. R. V. and A. J. E., "The floating potential of spherical probes and dust grains. part 1. radial motion theory," *Journal of Plasma Physics*, vol. 67.4, pp. 243–50, 2002.

[2] D. M. Thomas, "Theory and simulation of the charging of dust in plasmas," Physics PhD Thesis, Imperial College London, March 2016.

[3] P. C. Stangeby, *The Plasma Sheath*. Boston, MA: Springer US, 1986, pp. 41–97. [Online]. Available: https://doi.org/10.1007/978-1-4757-0067-1_3

9 Appendix

9.1 Symbol dictionary

e	Electron charge
ε_0	Permittivity of free space
k_B	Boltzmann's constant
a	Dust radius
α	Normalised dust radius
$_a$	Subscript indicating a quantity at the dust grain surface
r	Distance from the centre of the dust grain
ρ	Normalised distance from the centre of the dust grain
λ_D	Debye length
m_j	Mass
n_j	Density
T_j	Temperature
I_j	Current
$_j$	Subscript indicating a plasma particle
$_i$	Subscript indicating an ion quantity
$_e$	Subscript indicating an electron quantity
$_0$	Subscript indicating an electron quantity at infinity
μ	Root mass ratio
Θ	Ratio of ion to electron temperature
γ	Heat capacity ratio
u	Flow velocity
υ	Normalised flow velocity
Γ	ABR correction factor
Z	Ion charge number
Q	Dust grain charge
ϕ	Electric potential
Φ	Normalised electric potential