№ 18

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$\sigma(z) = \frac{1}{1+e^{-z}}$; $\sigma'(z) = \frac{-(-e^{-z})}{(1+e^{-z})^2} =$

$= \frac{e^{-z}}{(1+e^{-z})^2}$; $\sigma(1-\sigma) = \frac{1}{1+e^{-z}} \cdot \frac{e^{-z}}{1+e^{-z}} =$

$= \frac{e^{-z}}{(1+e^{-z})^2} = \sigma'(z)$

№ 19

1) $g_k(s_1, \dots s_k) = \frac{e^{s_k}}{\sum\limits_{i=1}^{k} e^{s_i}}$,

$R^{(i)} = -\sum\limits_{k=1}^{K} I(y^{(i)} = k) \cdot \ln g_k(s_1, \dots s_k)$

2) $\frac{\partial g_k}{\partial g_l} = \begin{cases} e^{s_k}\left(-\frac{e^{s_l}}{(\sum_i e^{s_i})^2}\right) = \frac{-e^{s_l}}{\sum e^{s_i}} = -g_l \cdot g_k, & k \neq l \\ \frac{e^{s_k}(\sum_i e^{s_i}) - e^{s_k} \cdot e^{s_l}}{(\sum_i e^{s_i})^2} = \frac{e^{s_k}}{\sum_i e^{s_i}} - \frac{e^{s_k}}{\sum_i e^{s_i}} \cdot \frac{e^{s_l}}{\sum_i e^{s_i}} = g_k \cdot (1-g_l), & k = l \end{cases}$

$$I(k=\ell)=\begin{cases}1, & k=\ell\\ 0, & k\neq\ell\end{cases} \Rightarrow \frac{\partial g_k}{\partial s_\ell}=g_k\cdot$$

$$(I(k=\ell)-g_\ell)$$

2) $$\frac{\partial R^{(i)}}{\partial g_k}=-I(y^{(i)}=k)(\ln g_k(s_1,\dots s_k))'=$$

$$=\frac{-I(y^{(i)}=k)}{g_k(s_1,\dots s_k)}$$

3) $$\frac{\partial R^{(i)}}{\partial s_\ell}=\sum_k \frac{\partial R^{(i)}}{\partial g_k}\cdot\frac{\partial g_k}{\partial s_\ell}=$$

$$=-\sum_k \frac{I(y^{(i)}=k)}{g_k(s_1,\dots s_k)}\cdot g_k(I(k=\ell)-g_\ell)=$$

$$=-(I(y^{(i)}=\ell)-g_\ell)=g_\ell-I(y^{(i)}=\ell)$$