

# Statistical models on phylogenetic networks

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June 3, 2022



<https://solislemuslab.github.io/>



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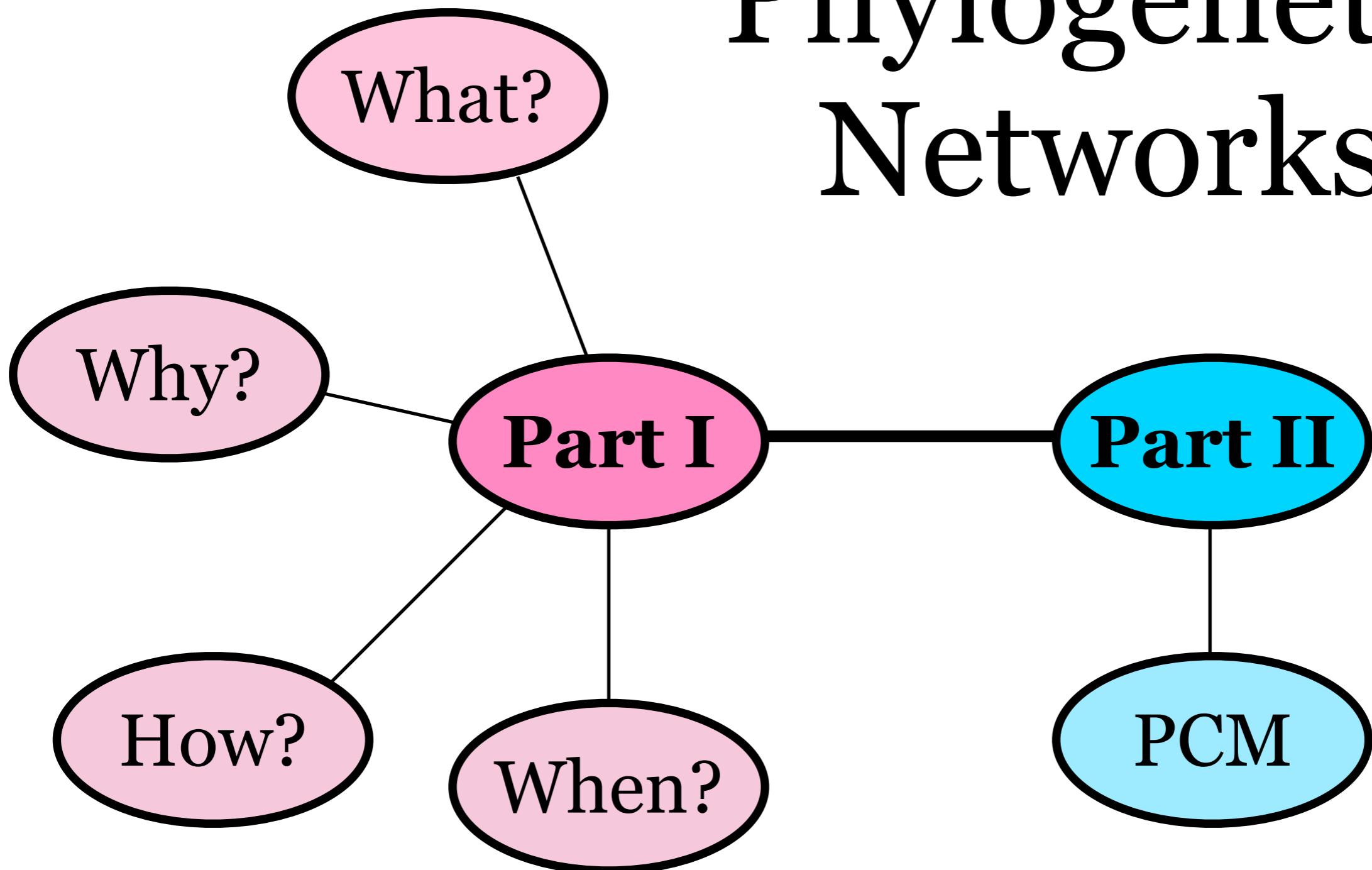


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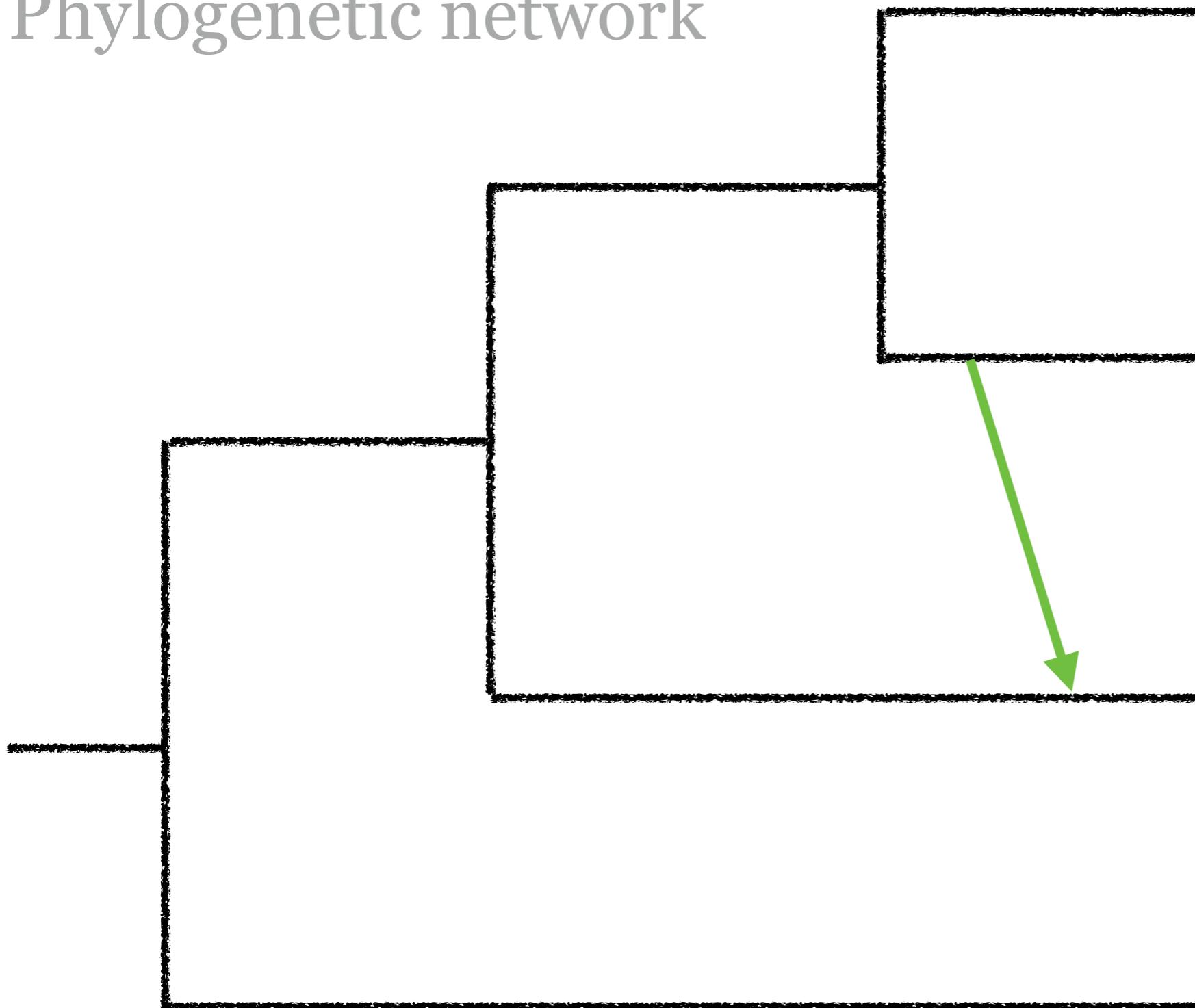
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# Phylogenetic Networks



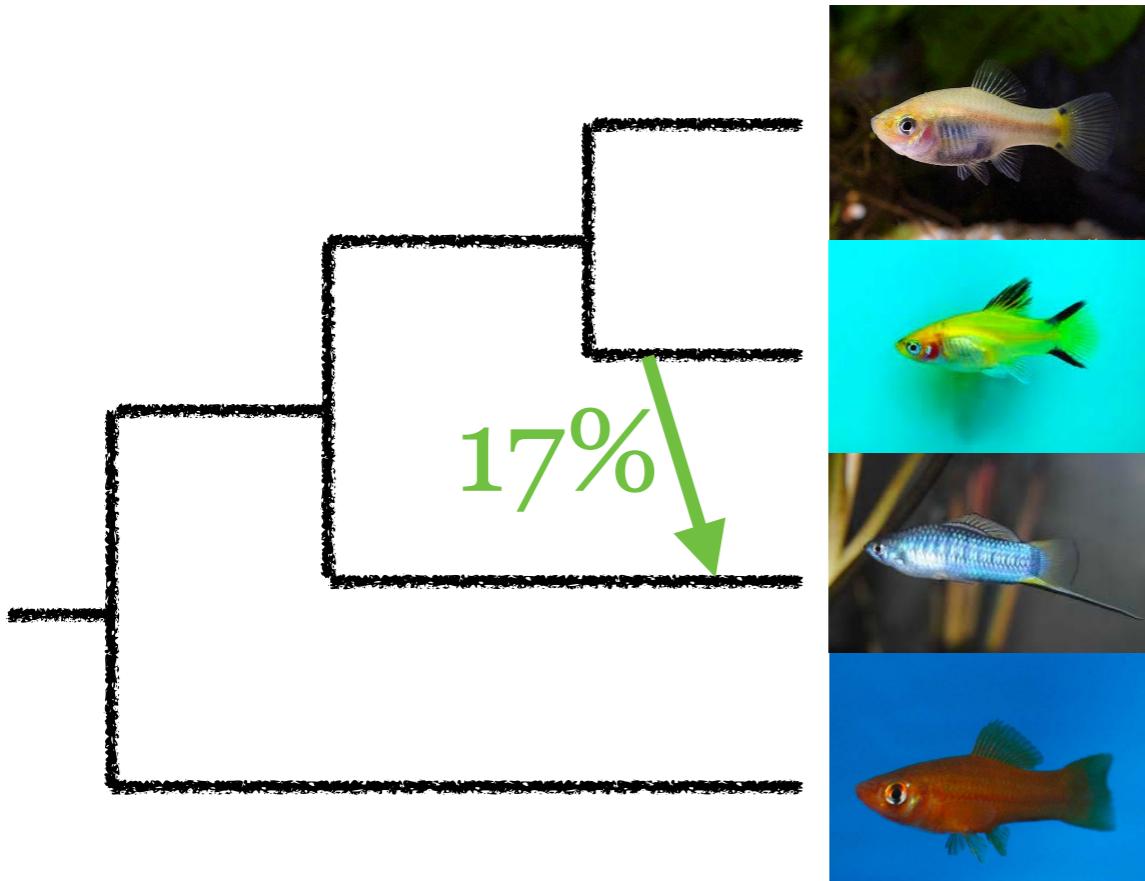
# What?

# Phylogenetic network

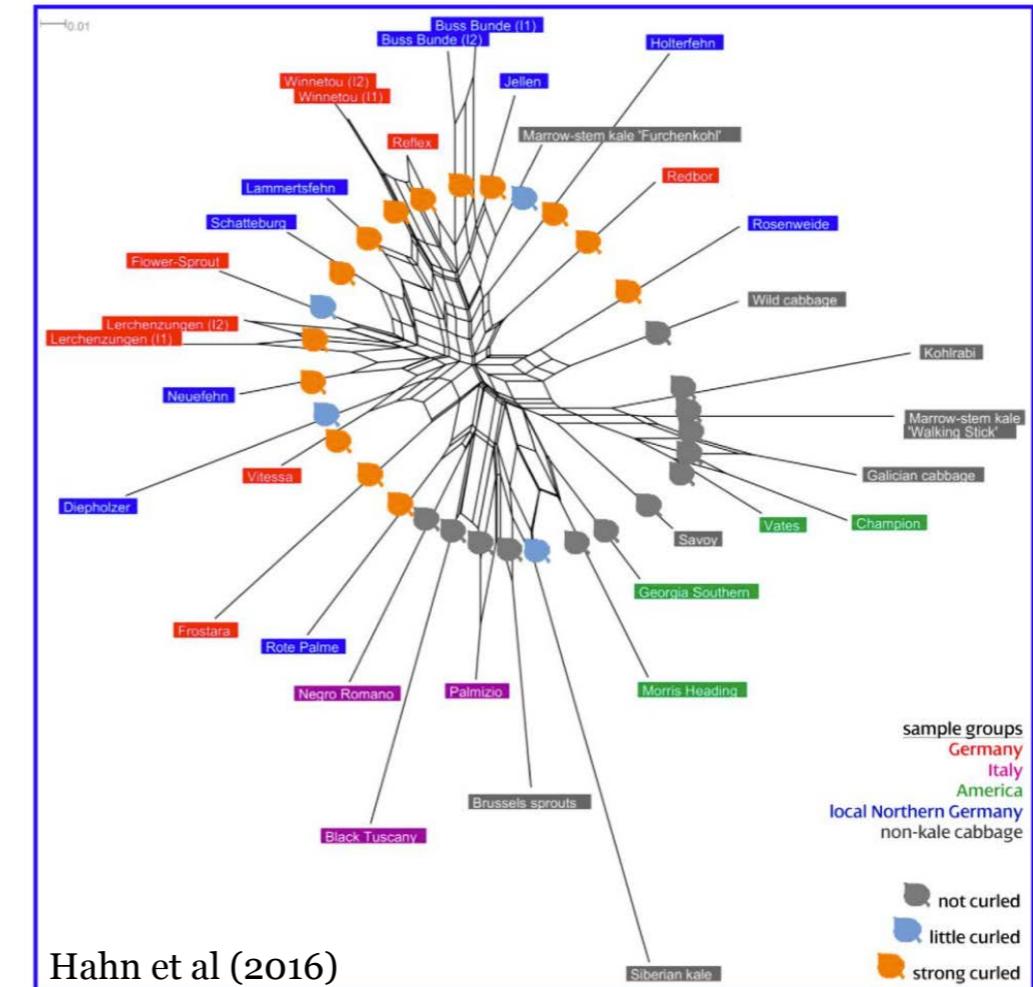


# What?

## Phylogenetic network



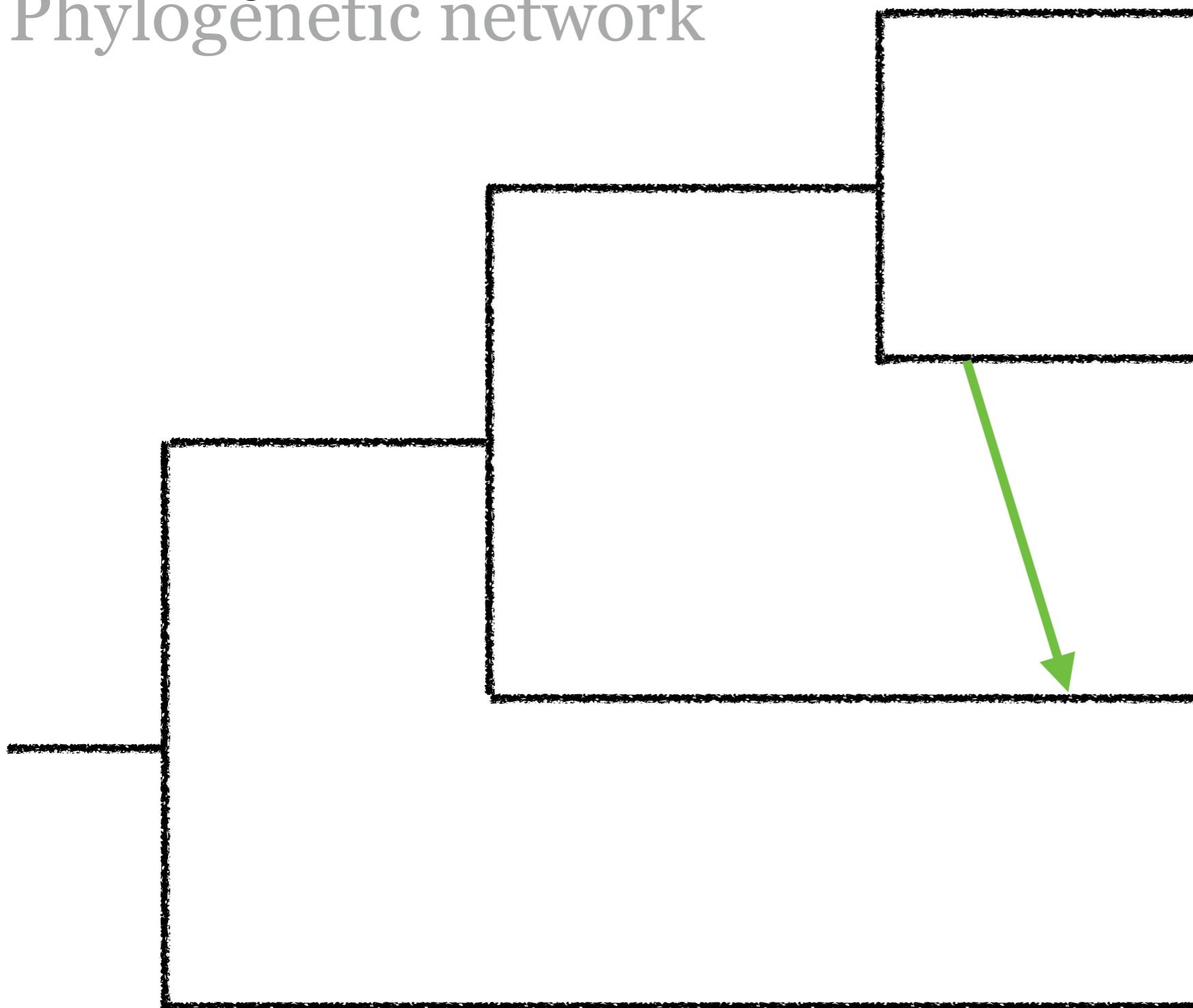
Explicit



Implicit

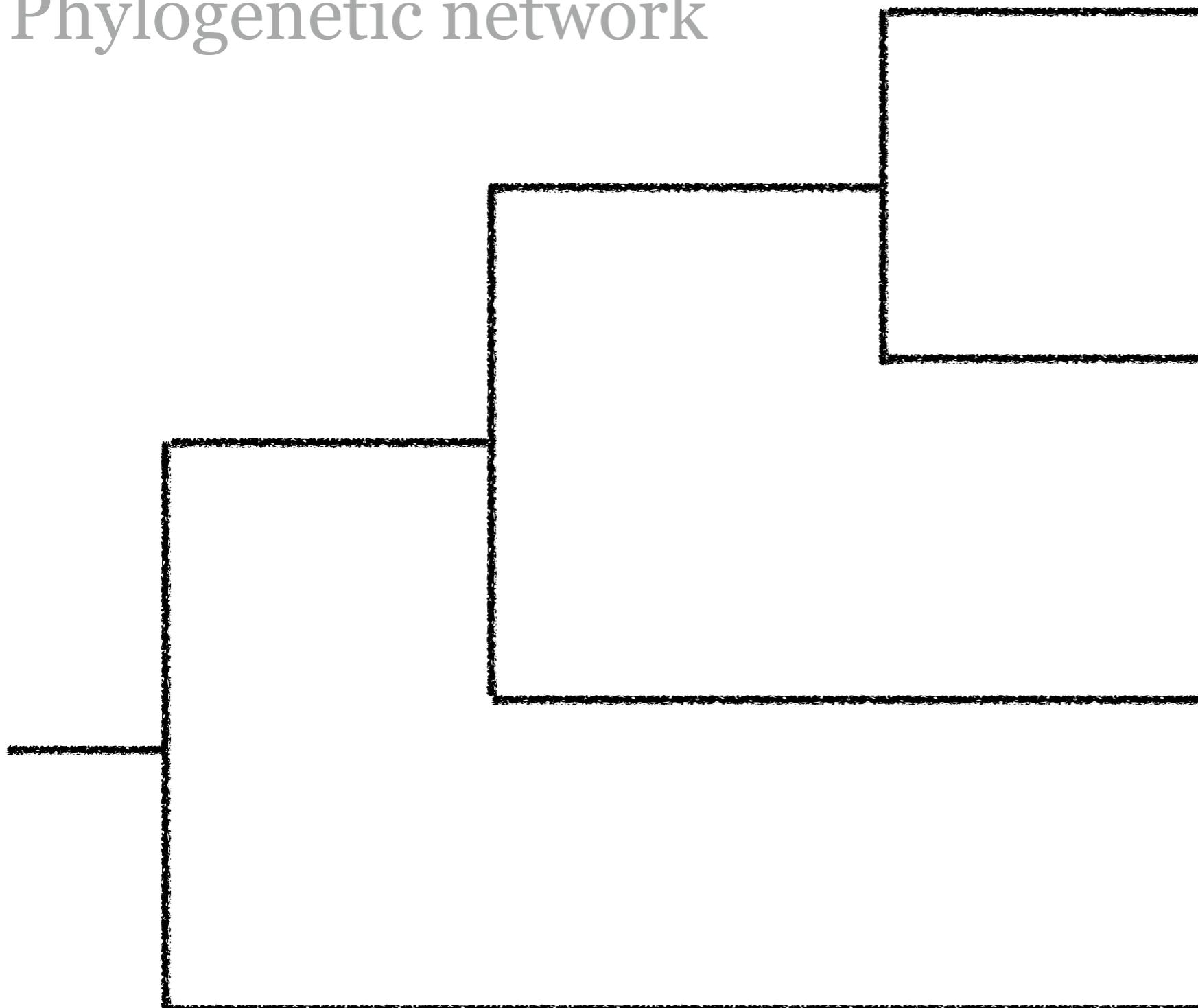
# Why?

Phylogenetic network



# Why?

Phylogenetic network



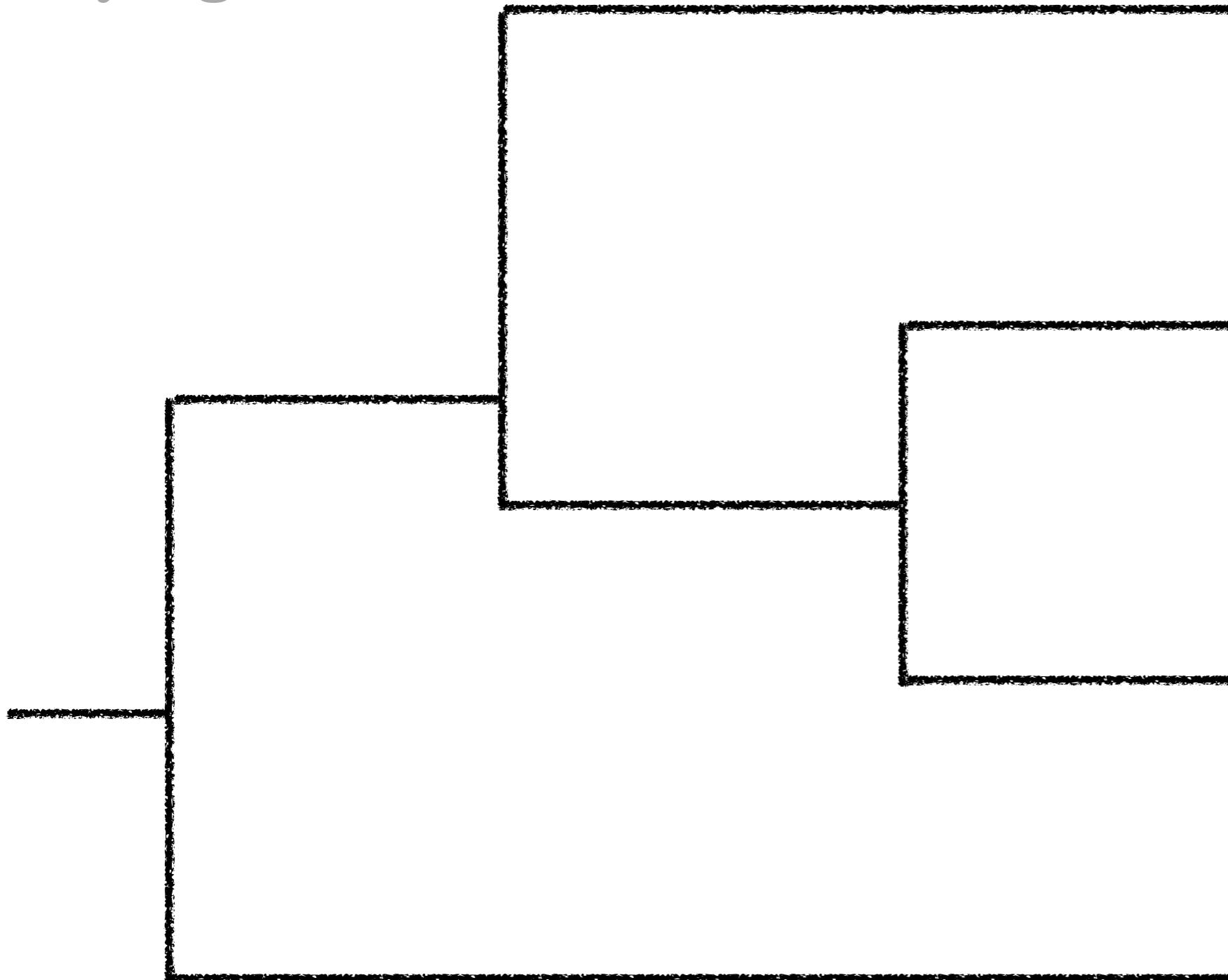
Main tree



# Why?

Phylogenetic network

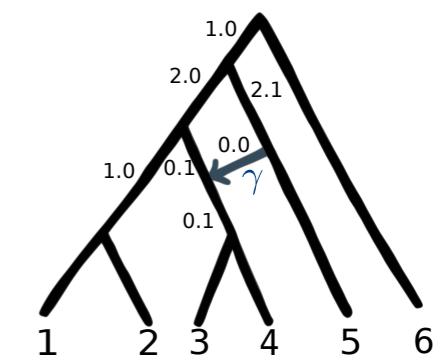
Ignore gene flow  
=>Wrong tree!



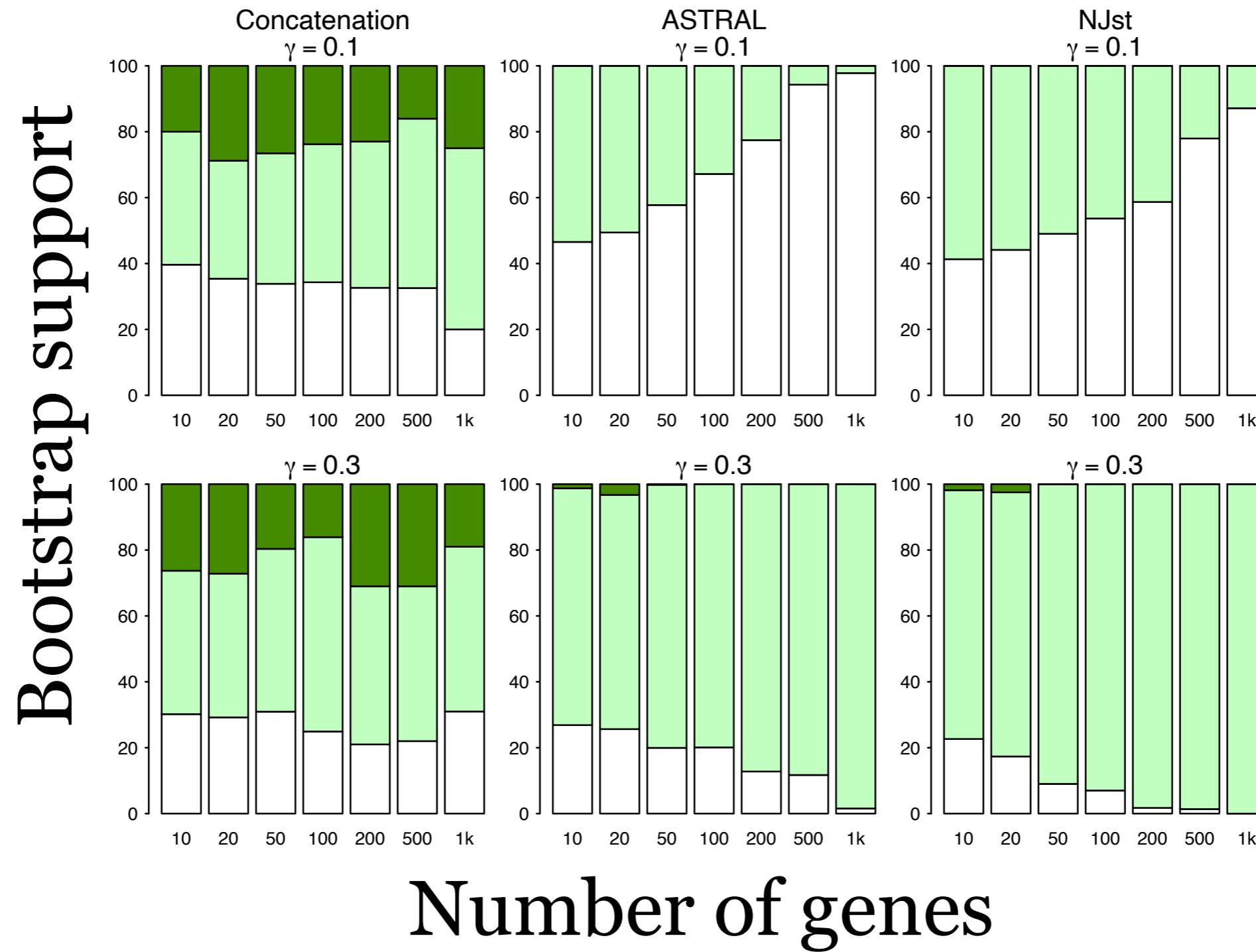
# Why?

Phylogenetic network

Coalescent tree methods  
not robust to gene flow



White:  
true tree

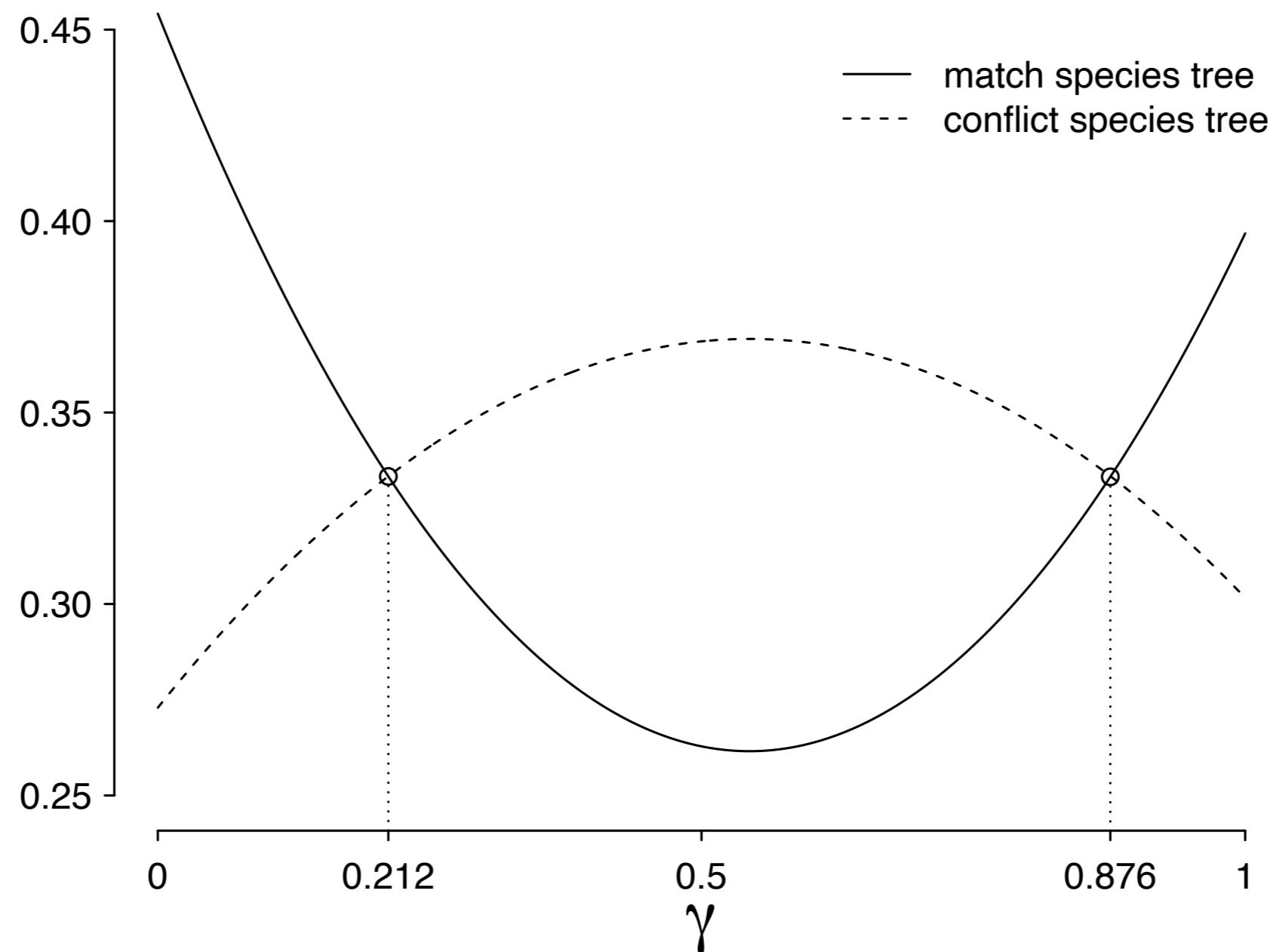


(S.-L., Yang, Ané, 2016, Syst Bio)

ASTRAL (Mirarab et al, 2014)  
NJst (Liu&Yu, 2011)

# Why? Phylogenetic network

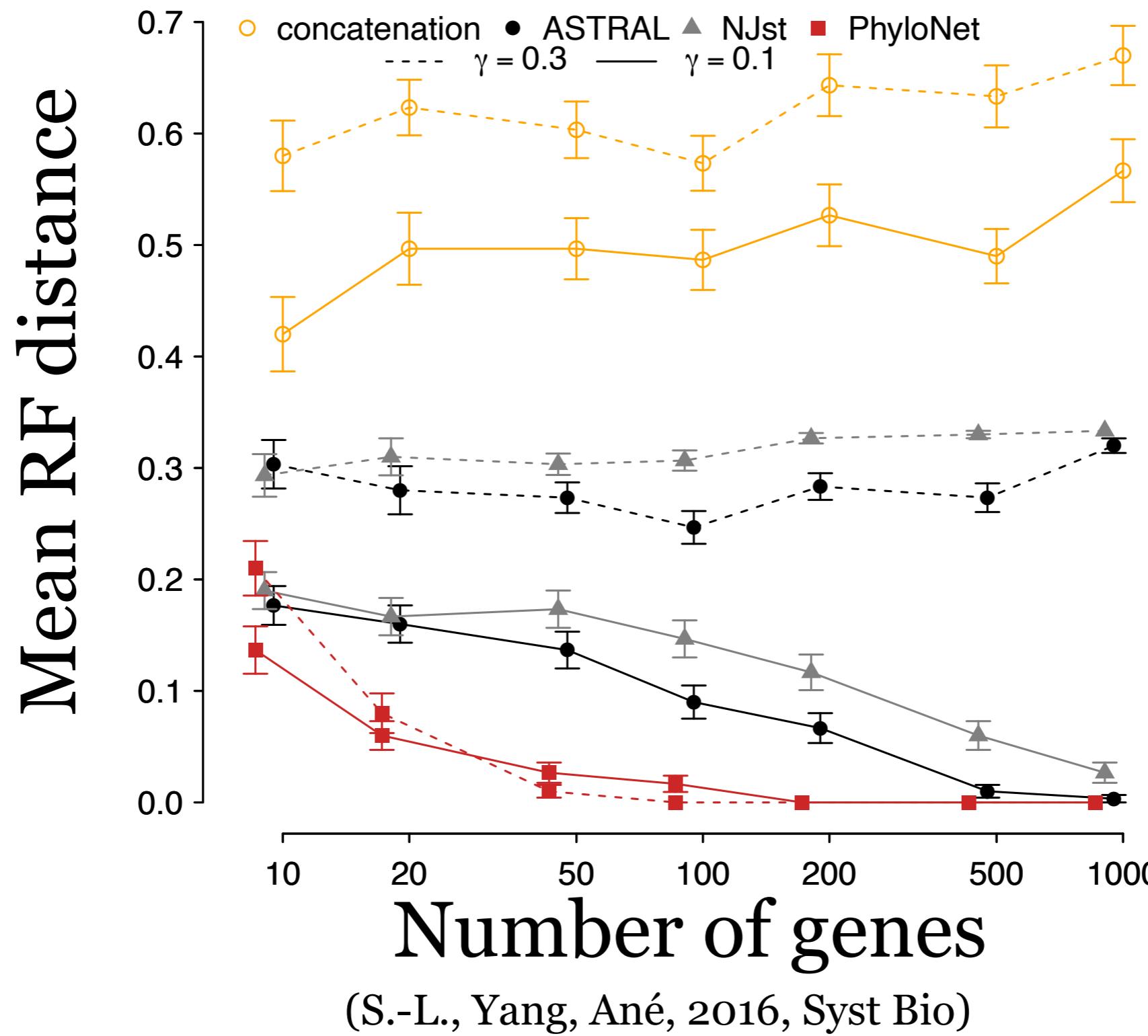
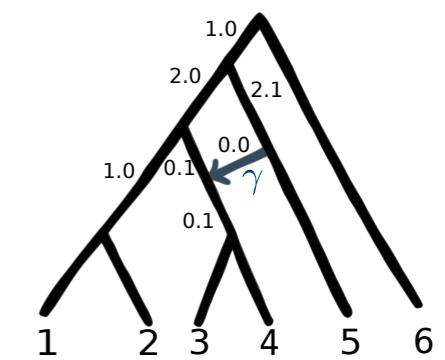
Anomaly zone with  
gene flow



# Why?

## Phylogenetic network

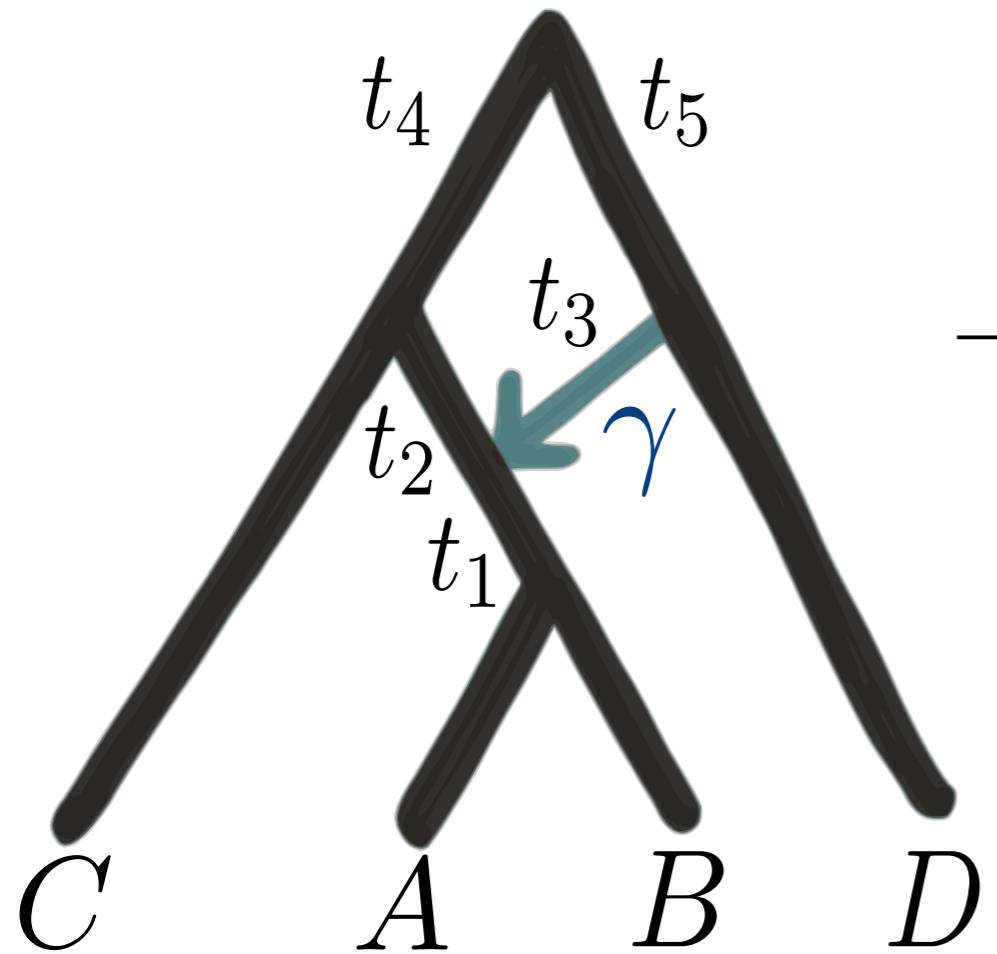
Coalescent tree methods  
not robust to gene flow



# Why?

Phylogenetic network

## Anomalous unrooted gene trees with gene flow



Frequency among gene trees

| Quartet | $\gamma = 0.0$ | $\gamma = 0.1$ | $\gamma = 0.3$ |
|---------|----------------|----------------|----------------|
| $AB CD$ | <b>0.347</b>   | <b>0.298</b>   | <b>0.260</b>   |
| $CA BD$ | 0.327          | 0.351          | 0.370          |
| $CB AD$ | 0.327          | 0.351          | 0.370          |

$$t_1 = t_2 = 0.01, t_3 = t_4 = t_5 = 1$$

- **ILS**: no AUGT on 4 taxa (Degnan, 2013)
- **ILS+HGT**: AUGT on 4 taxa (S.-L., Yang, Ané, 2016, Syst Bio)

# So far...

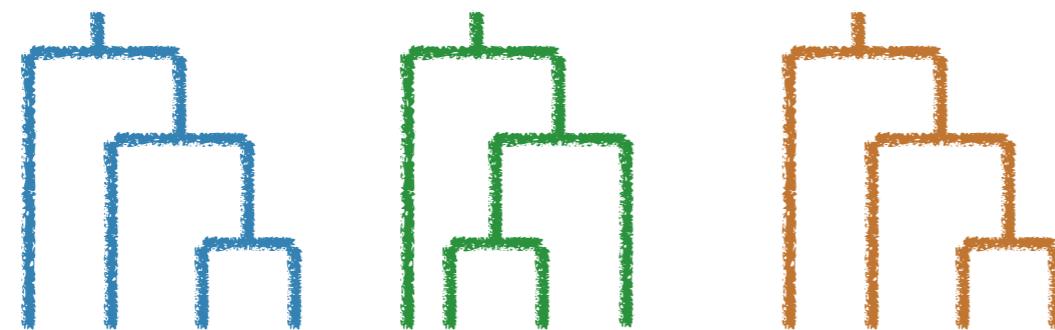
- Networks are good
- Explicit networks are better
- If you ignore gene flow, you can estimate the wrong tree

# How?

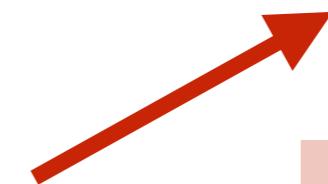
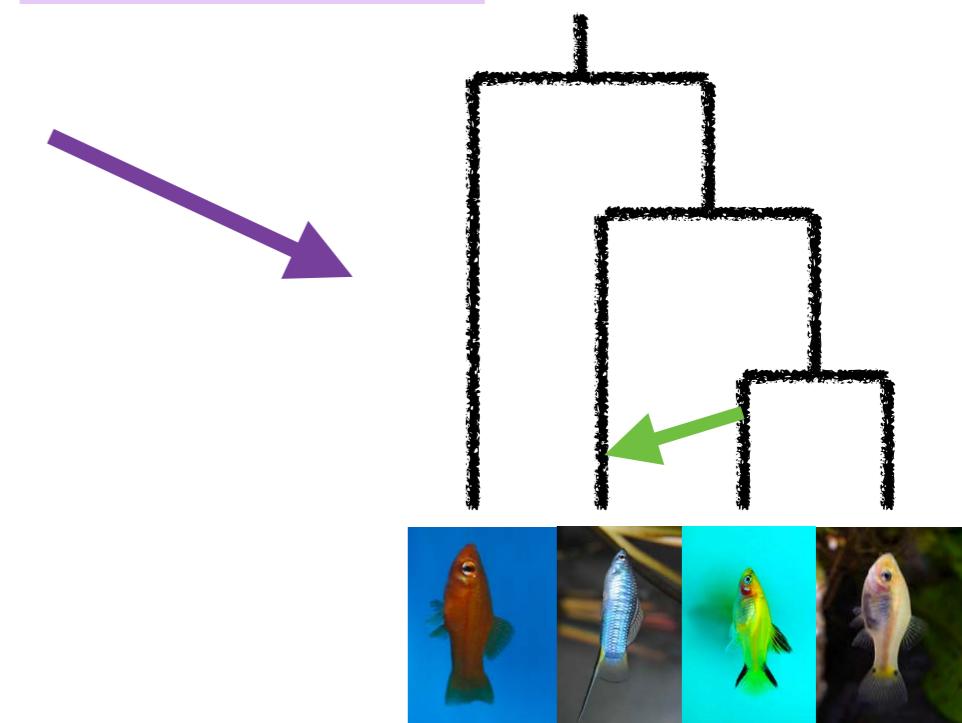
## Phylogenetic network



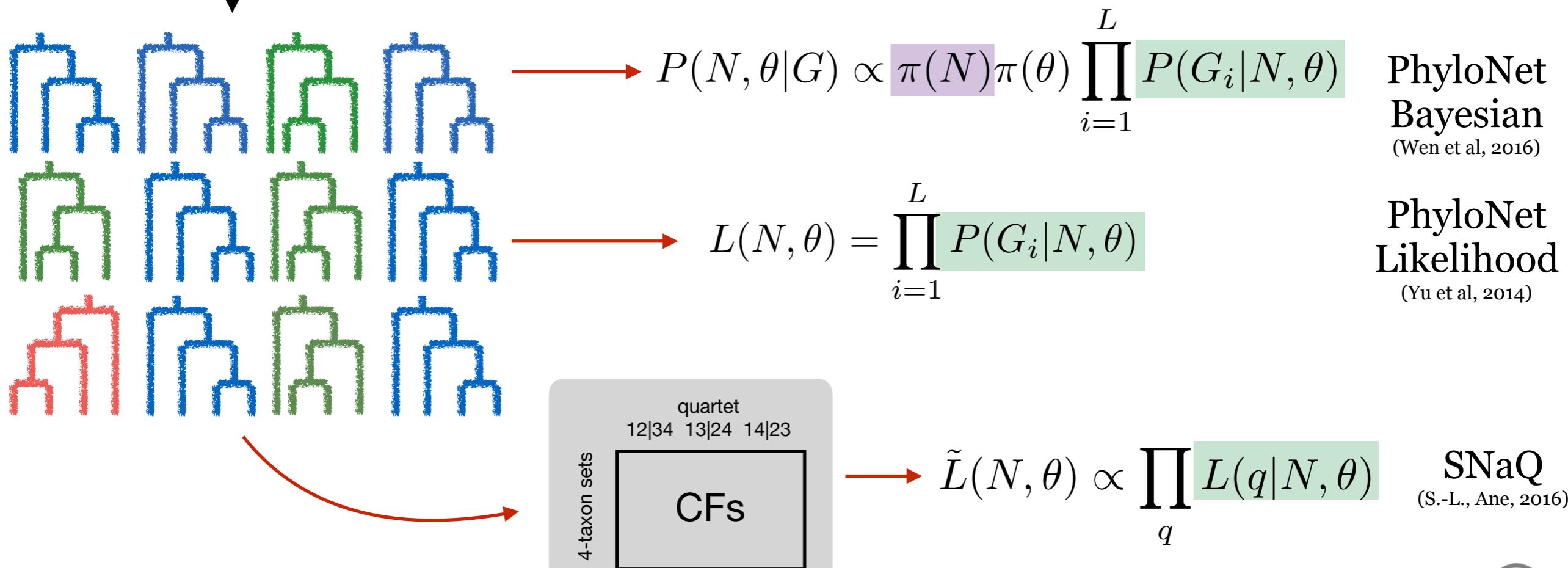
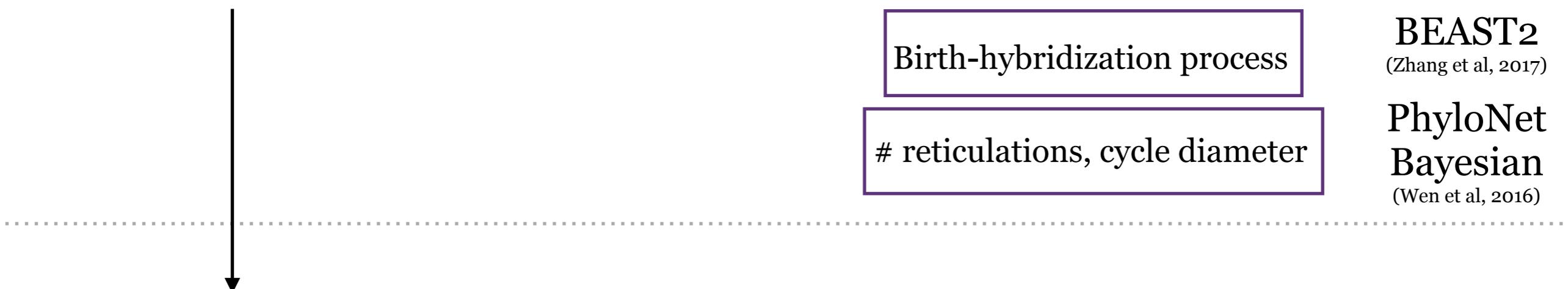
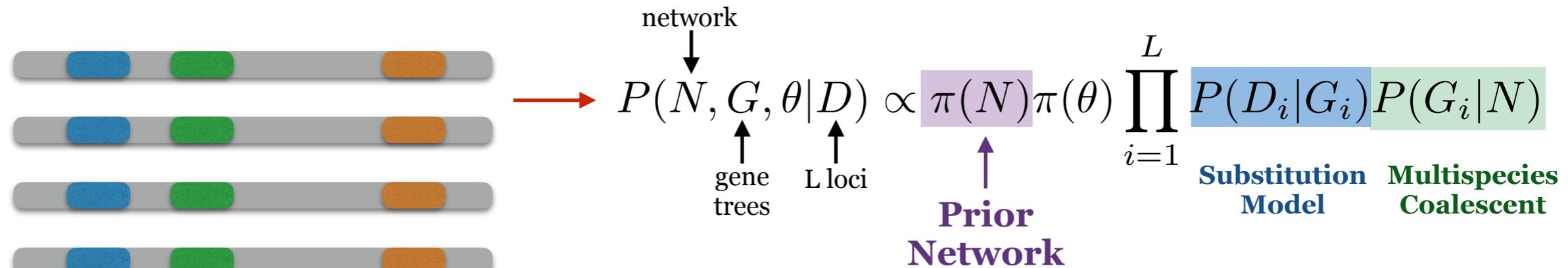
MrBayes  
(Huelsenbeck, Ronquist, 2001)  
RAxML  
(Stamatakis, 2014)  
PhyML  
(Guindon et al, 2010)  
RevBayes  
(Hoehna et al, 2016)  
IQ-TREE  
Nguyen et al. (2015)



BEAST2  
(Zhang et al, 2017)  
PhyloNet  
(Wen et al, 2016)



SNaQ  
(S.-L., Ane, 2016)  
PhyloNet  
(Yu et al, 2014)



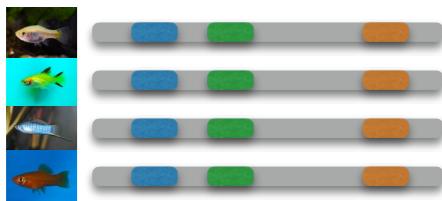
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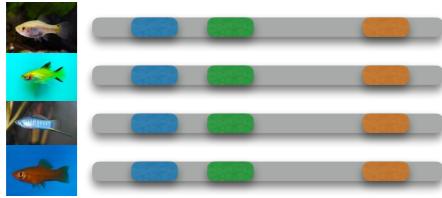
crsl4



**BEAST2**  
(Zhang et al, 2017)

Birth-hybridization process

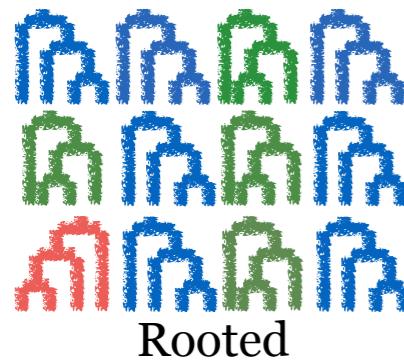
Most accurate,  
not scalable



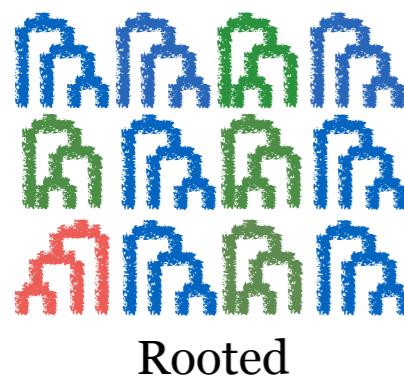
**PhyloNet Bayesian**  
(Wen et al, 2016)

# reticulations,  
cycle diameter

**MCMC:**  
Network  
moves,  
mixing

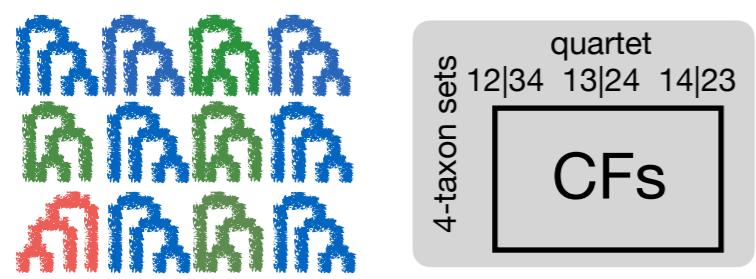


**PhyloNet Bayesian**  
(Wen et al, 2016)



**PhyloNet Likelihood**  
(Yu et al, 2014)

**Heuristic search:**  
Network  
moves



**SNaQ**  
(S.-L., Ane, 2016)

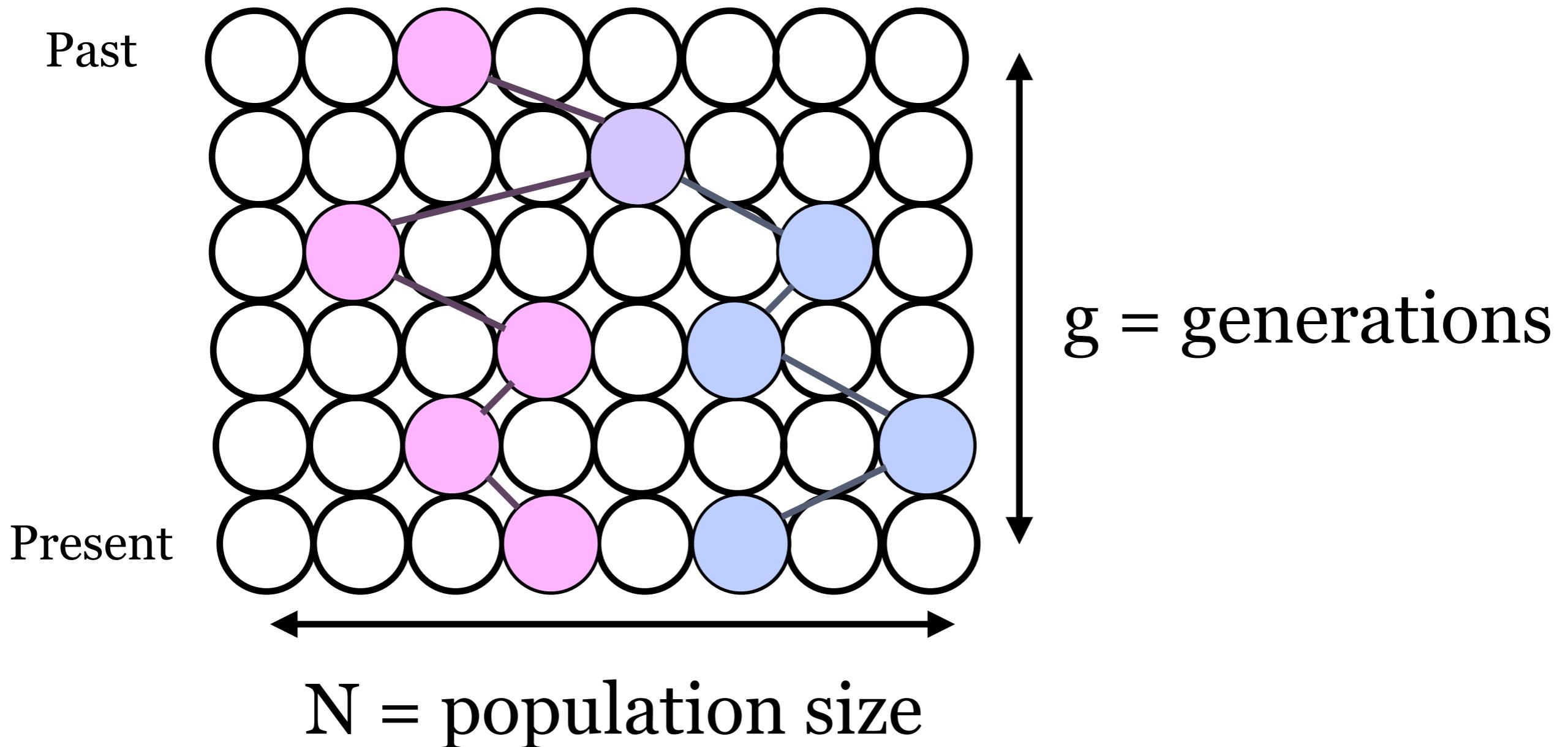
Level-1  
networks

More scalable,  
Robust

Unrooted

|                              |                              |                       |                                      |
|------------------------------|------------------------------|-----------------------|--------------------------------------|
| STEM-hy                      | gene trees<br>rooted, BL     | likelihood            | hybridization b/w<br>sister lineages |
| PhyloNet<br>InferNetwork_ML  | gene trees<br>rooted         | likelihood            |                                      |
| PhyloNet<br>InferNetwork_MPL | gene trees<br>rooted         | triplet<br>likelihood |                                      |
| PhyloNetworks<br>SNaQ        | gene trees<br>or quartet CFs | quartet<br>likelihood | level-1 network                      |
| PhyloNet<br>MCMC_GT          | gene trees<br>rooted         | Bayesian              | compound prior                       |
| PhyloNet<br>MCMC_SEQ         | alignments                   | Bayesian              | compound prior<br>no rate variation  |
| BEAST2<br>SpeciesNetwork     | alignments                   | Bayesian              | birth-hyb prior                      |
| PhyloNet<br>MLE_BiMarkers    | biallelic sites              | likelihood            | compound prior                       |
| PhyloNet<br>MCMC_BiMarkers   | biallelic sites              | Bayesian              | compound prior                       |
| HyDe                         | sites                        | invariants            | 4 taxa, 1 hyb.                       |

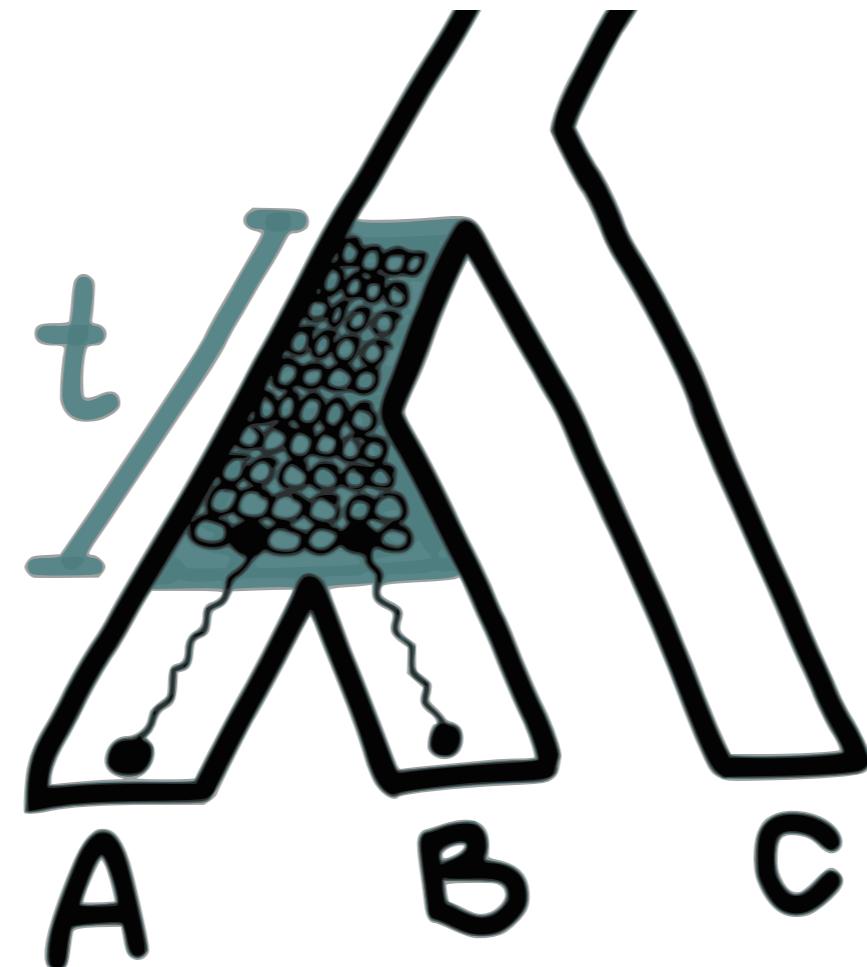
# Coalescent model within 1 population



Probability of no coalescence in  $g$  generations:

$$\left(1 - \frac{1}{N}\right)^g$$
$$t = g/N \Rightarrow \left(1 - \frac{t}{Nt}\right)^{Nt} \xrightarrow[N \rightarrow \infty]{} e^{-t}$$

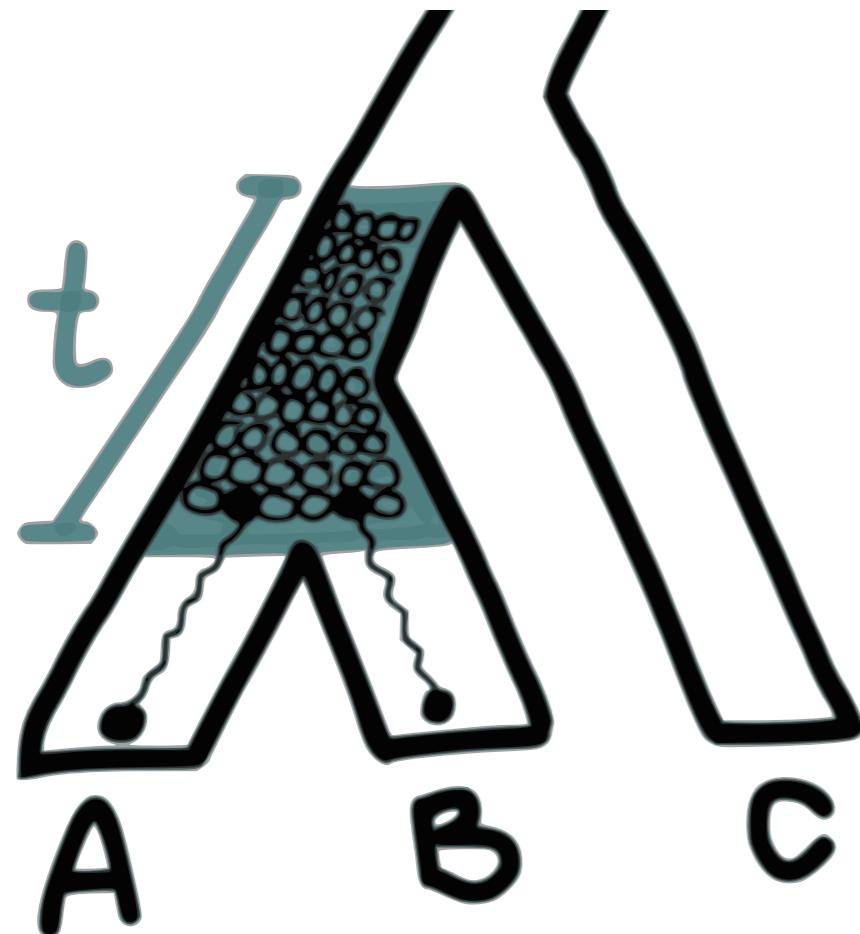
# Multispecies coalescent on a tree



$$P(T > t) = e^{-t}$$

$$T = \frac{g}{N} \text{ coalescent units} \sim \text{Exp}(1)$$

# Multispecies coalescent on a tree

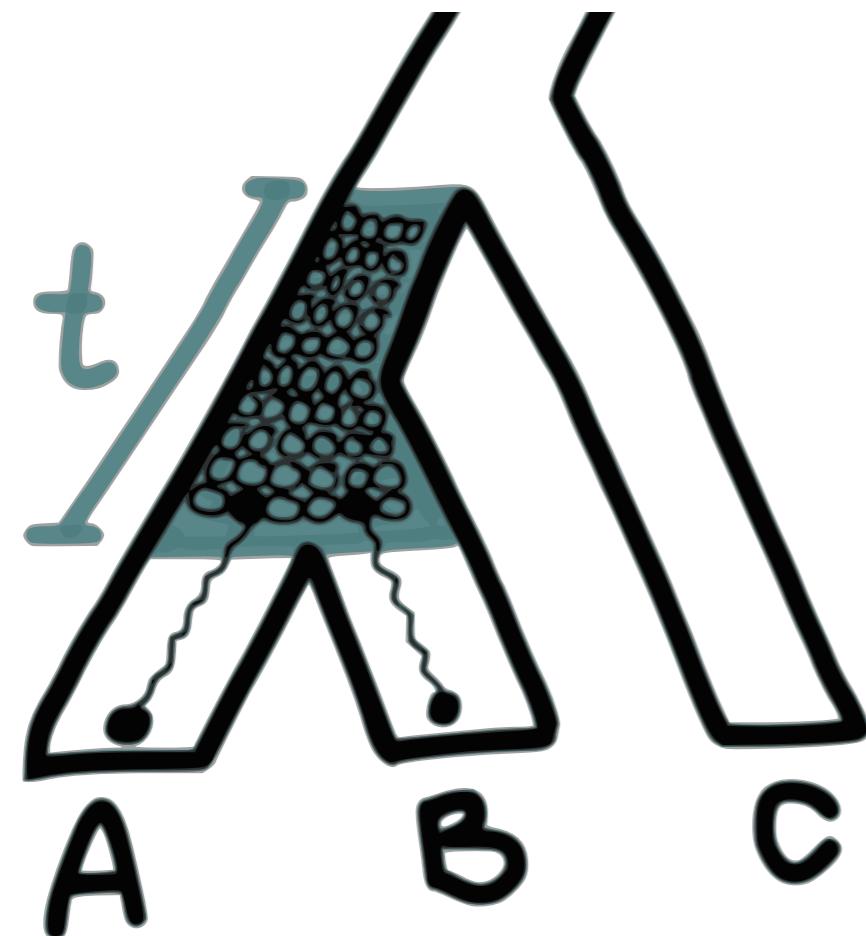


$$P(\text{ } \text{ } \text{ } \text{ } \text{ } \text{ } ) =$$

A large black letter P followed by a large black parenthesis containing a phylogenetic tree with three tips labeled A, B, and C. To the right of the parenthesis is an equals sign (=).

$$P(T > t) = e^{-t}$$

# Multispecies coalescent on a tree

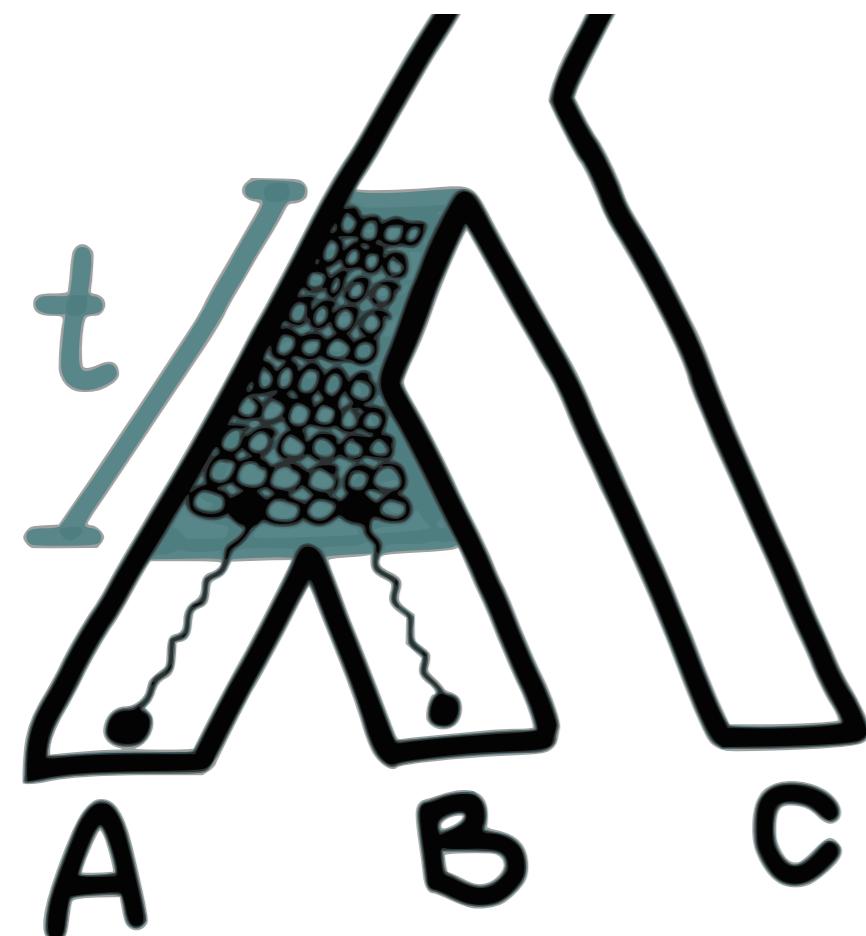


$$P(\text{ } \text{ } \text{ } \text{ } \text{ } \text{ } ) = 1 - e^{-t}$$

The probability of finding the tree configuration shown above at time  $t$  is given by the formula  $P(\text{ } \text{ } \text{ } \text{ } \text{ } \text{ } ) = 1 - e^{-t}$ .

$$P(T > t) = e^{-t}$$

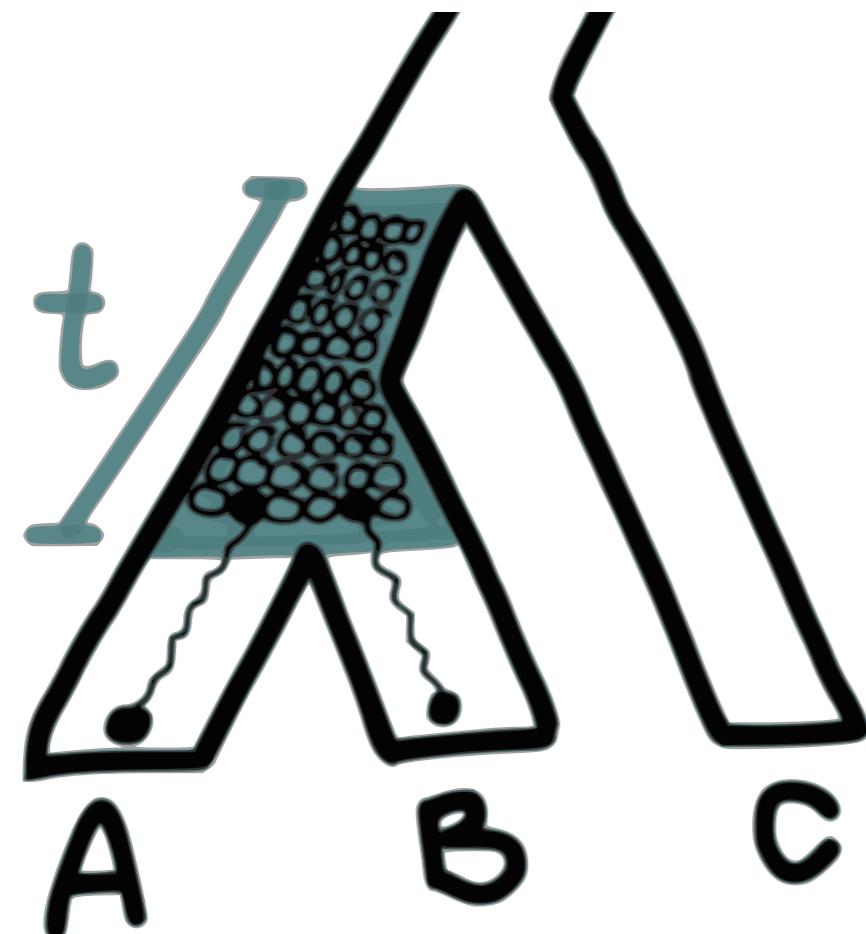
# Multispecies coalescent on a tree



$$P(\text{ } \text{ } \text{ } \text{ } \text{ } \text{ } ) = \\ 1 - e^{-t} \\ +$$

$$P(T > t) = e^{-t}$$

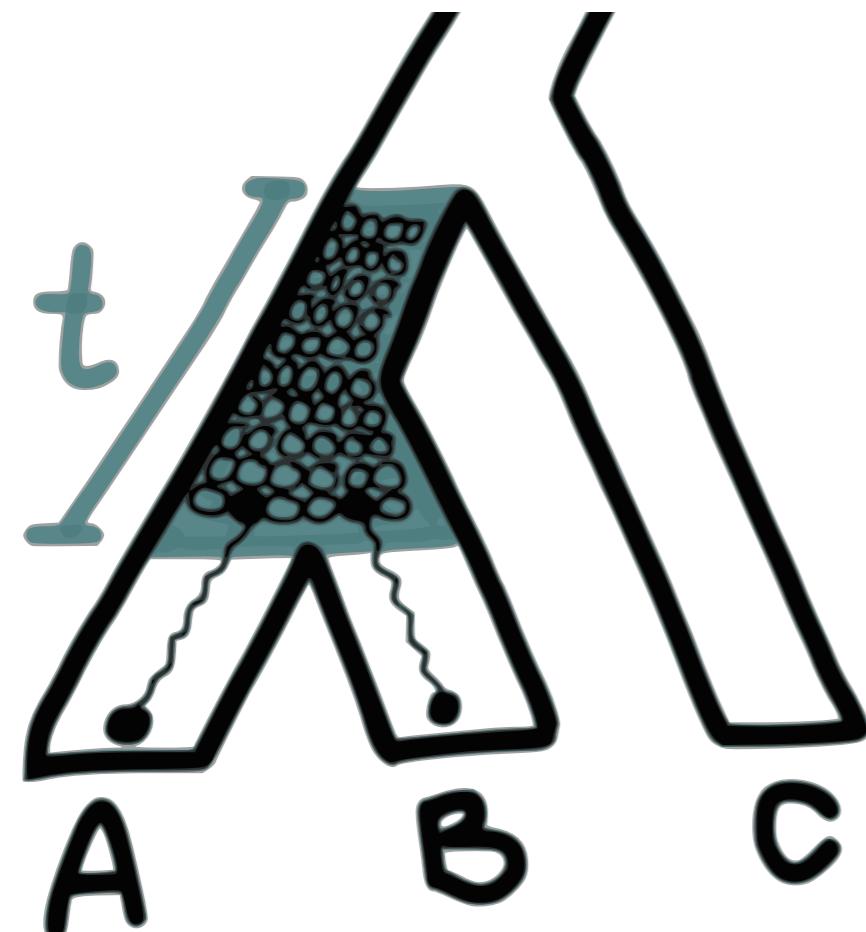
# Multispecies coalescent on a tree



$$P(\text{ } \text{ } \text{ } \text{ } \text{ } \text{ } ) = \\ 1 - e^{-t} \\ + \\ e^{-t} \times 1/3$$

$$P(T > t) = e^{-t}$$

# Multispecies coalescent on a tree



$$P(\text{ } \text{ } \text{ } \text{ } \text{ } \text{ } ) =$$

$$1 - e^{-t}$$

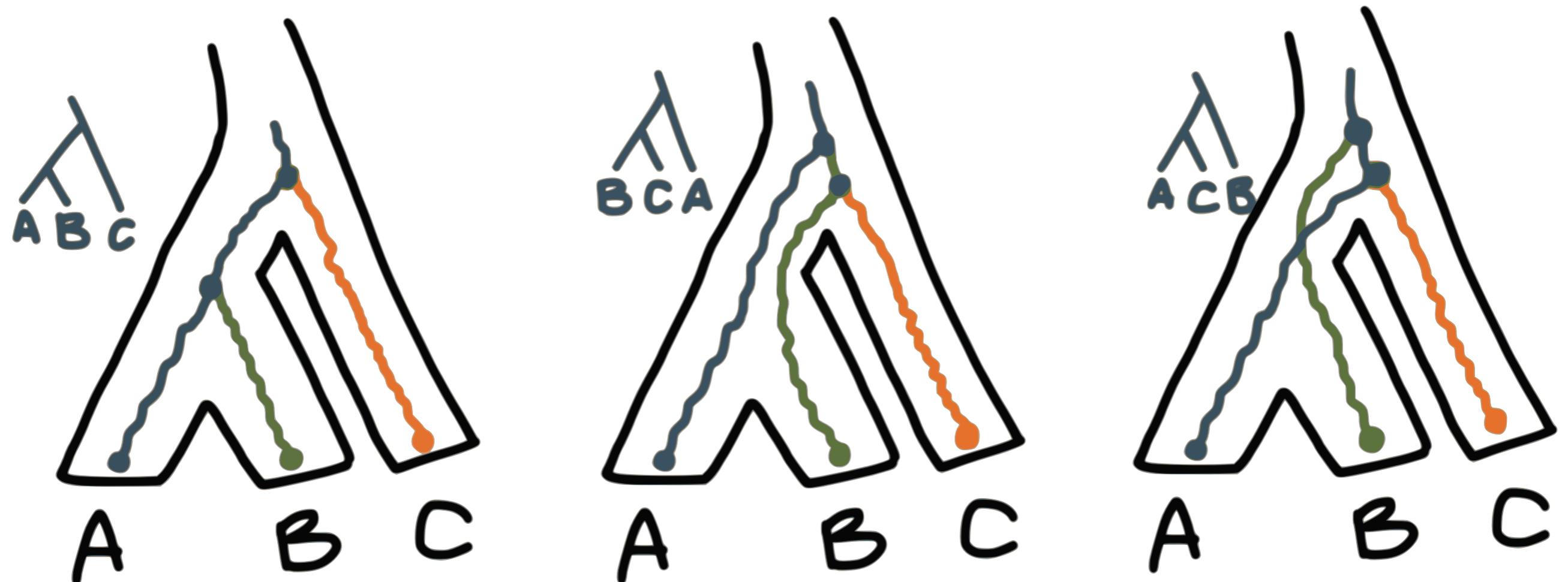
+

$$e^{-t} \times 1/3$$

$$= 1 - \frac{2}{3}e^{-t}$$

$$P(T > t) = e^{-t}$$

# Multispecies coalescent on a tree

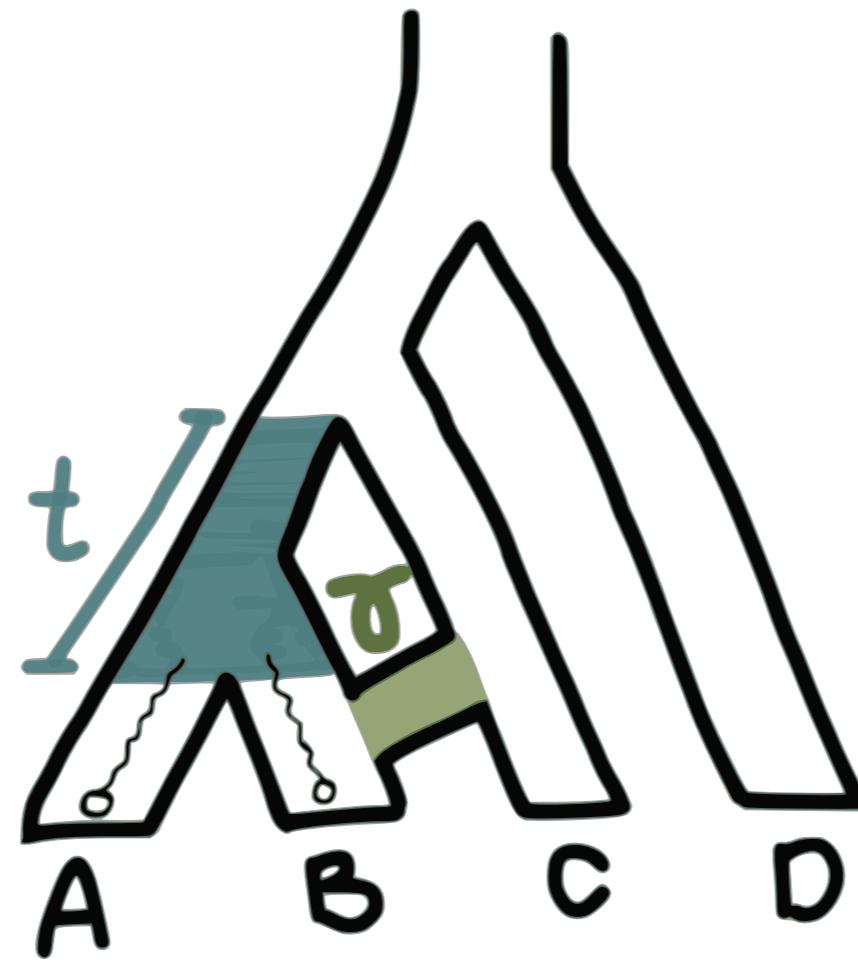


$$1 - \frac{2}{3}e^{-t}$$

$$\frac{1}{3}e^{-t}$$

$$\frac{1}{3}e^{-t}$$

# Multispecies coalescent on a network



(Meng, Kubatko, 2009)  
(Yu, Degnan, Nakhleh, 2012)

# Multispecies coalescent on a network



(Meng, Kubatko, 2009)  
(Yu, Degnan, Nakhleh, 2012)

# Multispecies coalescent on a network



$$(1 - \gamma) \frac{1}{3} e^{-t} + \gamma (1 - \frac{2}{3} e^{-t_2})$$

(Meng, Kubatko, 2009)  
(Yu, Degnan, Nakhleh, 2012)

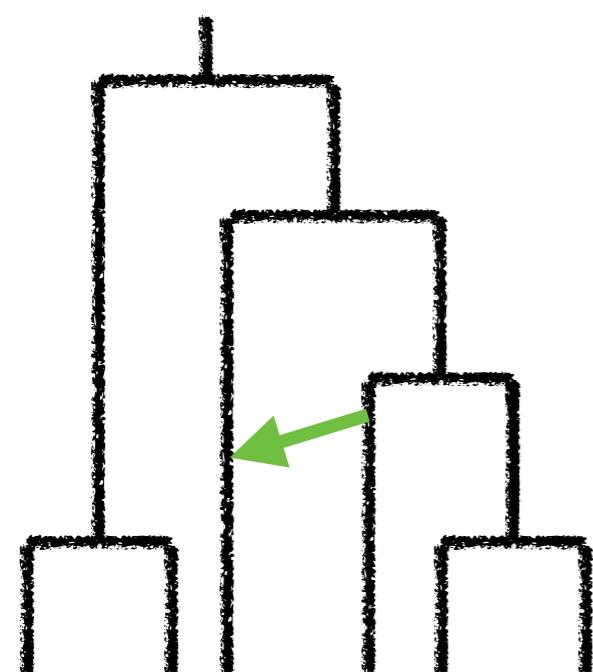
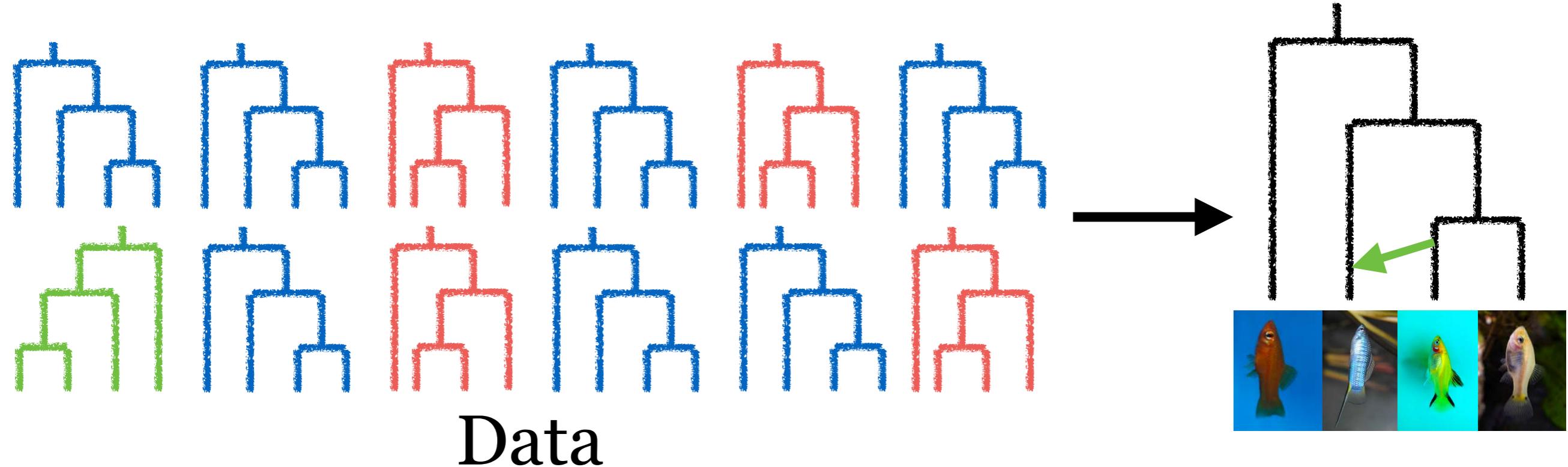
# Multispecies coalescent on a network



$$CF_{BC|AD}(t, t_2, \gamma) = (1 - \gamma) \frac{1}{3} e^{-t} + \gamma (1 - \frac{2}{3} e^{-t_2})$$

(Meng, Kubatko, 2009)  
(Yu, Degnan, Nakhleh, 2012)

# Maximum pseudolikelihood



Quartet-based inference

$$\tilde{L}(\textit{network}) = \prod L(\textit{quartet})$$

(S-L, Ané, 2016, PLoS Genetics)

[www.github.com/CRSL4/PhyloNetworks](https://www.github.com/CRSL4/PhyloNetworks)

snaQ julia



<https://solislemuslab.github.io/>

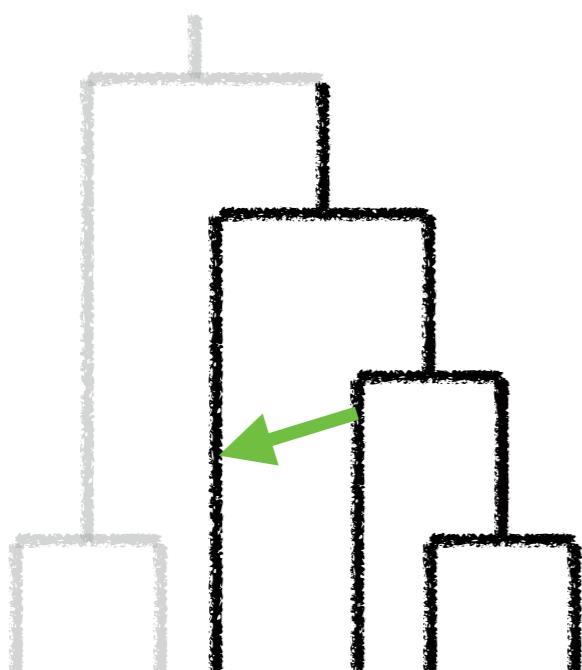
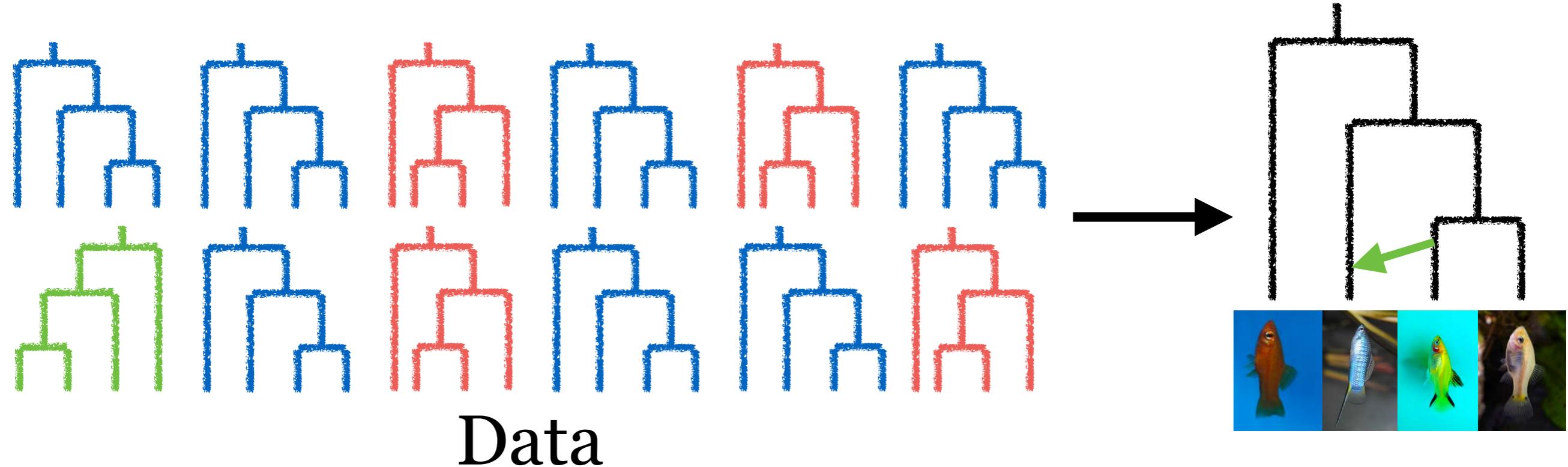


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# Maximum pseudolikelihood



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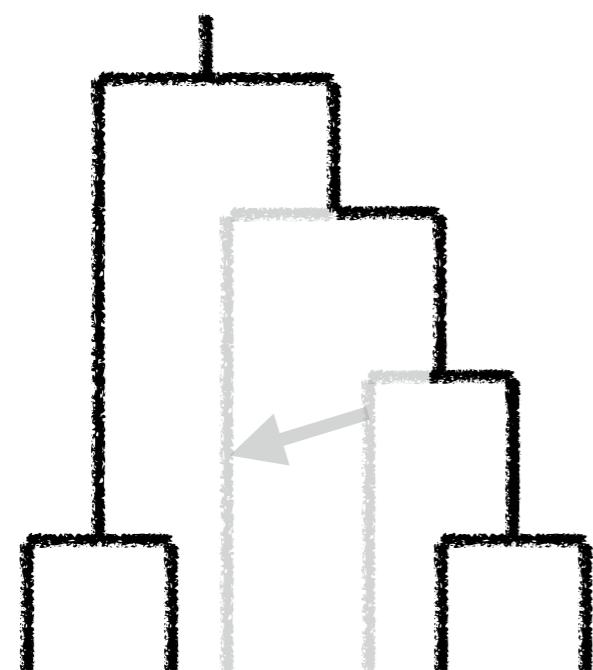
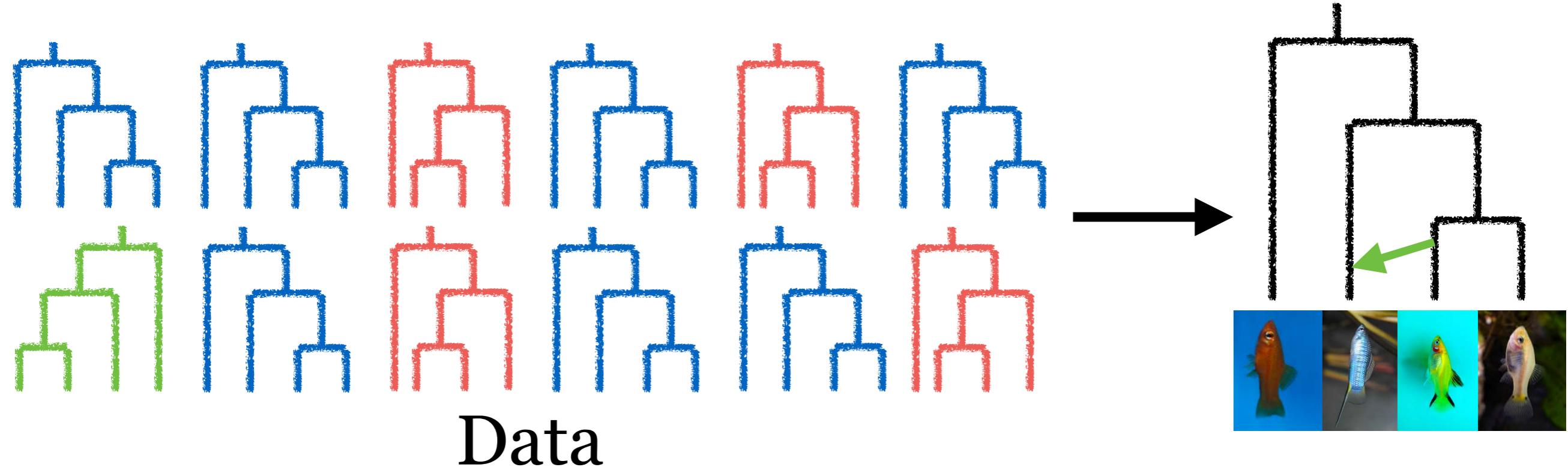


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# Maximum pseudolikelihood



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[www.github.com/CRSL4/PhyloNetworks](https://www.github.com/CRSL4/PhyloNetworks)

Quartet-based inference

snaQ julia

# Maximum pseudolikelihood

Unrooted gene trees

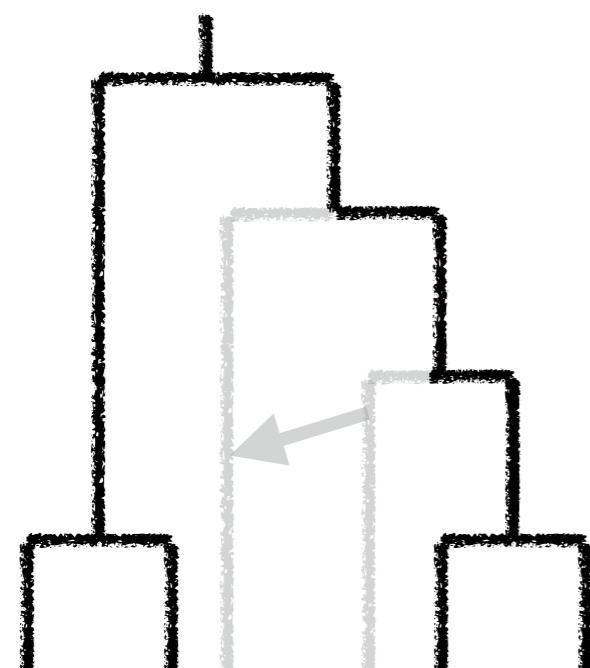
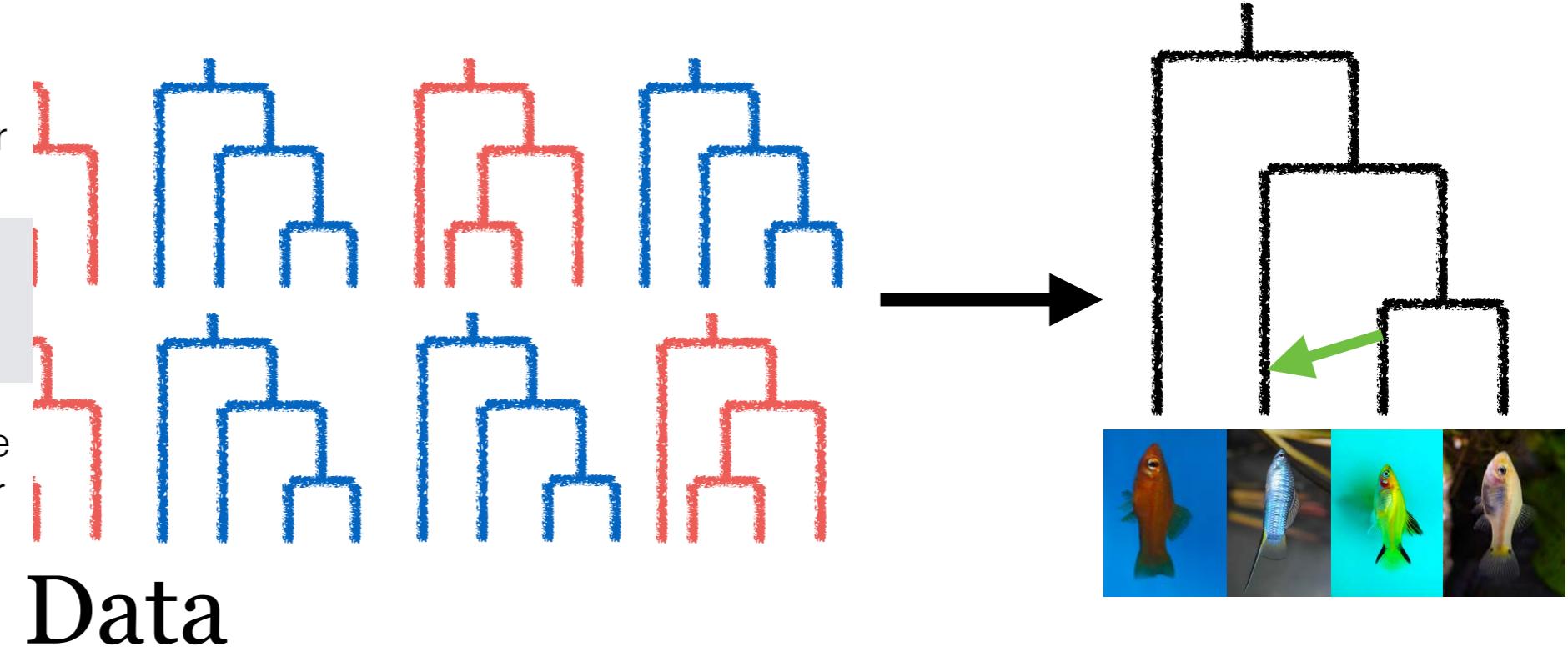
No branch lengths

Concordance factors

No rooting error

No molecular clock assumption

Account for tree estimation error



Quartet-based inference

$$\tilde{L}(\textit{network}) = \prod L(\textit{quartet})$$

(S-L, Ané, 2016, PLoS Genetics)

[www.github.com/CRSL4/PhyloNetworks](https://www.github.com/CRSL4/PhyloNetworks)

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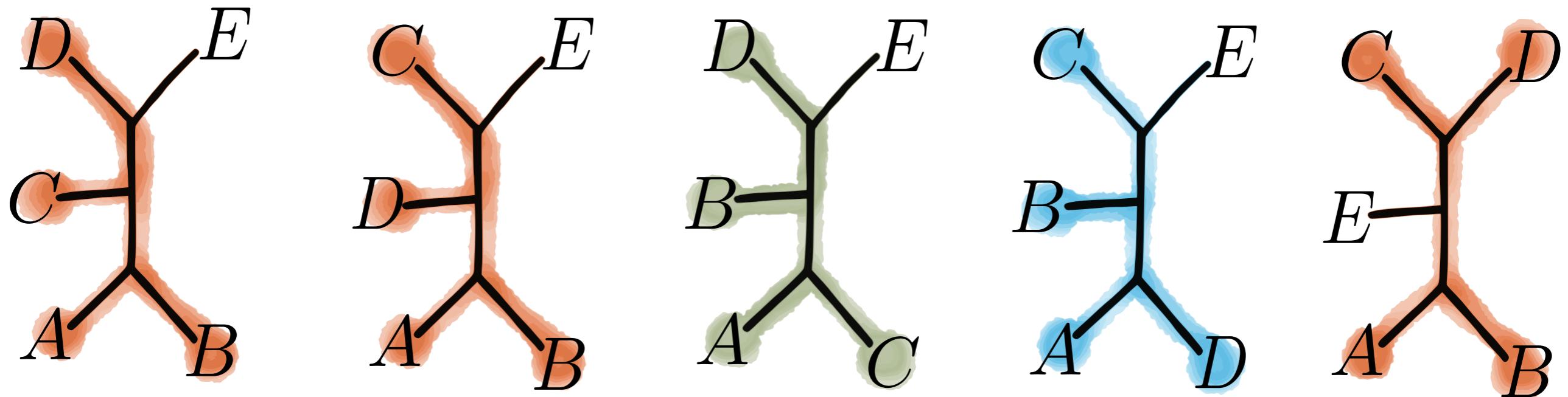


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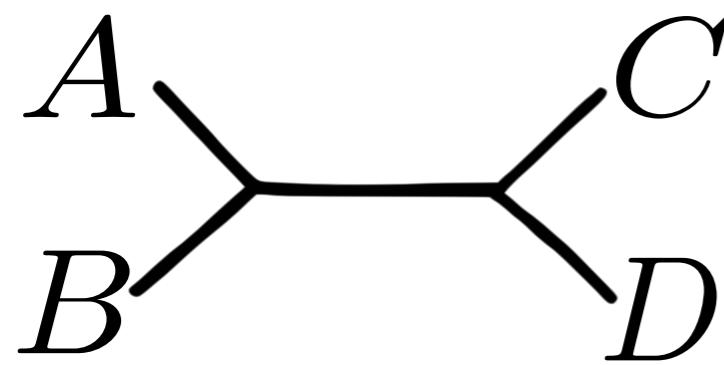


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# Quartet-based inference



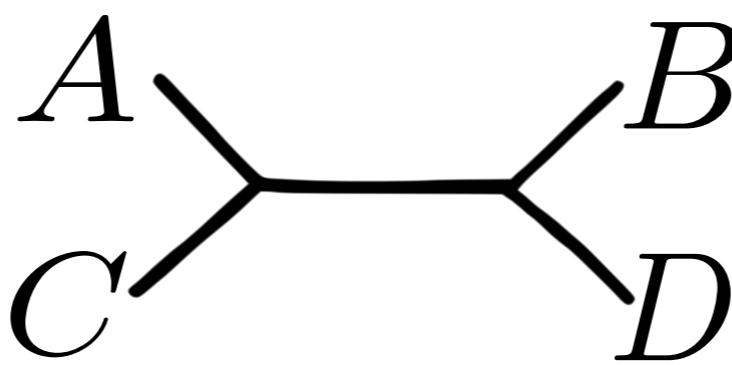
Concordance factors (CF):  
% of genes having the quartet in their tree



3/5



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1/5



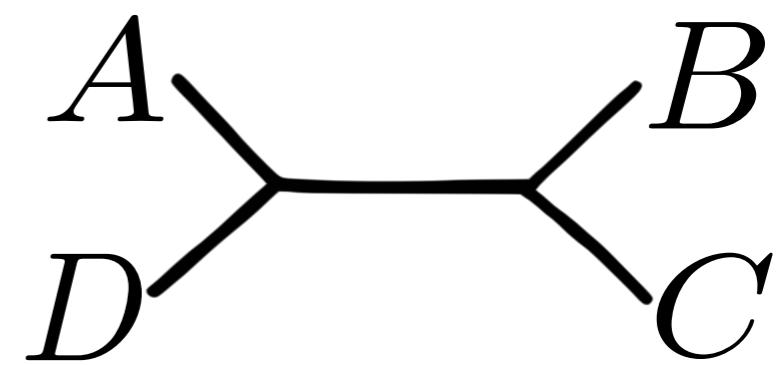
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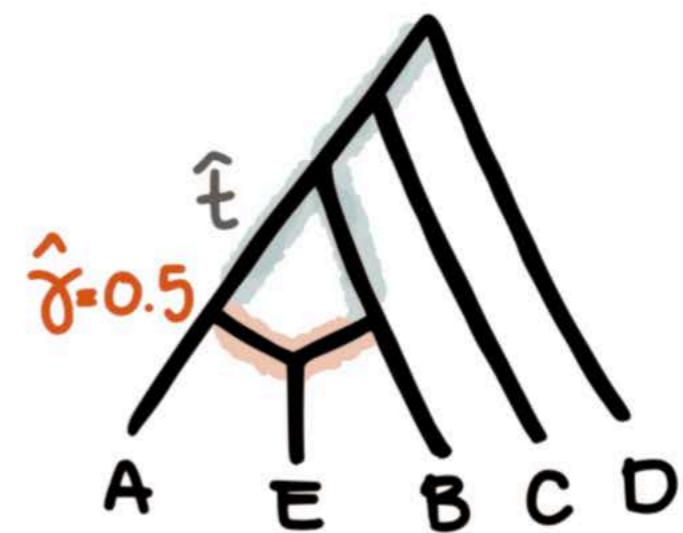
1/5

# Quartet-based inference

Observed **quartet** CFs:

| 4 taxon set | $CF_1$ | $CF_2$ | $CF_3$ |
|-------------|--------|--------|--------|
| A B C D     | .80    | .10    | .10    |
| A B C E     | .40    | .40    | .20    |
| A B D E     | .40    | .40    | .20    |
| A C D E     | .84    | .08    | .08    |
| B C D E     | .82    | .10    | .08    |

inferred network:



Maximum Pseudo-Likelihood:

$$\log \tilde{L} = \sum_{q \in Q(N)} CF_{in,1} \log(CF_{net,1}) + CF_{in,2} \log(CF_{net,2}) + CF_{in,3} \log(CF_{net,3})$$



<https://solislemuslab.github.io/>



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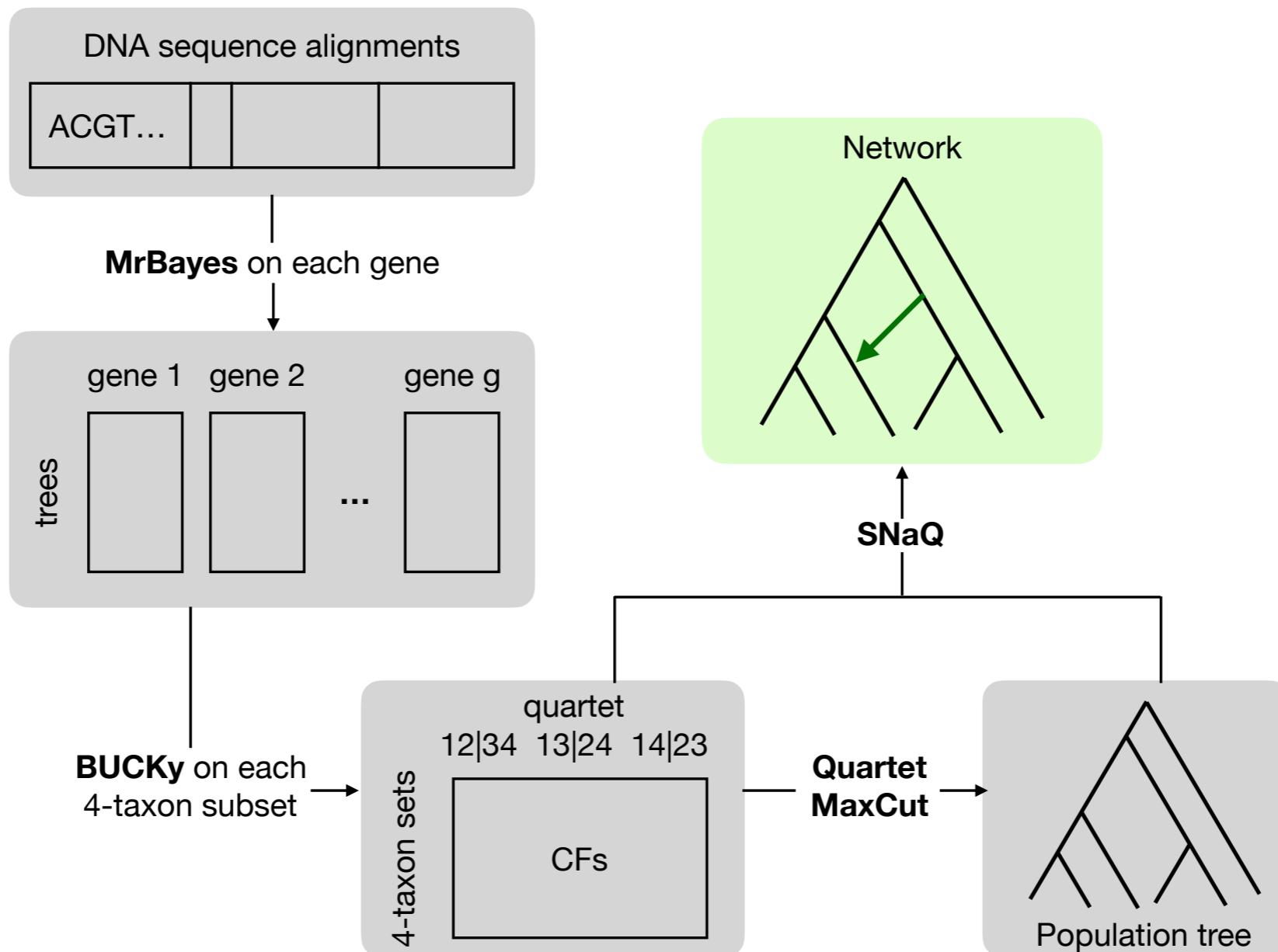
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# How?

## Phylogenetic network



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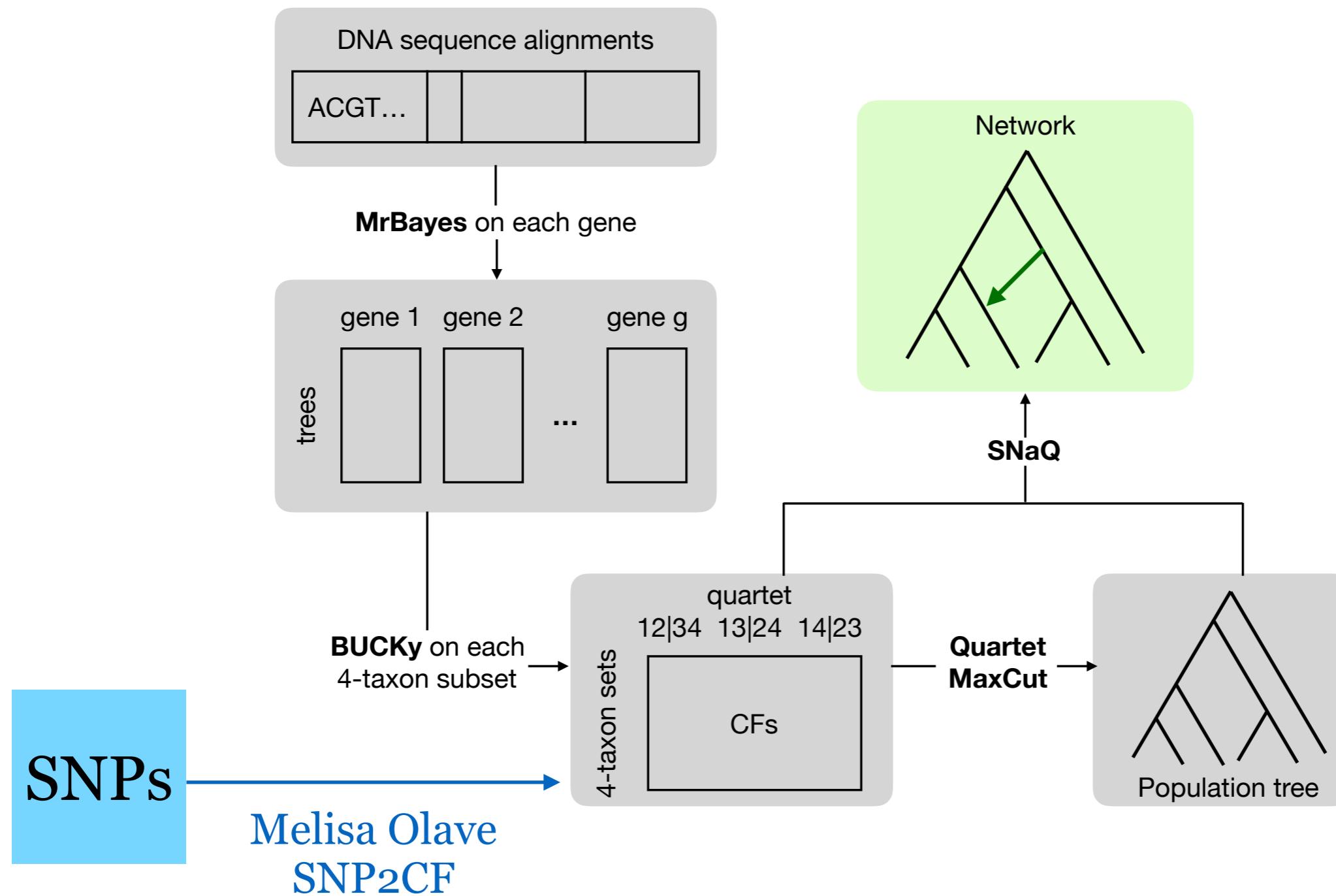
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# How?

## Phylogenetic network



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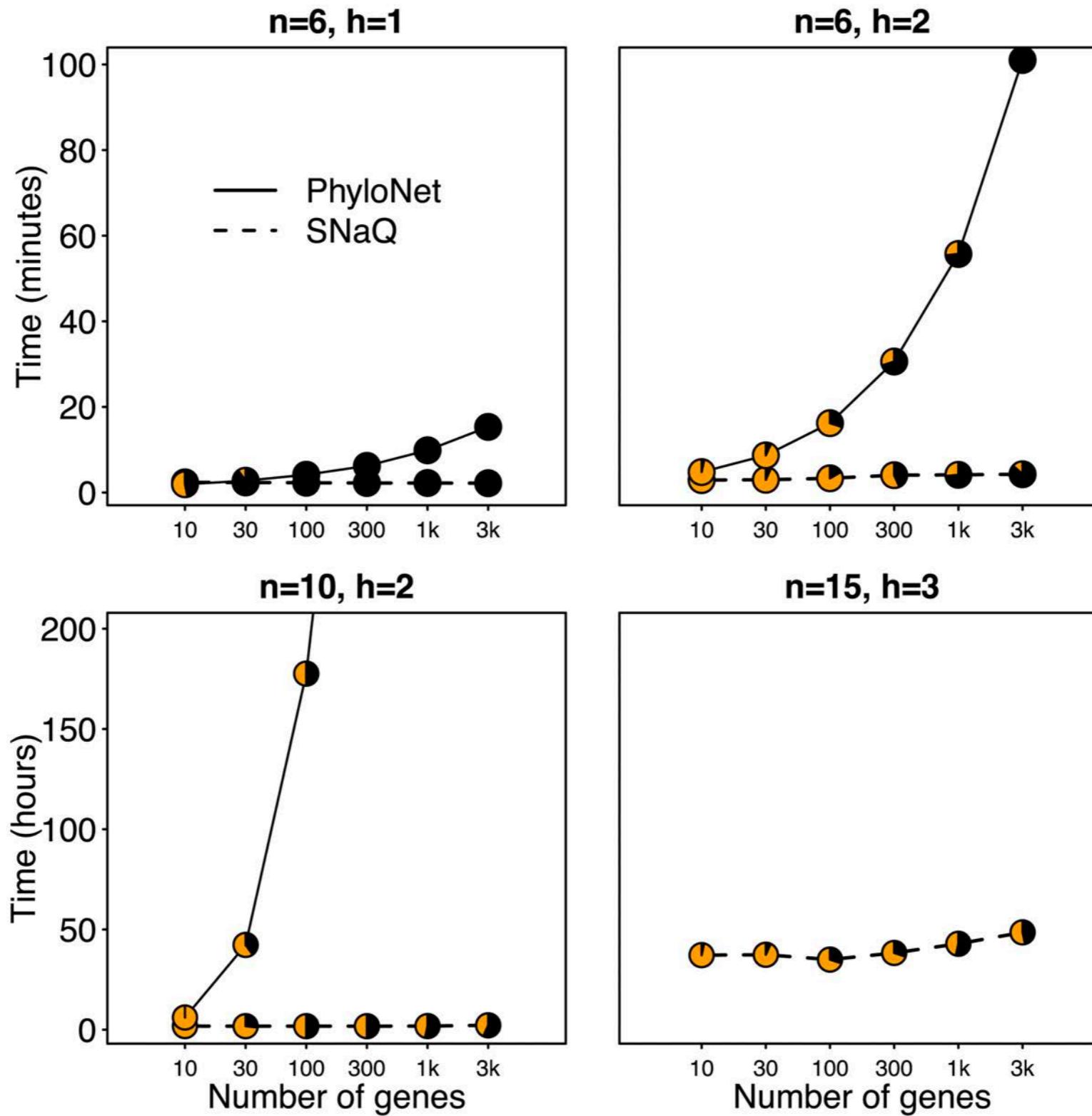
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# Network challenges

- Scalability
- Identifiability
- Network space
- Network comparison



# Scalability gains



(Solís-Lemus, Ané, 2016, PLoS Genetics)



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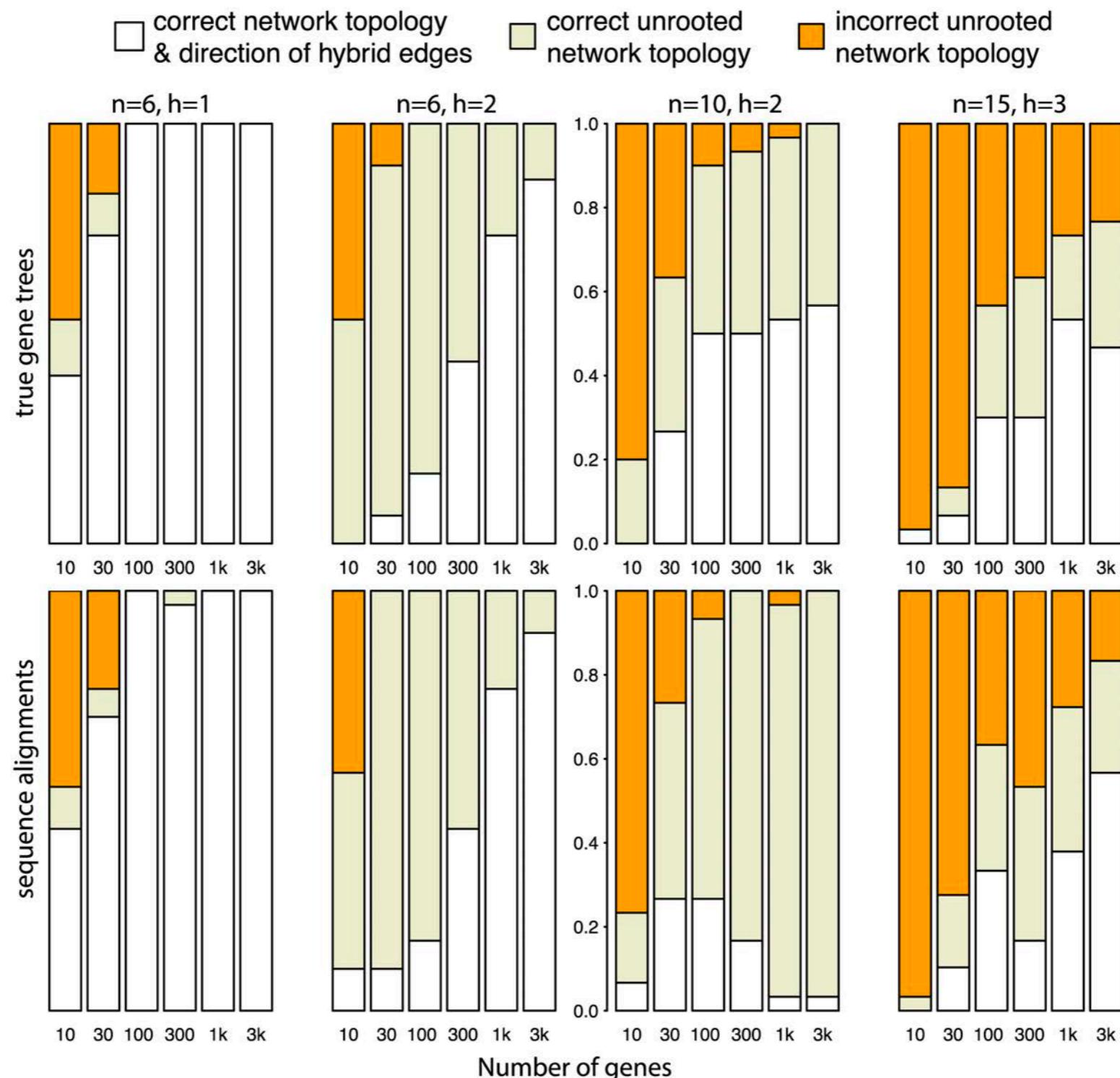


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# Accuracy



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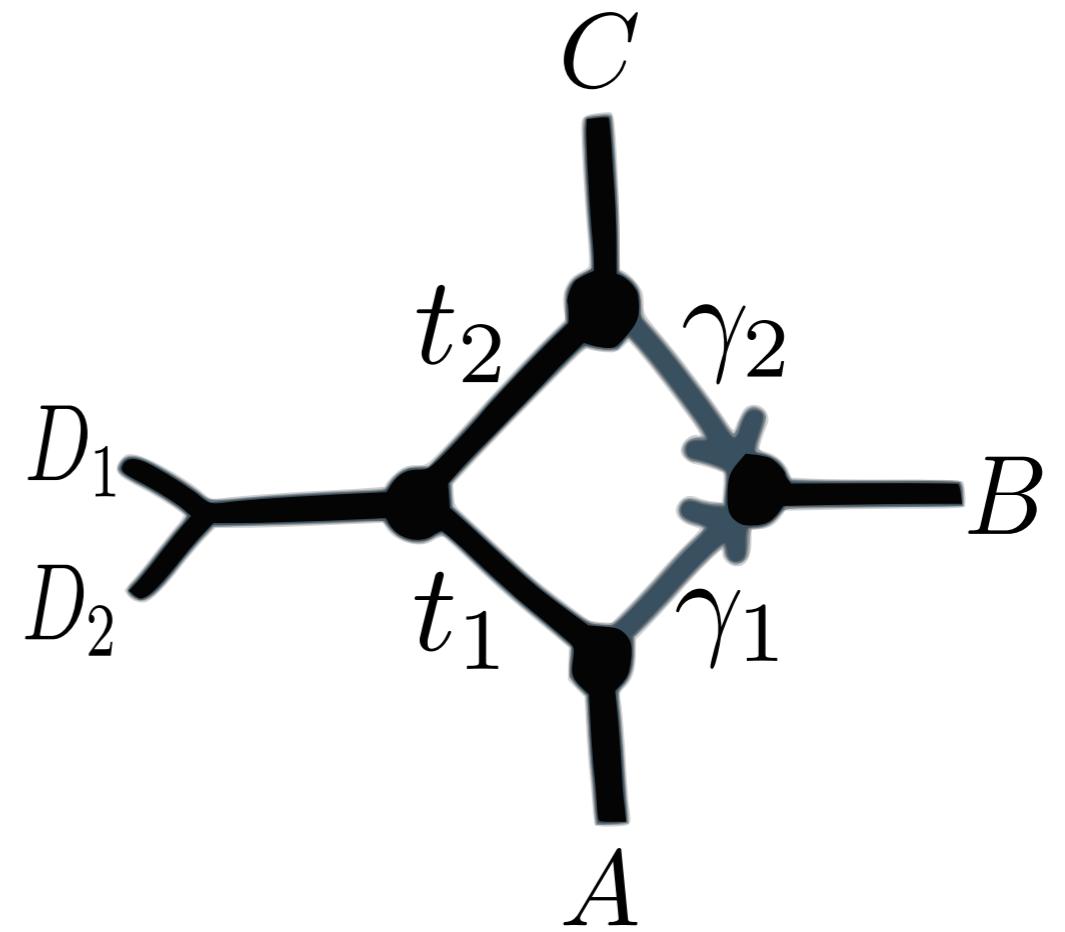
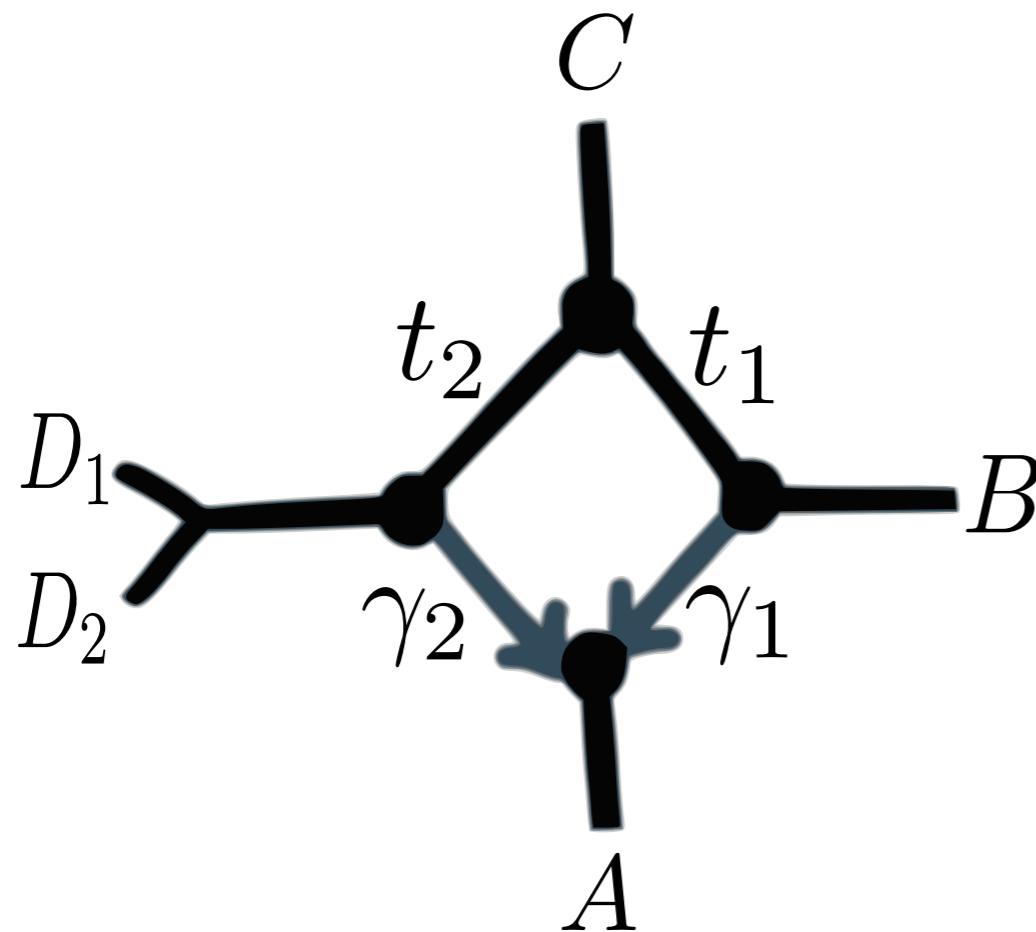


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# In practice: flat pseudolikelihood



(S.-L., Ané, 2016, PLoS Genetics)



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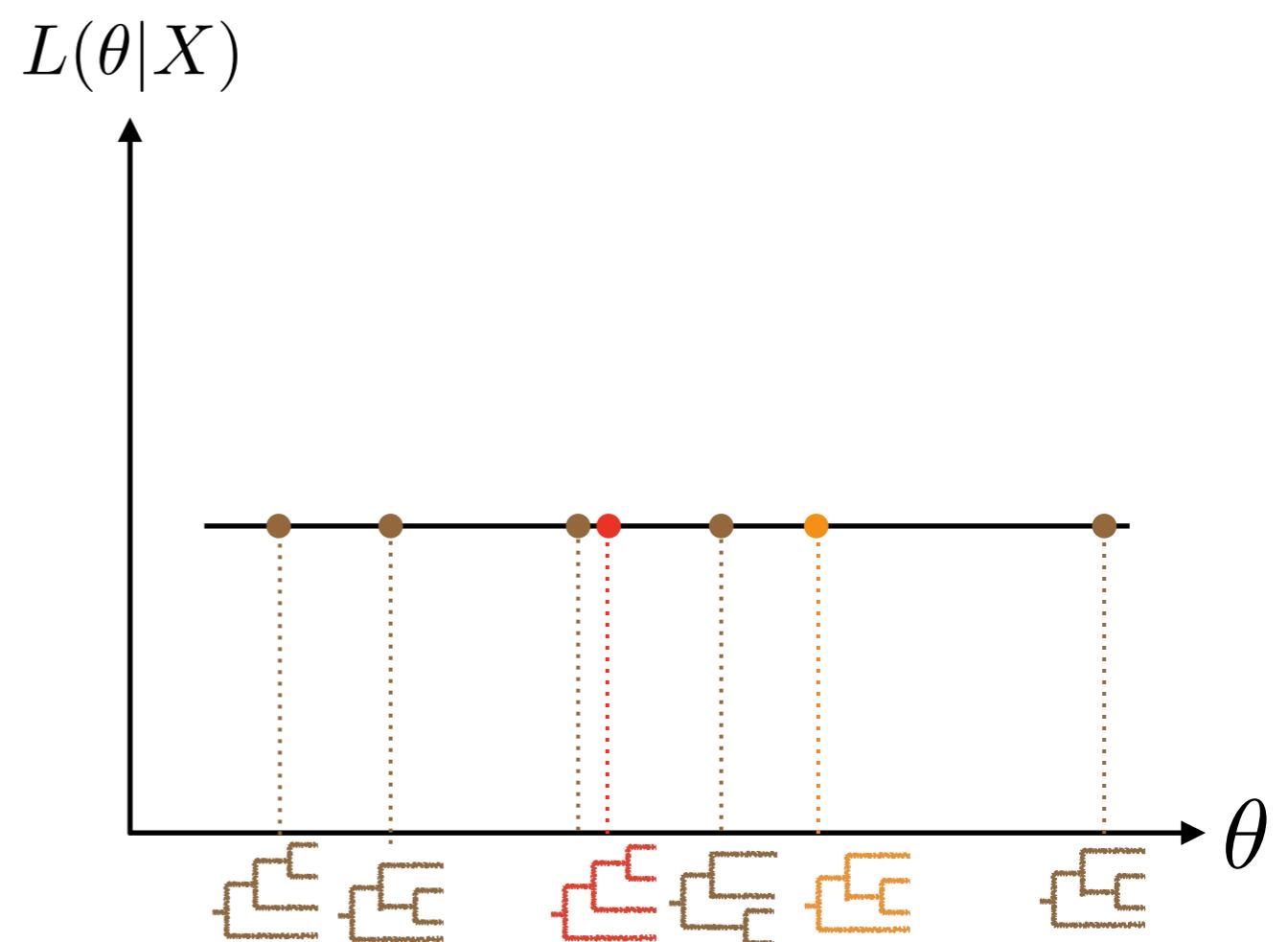
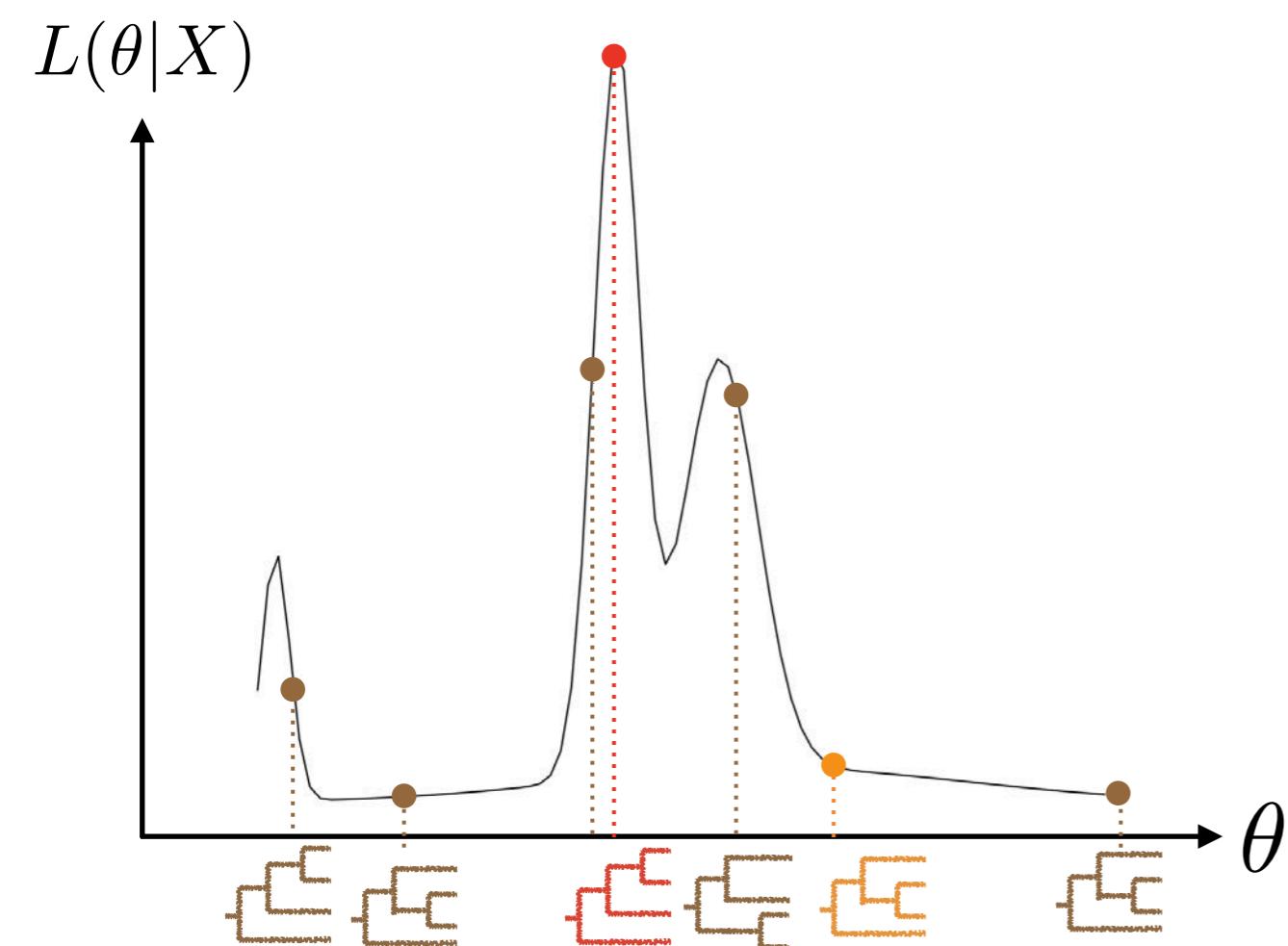


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# Identifiability

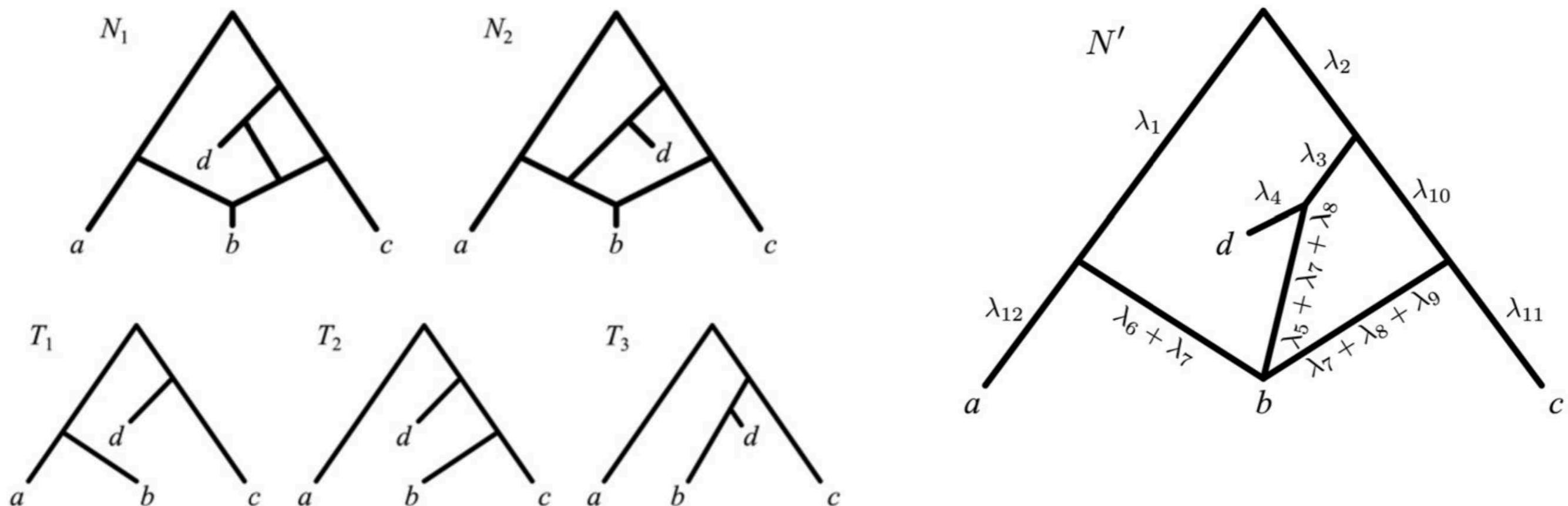


RESEARCH ARTICLE

# Reconstructible Phylogenetic Networks: Do Not Distinguish the Indistinguishable

Fabio Pardi<sup>1,3\*</sup>, Celine Scornavacca<sup>2,3</sup>

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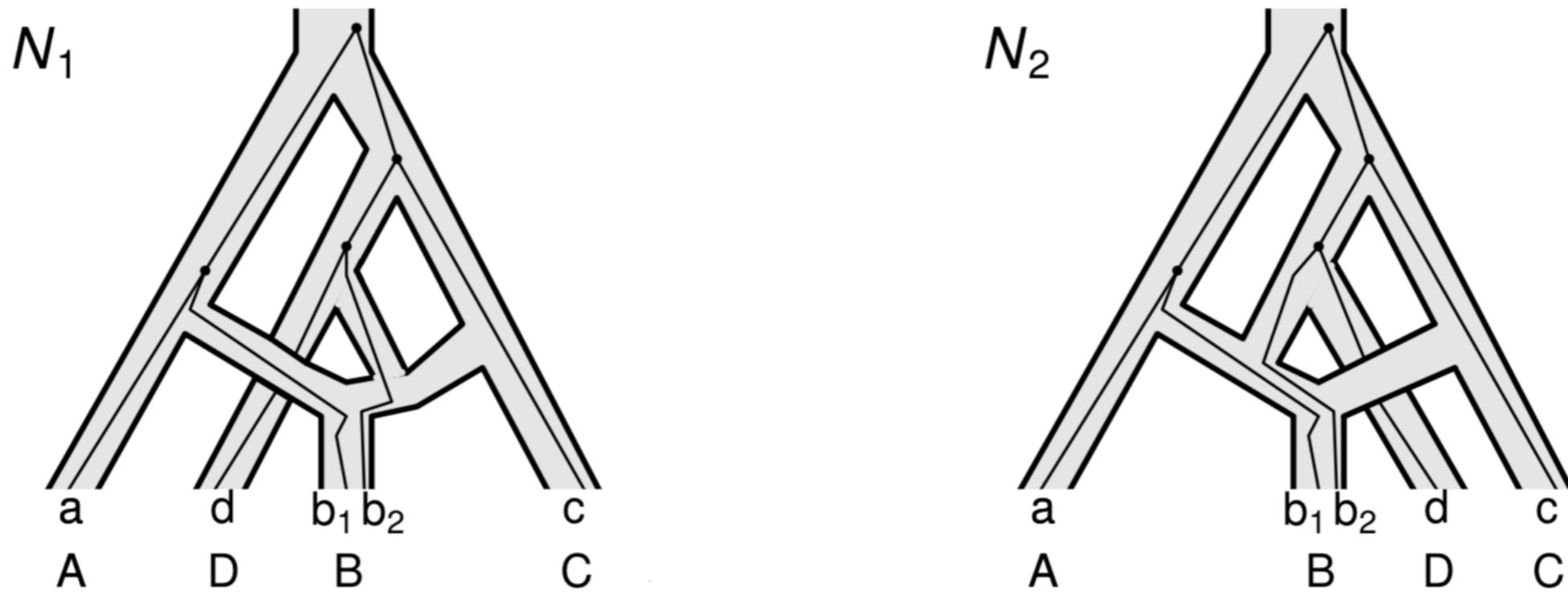


Undistinguishable with the  
“displayed trees” criterion

Solution: Canonical  
network (“unzipped”)

# Displayed Trees Do Not Determine Distinguishability Under the Network Multispecies Coalescent

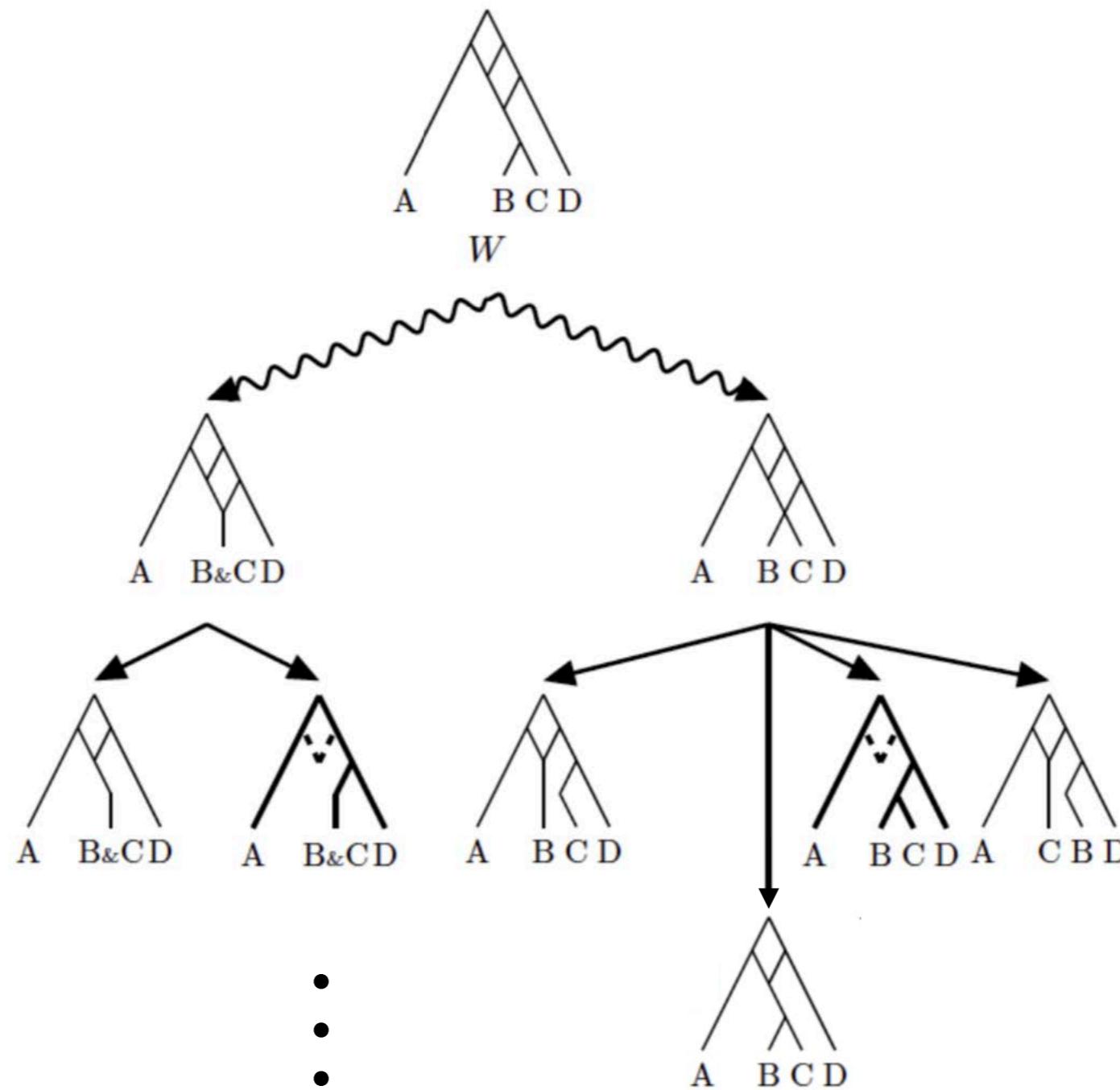
Sha Zhu<sup>1</sup>, James H. Degnan<sup>2</sup>



Distinguishable under the MSC

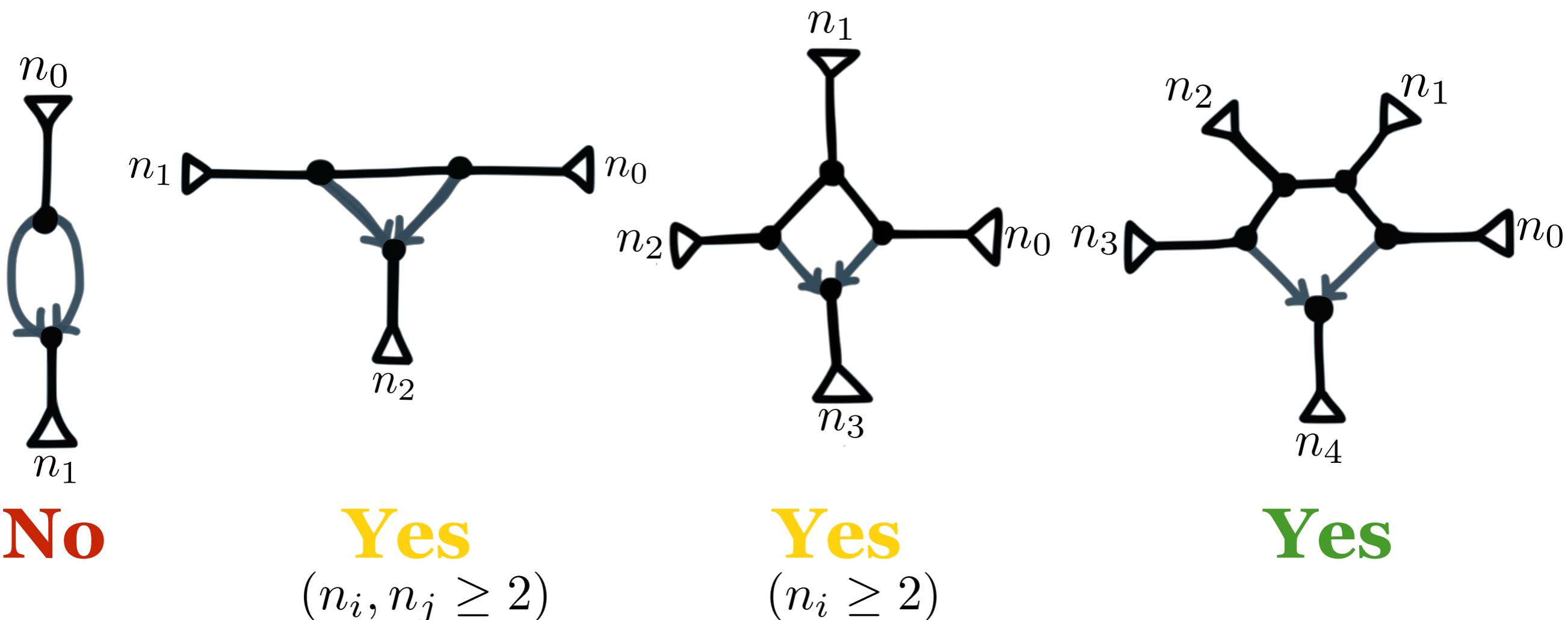
# Displayed Trees Do Not Determine Distinguishability Under the Network Multispecies Coalescent

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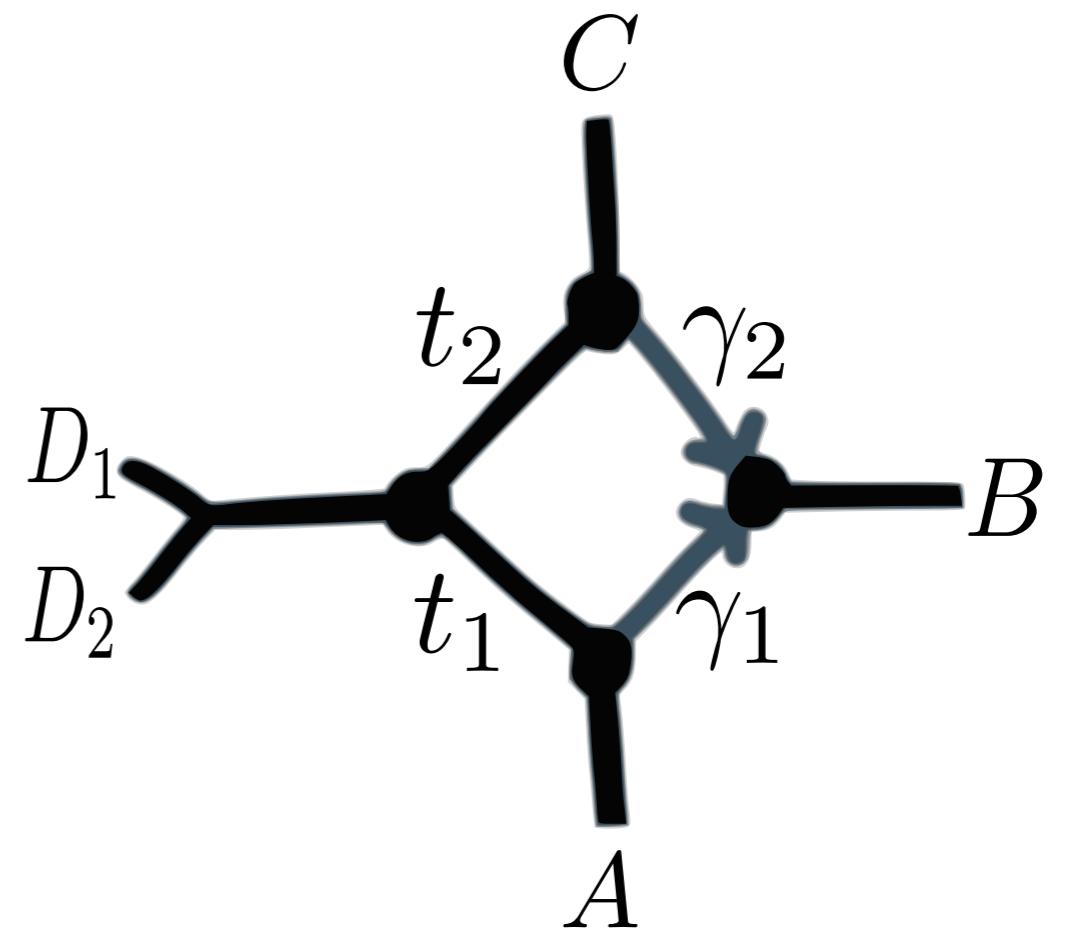
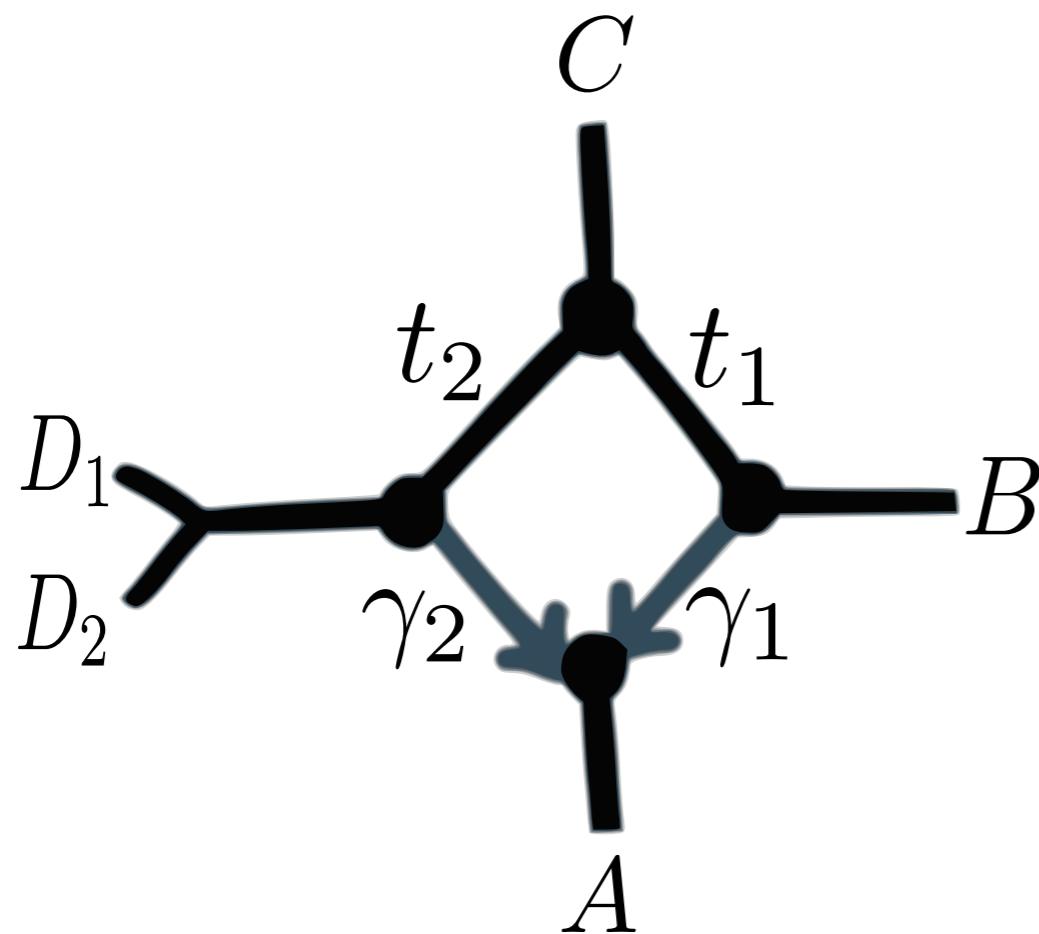


Decomposing network in **parental** trees

## RESEARCH ARTICLE

Inferring Phylogenetic Networks with  
Maximum Pseudolikelihood under  
Incomplete Lineage SortingClaudia Solís-Lemus<sup>1\*</sup>, Cécile Ané<sup>1,2</sup>Can we detect the  
presence of  
hybridization in level-1  
networks?

# In practice: flat pseudolikelihood

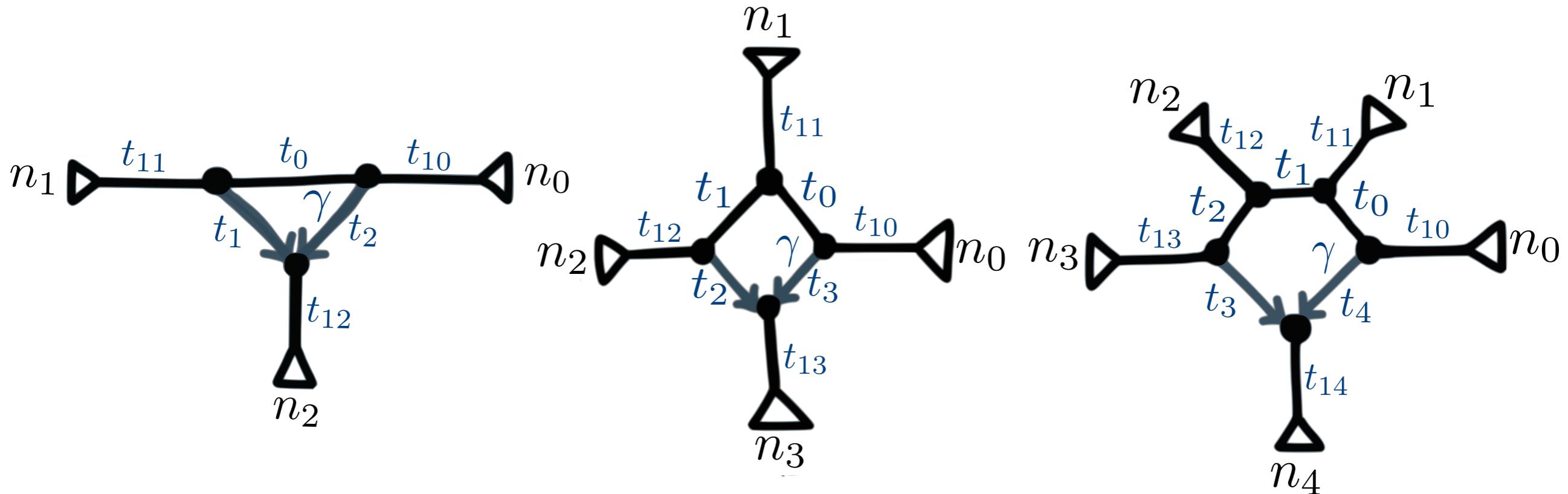


# Can we estimate numerical parameters?

RESEARCH ARTICLE

## Inferring Phylogenetic Networks with Maximum Pseudolikelihood under Incomplete Lineage Sorting

Claudia Solís-Lemus<sup>1\*</sup>, Cécile Ané<sup>1,2</sup>



No

Good triangle  
( $t_{12} = 0$ )

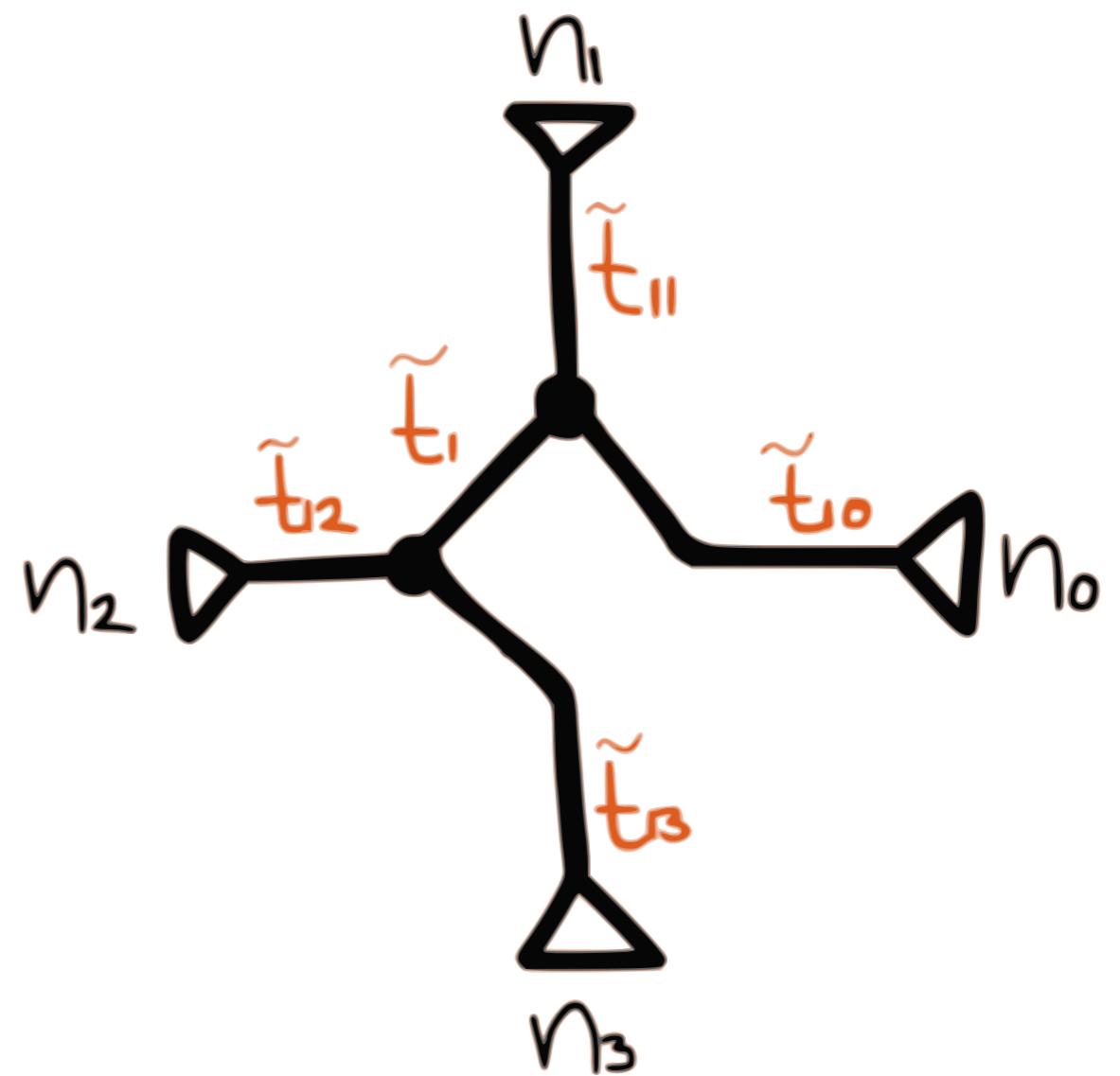
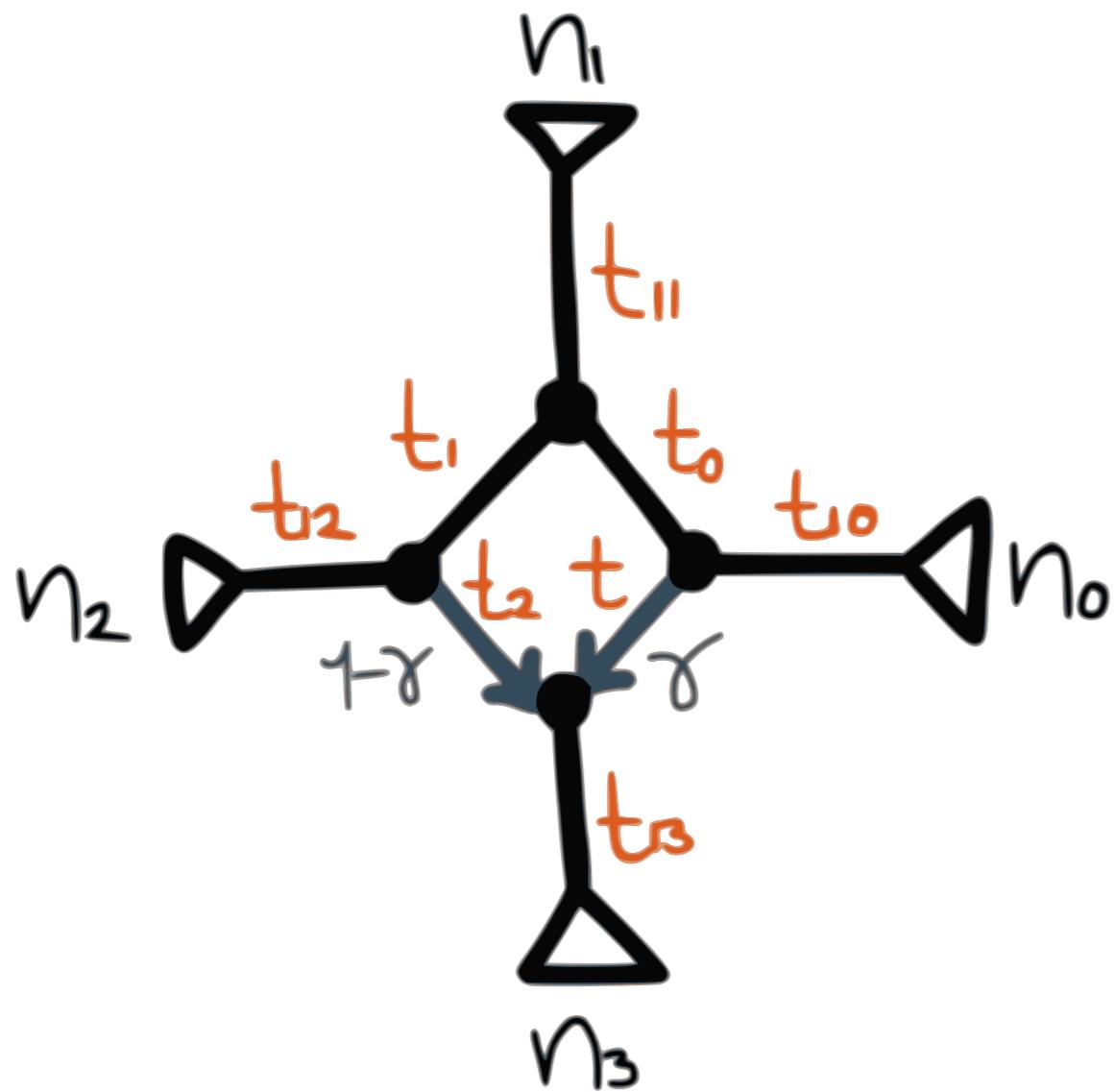
Yes

Good diamond  
( $n_0, n_2 \geq 2$ )

Generic Identifiability

$t_i \in (0, \infty), \gamma \in (0, 1)$

# Idea of proof of identifiability: hybridization



System of equations

{CF<sub>network</sub>}

(Solís-Lemus & Ané, 2016;  
Solís-Lemus et al, 2020)

System of equations

{CF<sub>tree</sub>}



<https://solislemuslab.github.io/>



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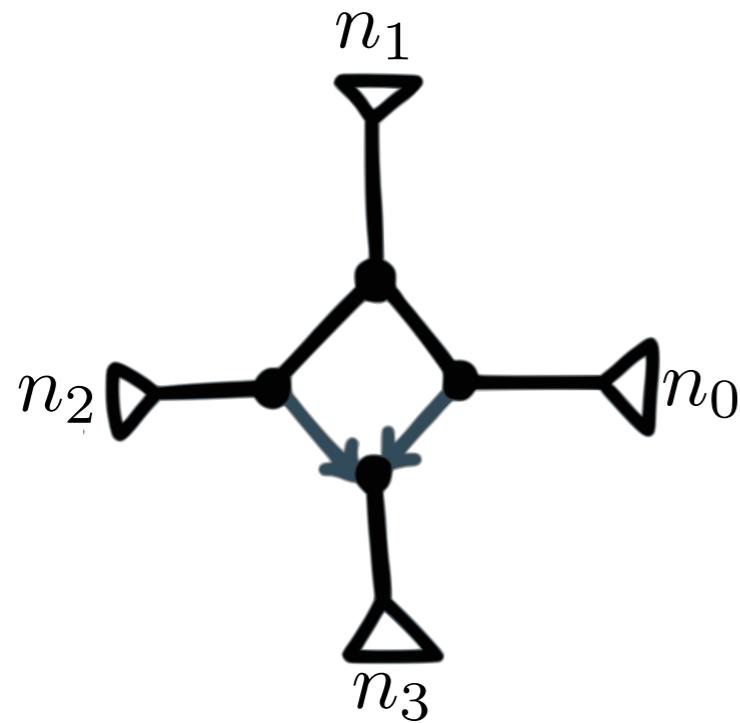
crsl4



@thestatistician

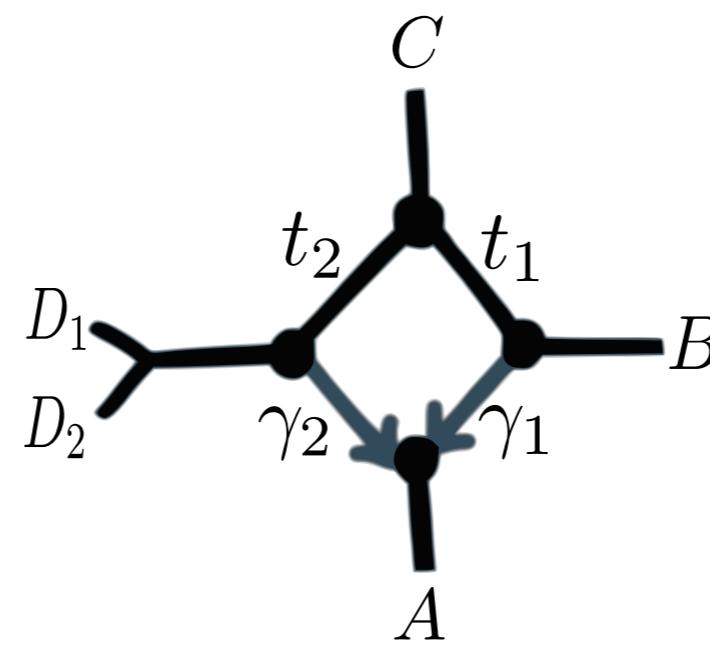
# Can we detect the presence of hybridization in level-1 networks?

In theory

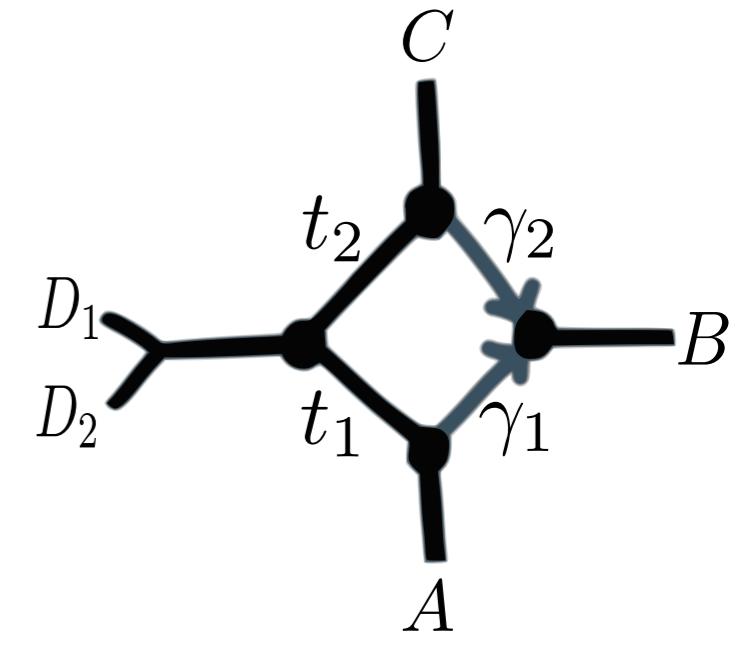


**Yes**  
 $(n_i \geq 2)$

In practice



**Sometimes**



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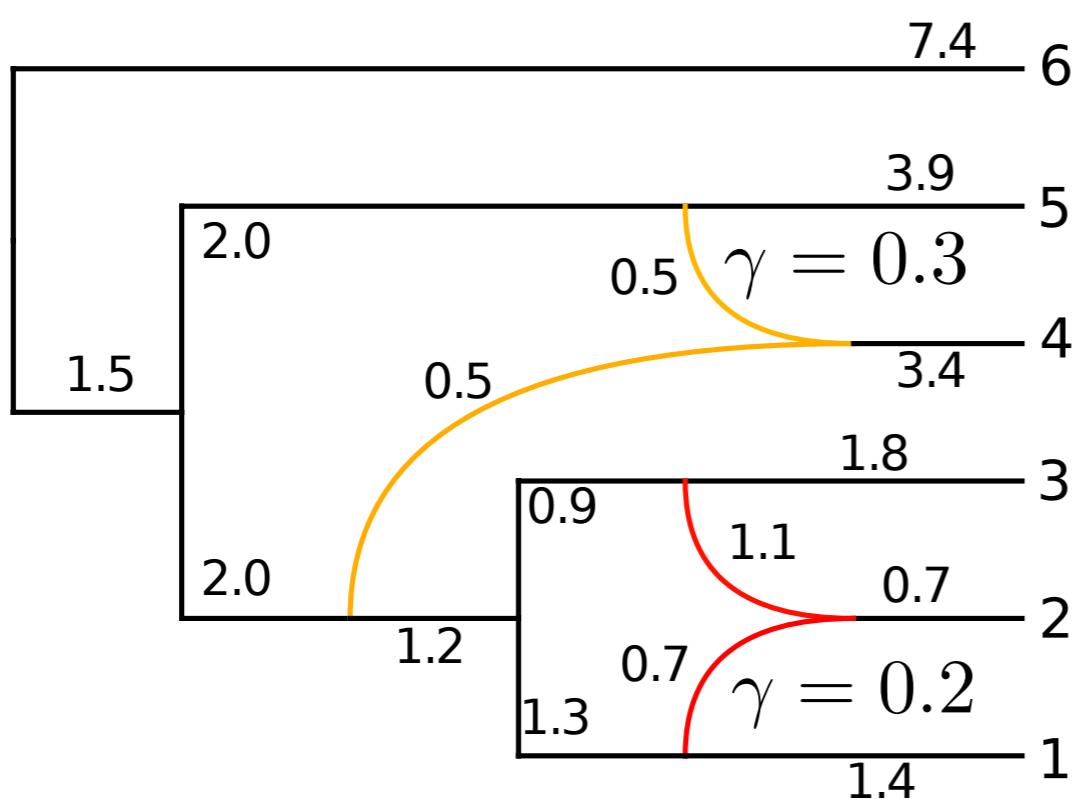
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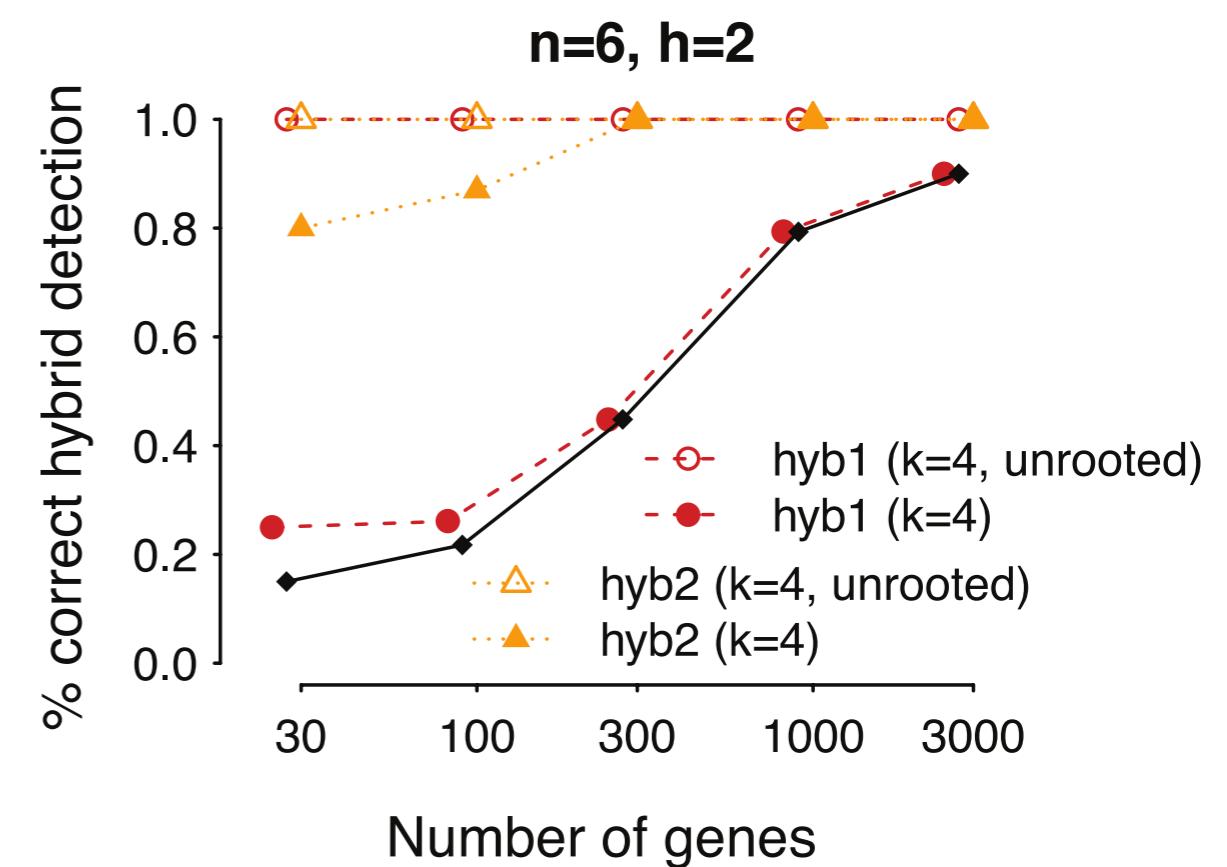
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# Identifiability matters: SNaQ performance

Good diamond



Bad diamond



# Network challenges

- Scalability
- Identifiability
- Network space
- Network comparison

Displayed vs Parental trees  
Level-1 semi-directed networks  
Hybridizations: case by case  
**Missing:** likelihood, level-k semi-directed



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K. Huber, V. Moulton, C. Scornavacca,...  
**Missing:** path through tree space, semi-directed

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# Network challenges

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K. Huber, V. Moulton, C. Scornavacca,...  
**Missing:** path through tree space, semi-directed

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**Missing:** distance function  
Hardwired-cluster distance only for rooted networks  
Summary of networks: clades!



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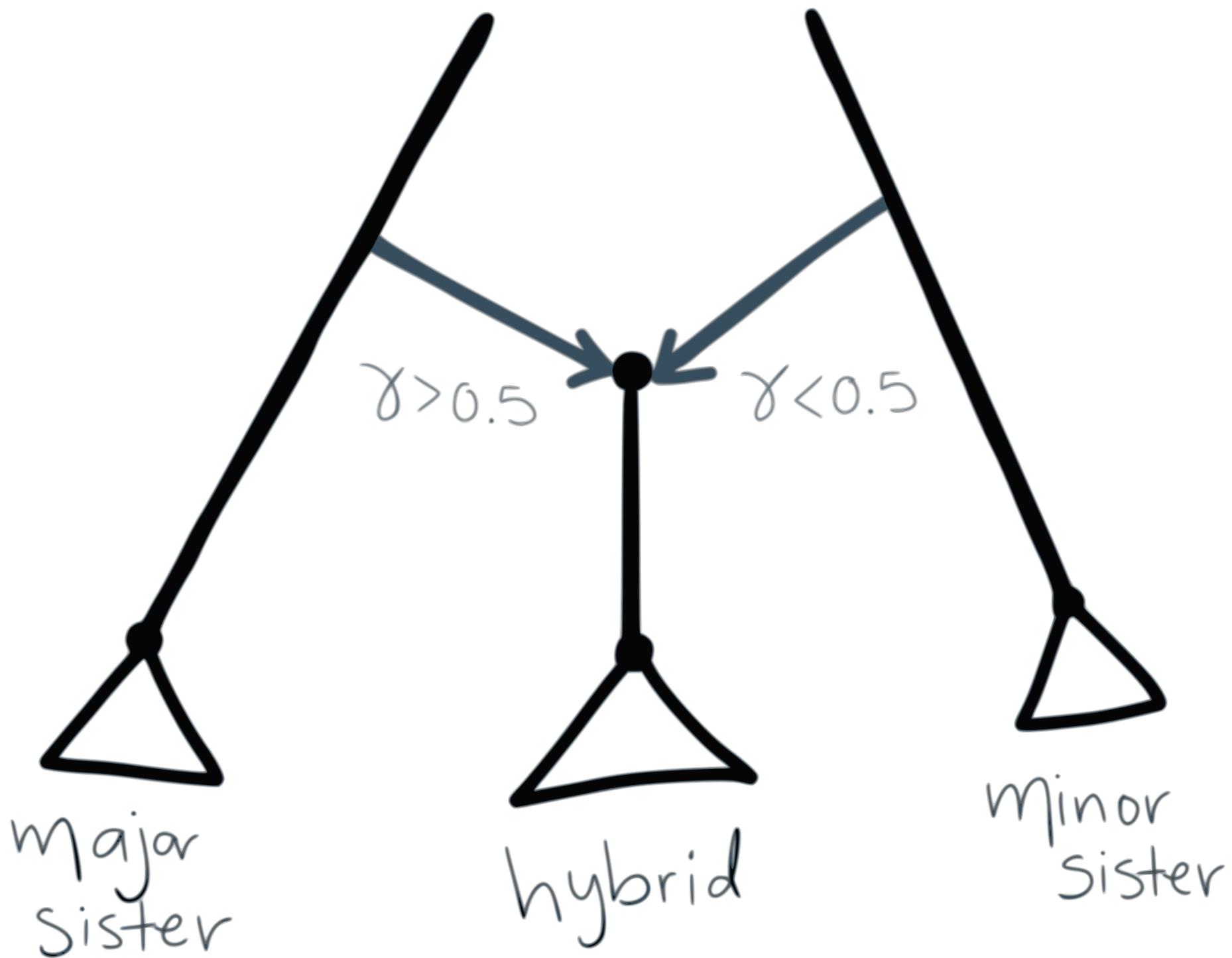


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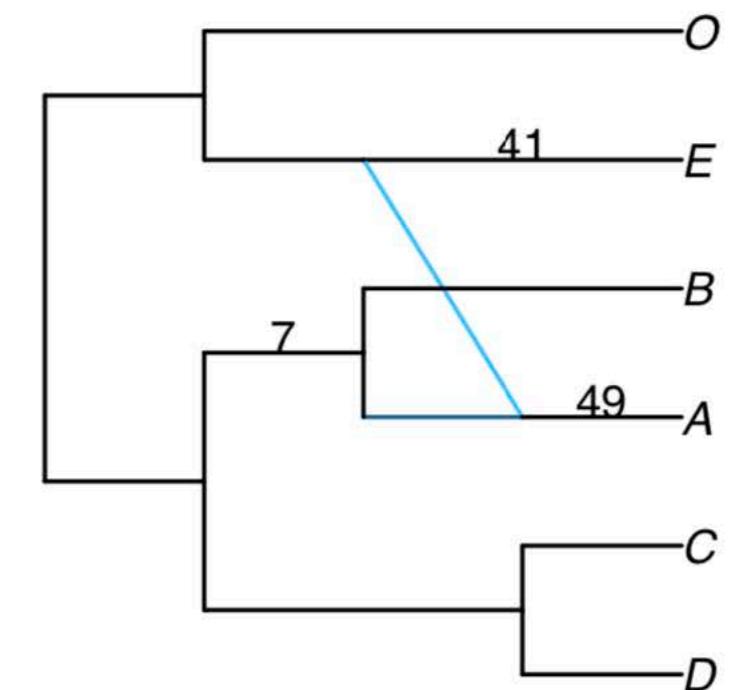
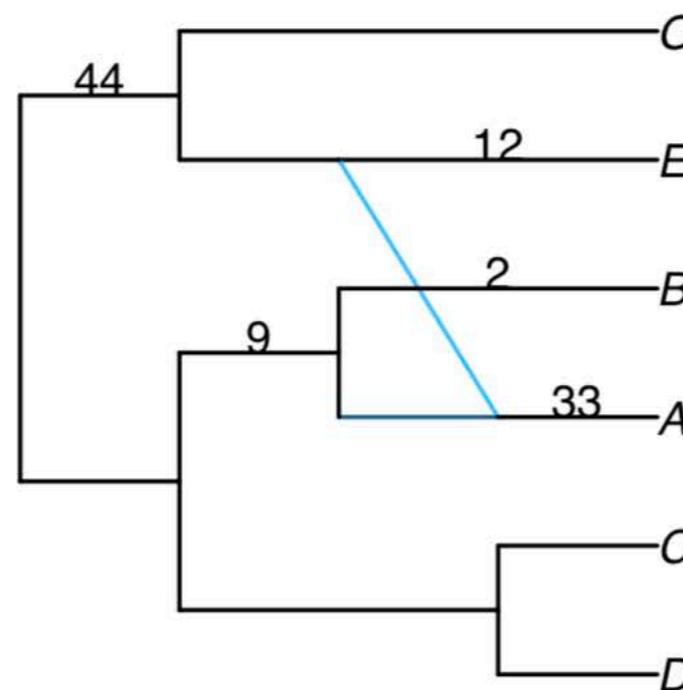
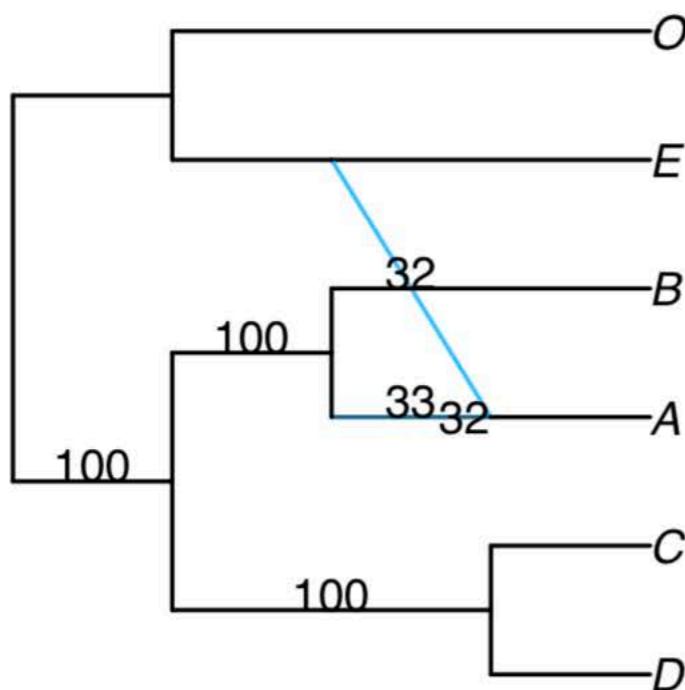
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# Network summary



(S.-L. et al, 2017, MBE)

# Network summary

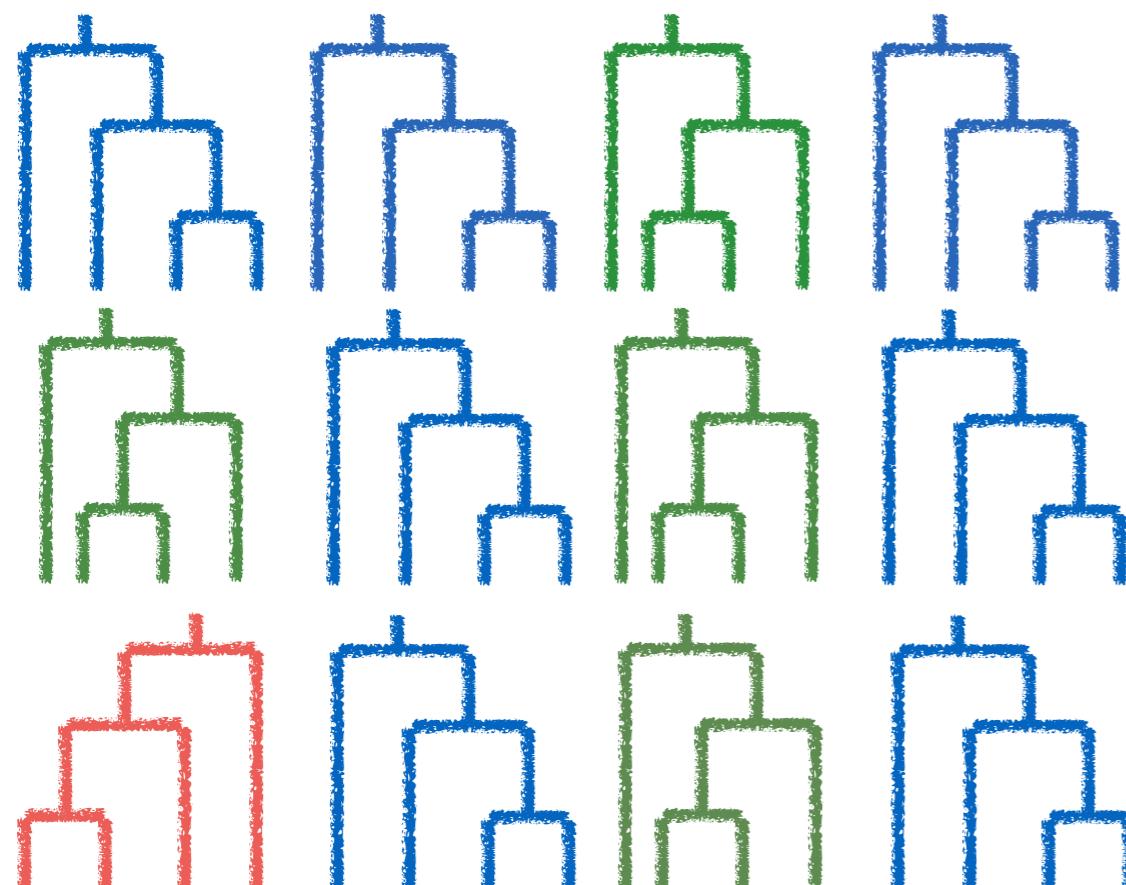
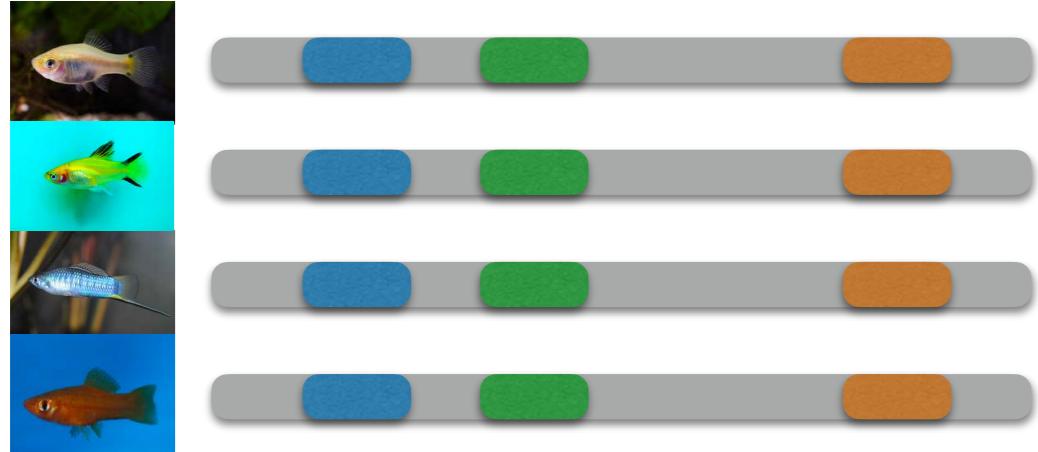


Hybrid  
clades

Minor  
sister  
clades

# When?

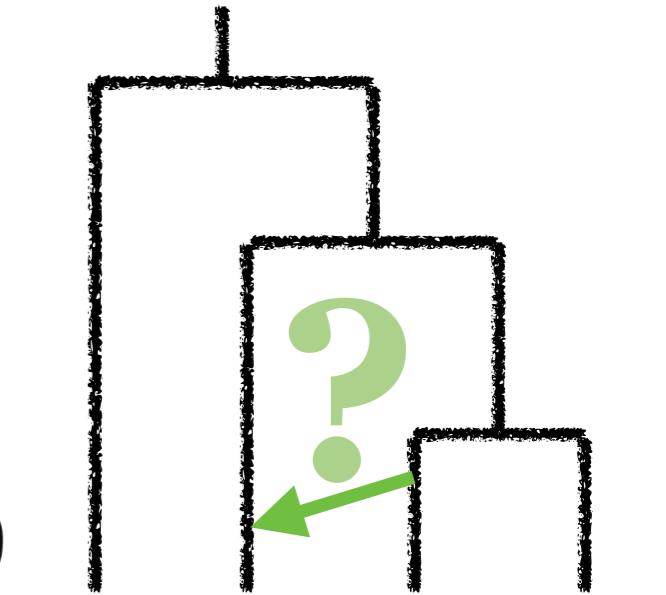
Phylogenetic network



Data

**Goodness-of-fit test**  
Hypothesis test:  
Is a tree a good fit?

TICR  
→  
GitHub



<https://github.com/nstenz/TICR>  
(Stenz et al, 2015, Syst Bio)

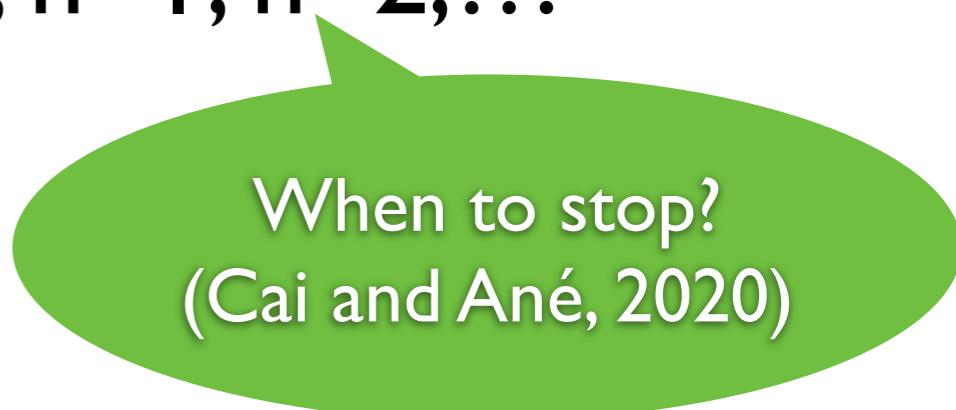
# Practical advice

- Do multiple runs
- Do bootstrap
- Check the .networks output file (especially if hybridization conflicts with outgroup)
- What is the quality of my input data (gene trees/CFs)?
- Run SNaQ sequentially:  $h=0, h=1, h=2, \dots$



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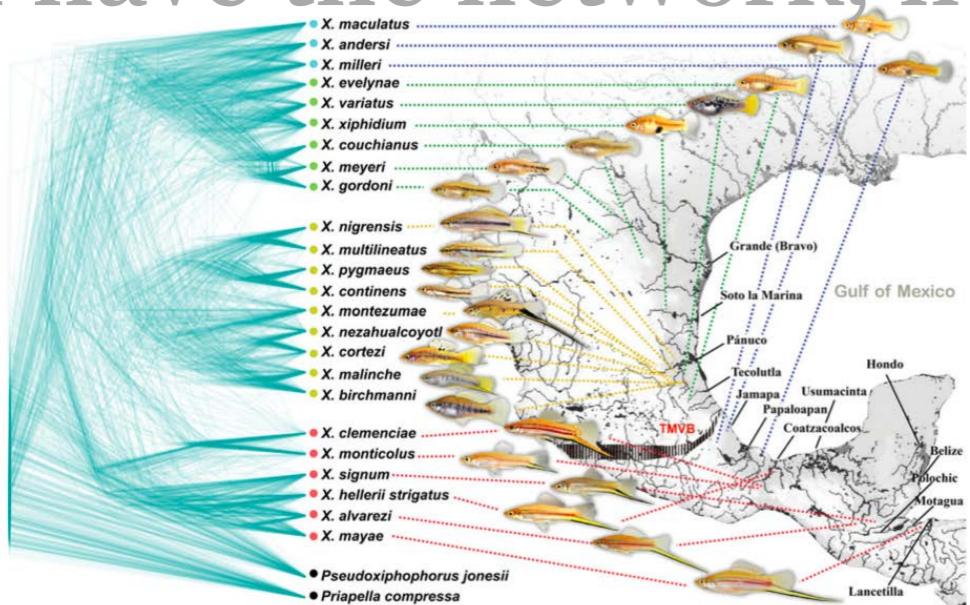


When to stop?  
(Cai and Ané, 2020)



# Part II

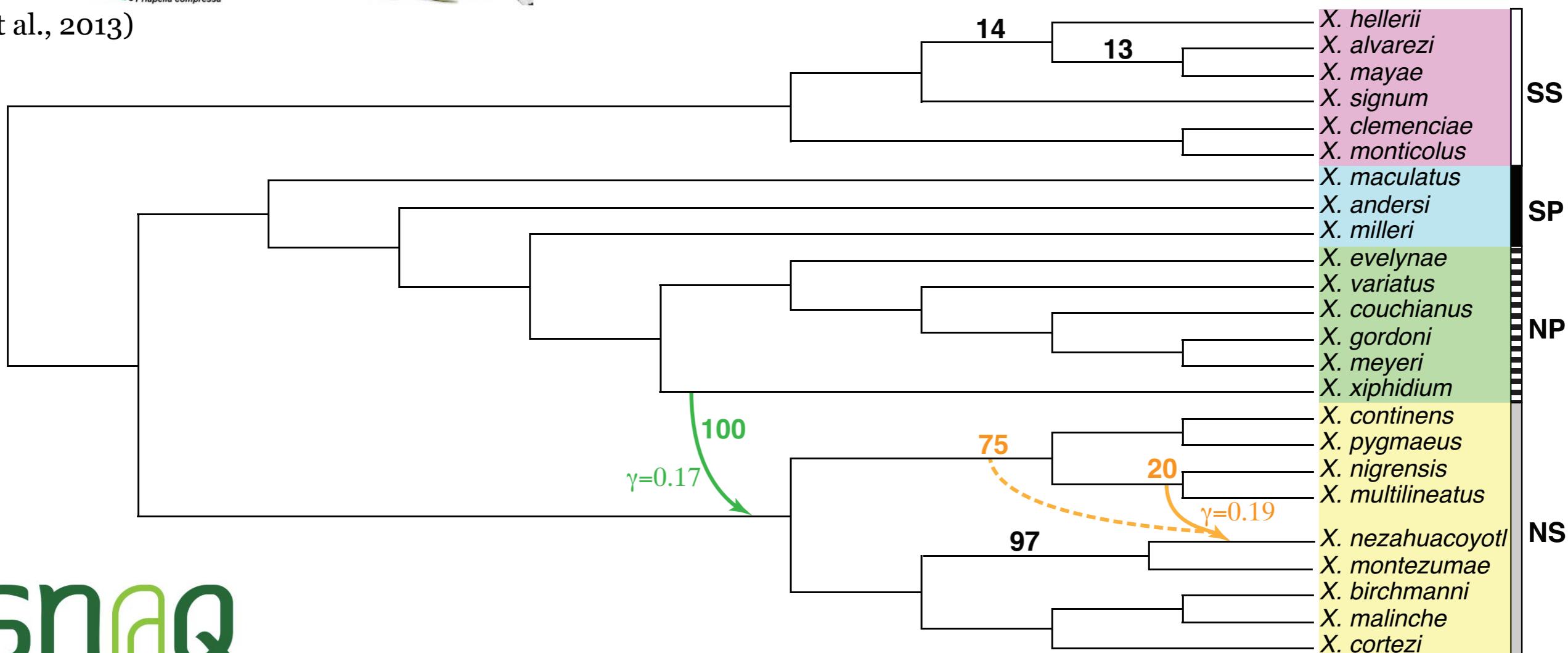
I have the network, now what?



(Cui et al., 2013)

# Xiphophorus fish data

1183 genes,  
24 swordtails  
and platyfish

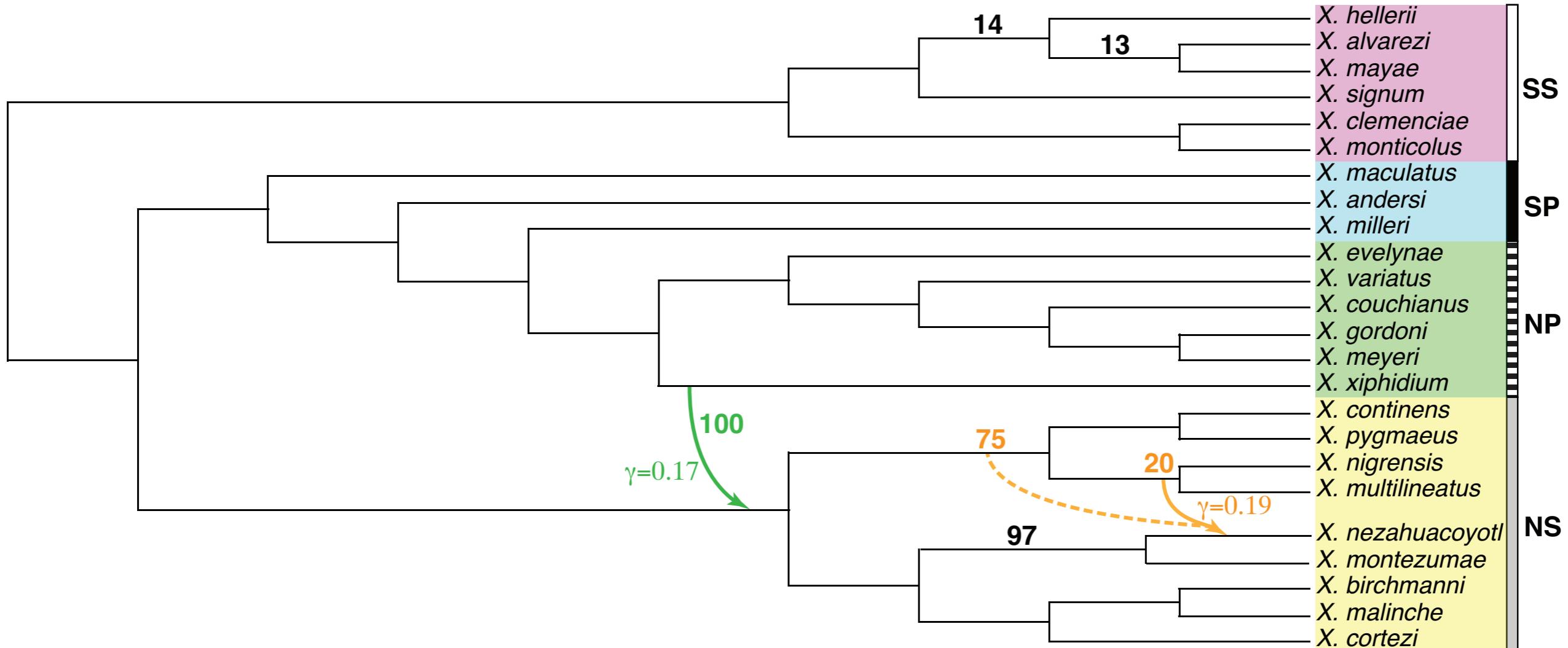


snaQ

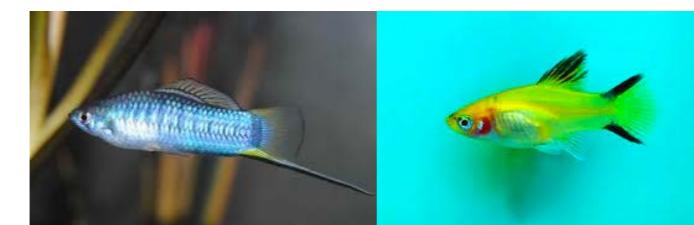
(Solís-Lemus, Ané, 2016, PLoS Genetics)

# Part II

I have the network, now what?

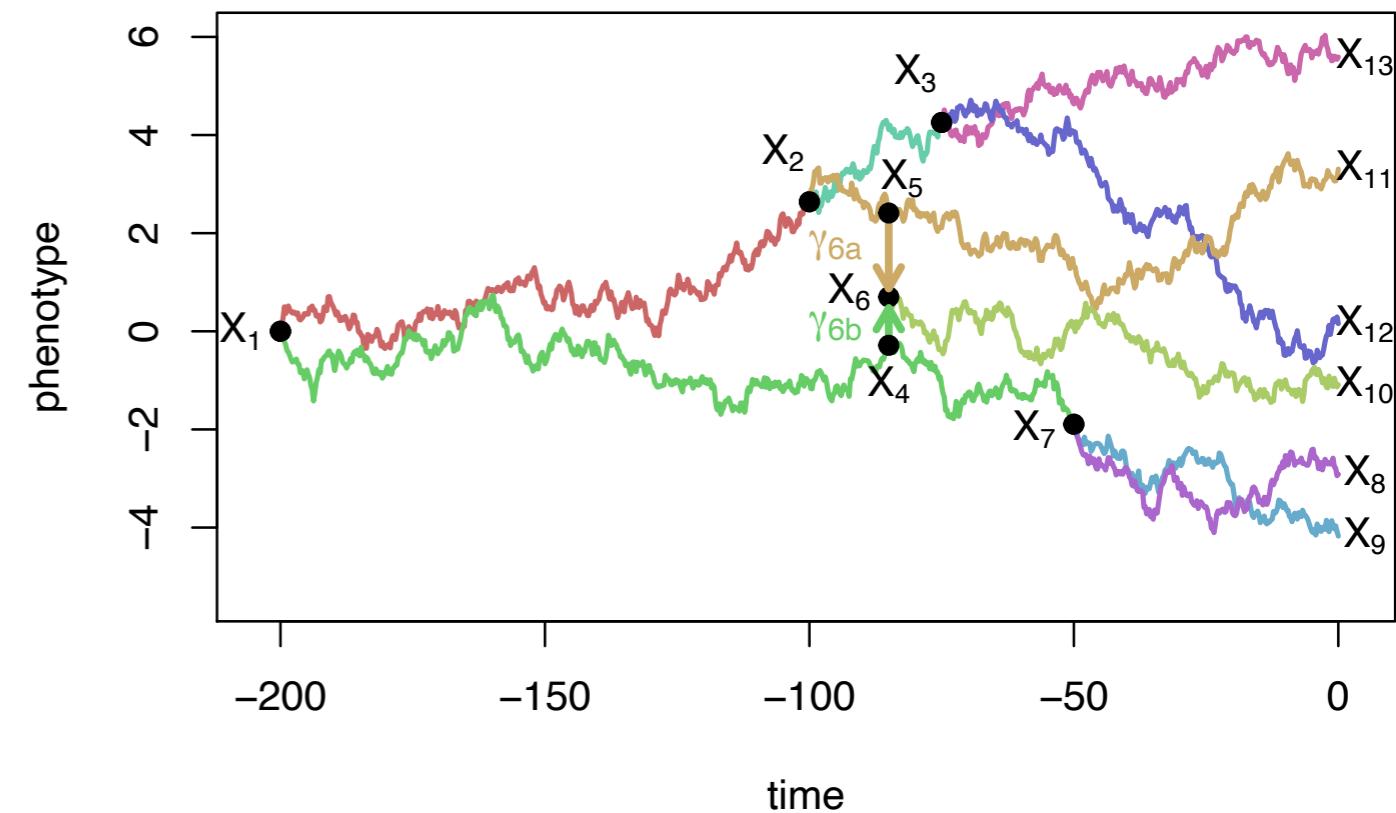
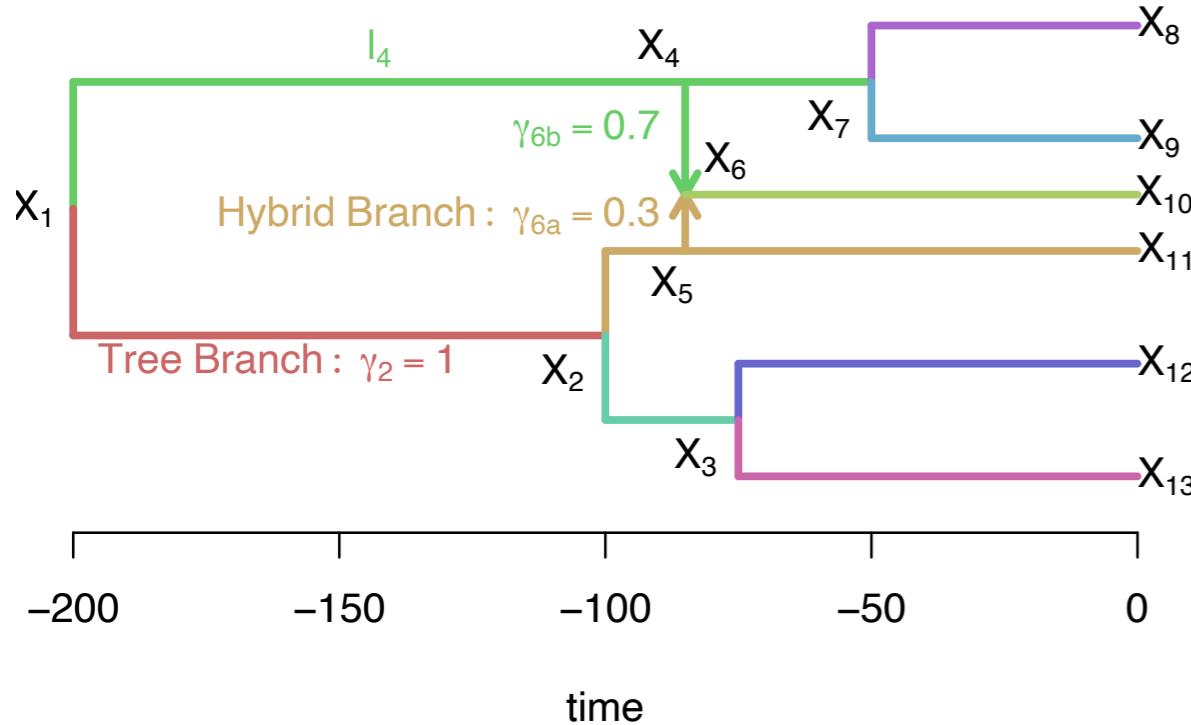


- Sword index
- Female preference



(Cui et al., 2013)  
(Solís-Lemus, Ané, 2016, PLoS Genetics)

# Trait models of evolution in networks



Brownian Motion  
+ weighted  
average in hybrid

$$X_h = \gamma_1 X_{p_1} + \gamma_2 X_{p_2}$$

(Bastide et al, 2018, Syst Bio)

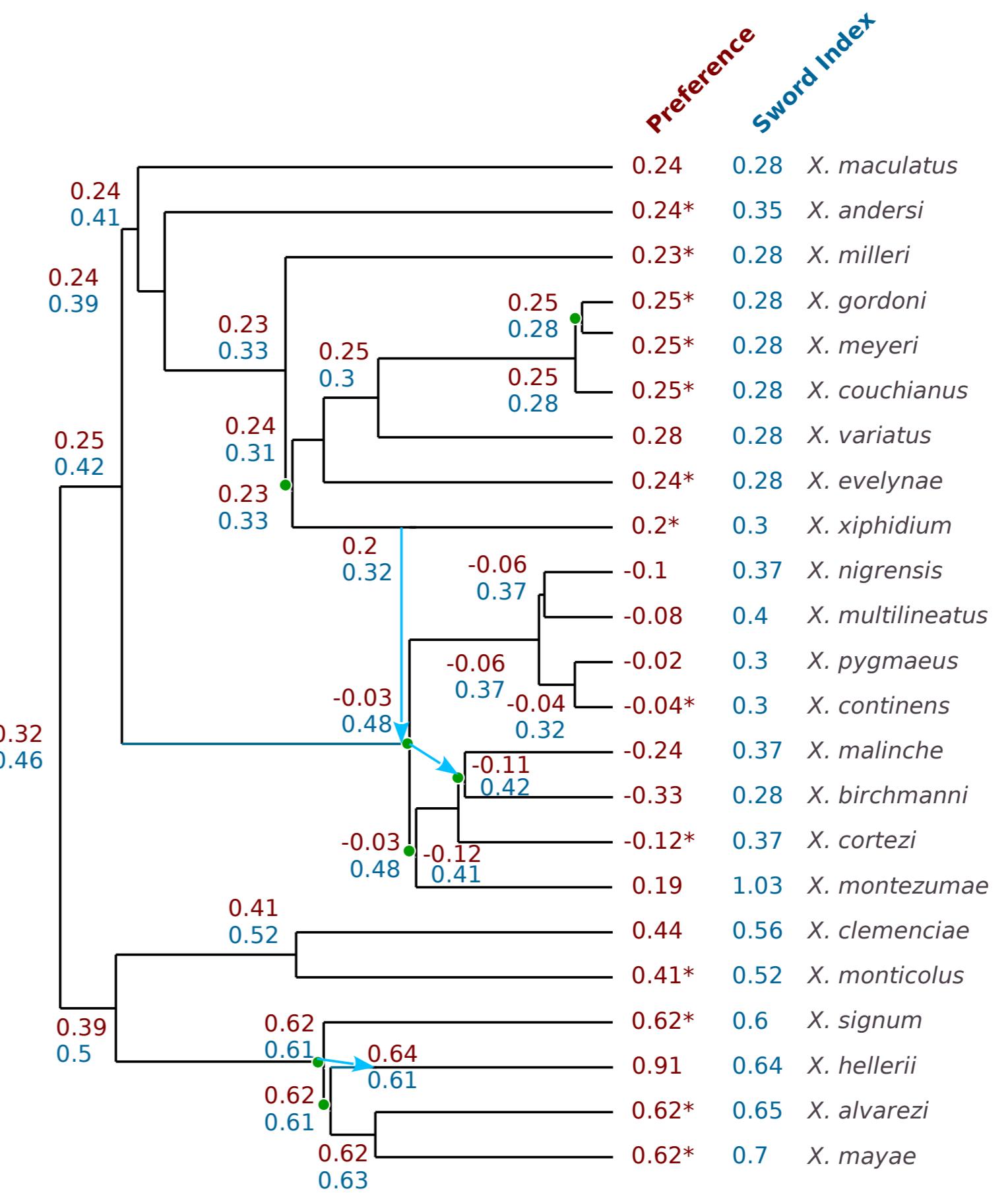
$$\mathbf{X} \sim N(X_{root}, \sigma^2 \mathbf{V})$$

- Phylogenetic signal
- Ancestral reconstruction
- Phylogenetic regression
- Phylogenetic ANOVA

- Sword index
- Female preference

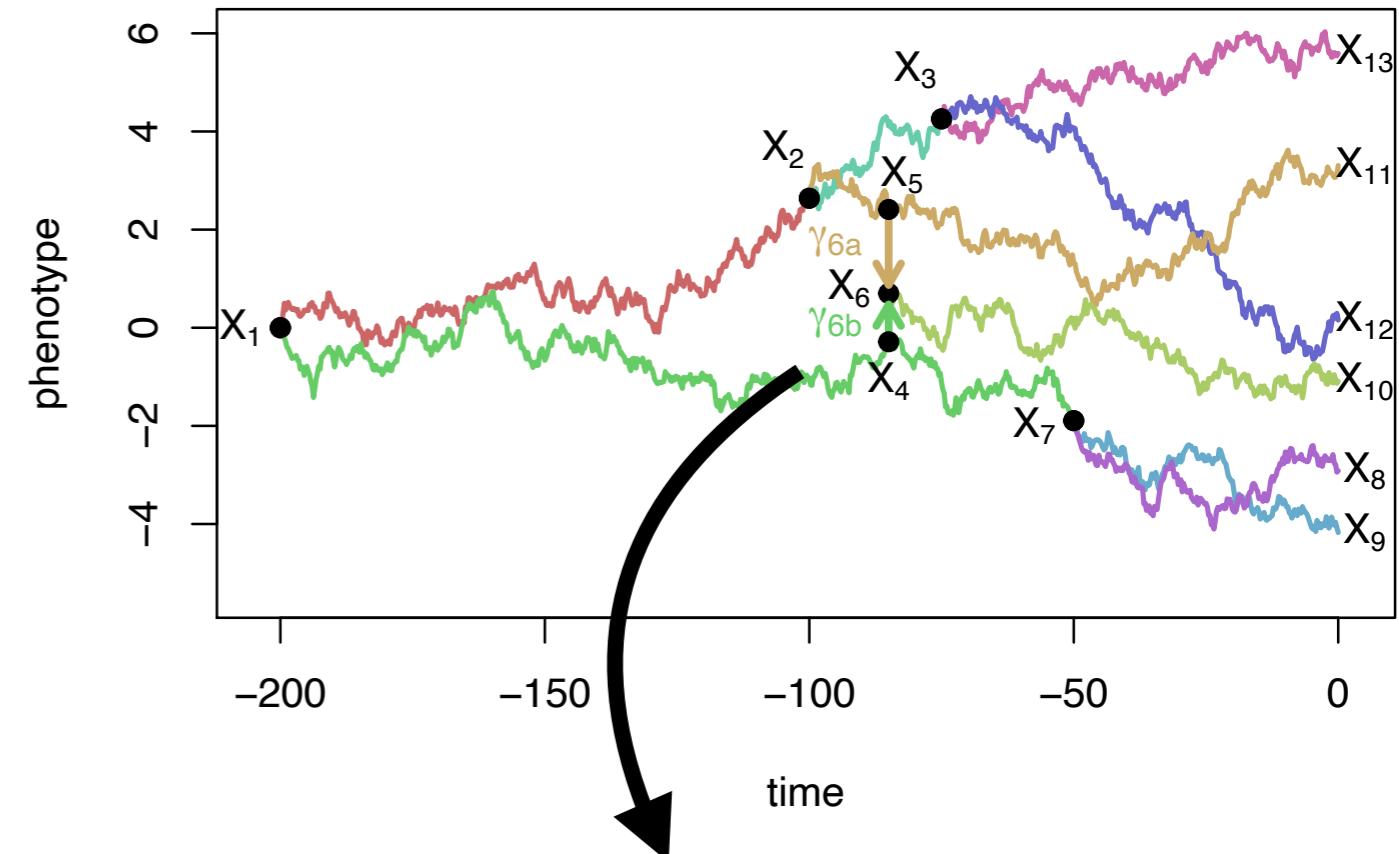
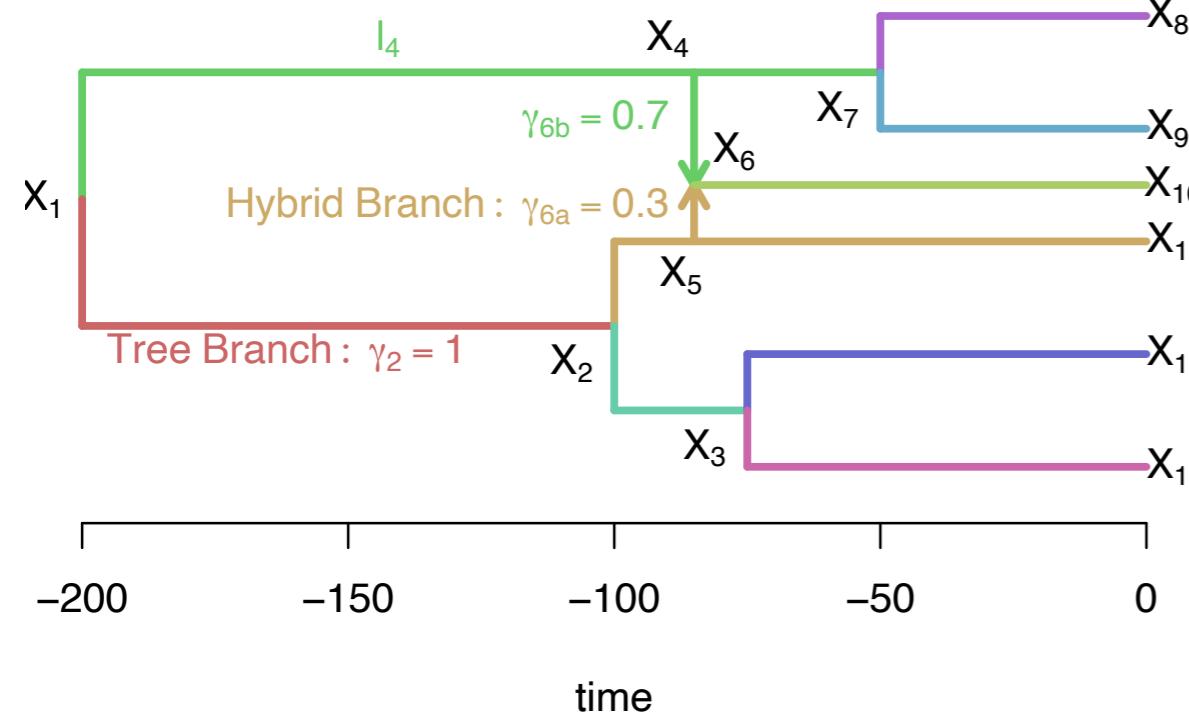


- **Ancestral reconstruction:** common ancestor likely had sword
- **Phylogenetic regression:** positive association between sword index and female preference but not significant ( $p = 0.106$ )



# Test for transgressive evolution

$$X_h = \gamma_1 X_{p_1} + \gamma_2 X_{p_2} + \Delta_h$$



$\Delta_h = 0$  No transgressive evolution

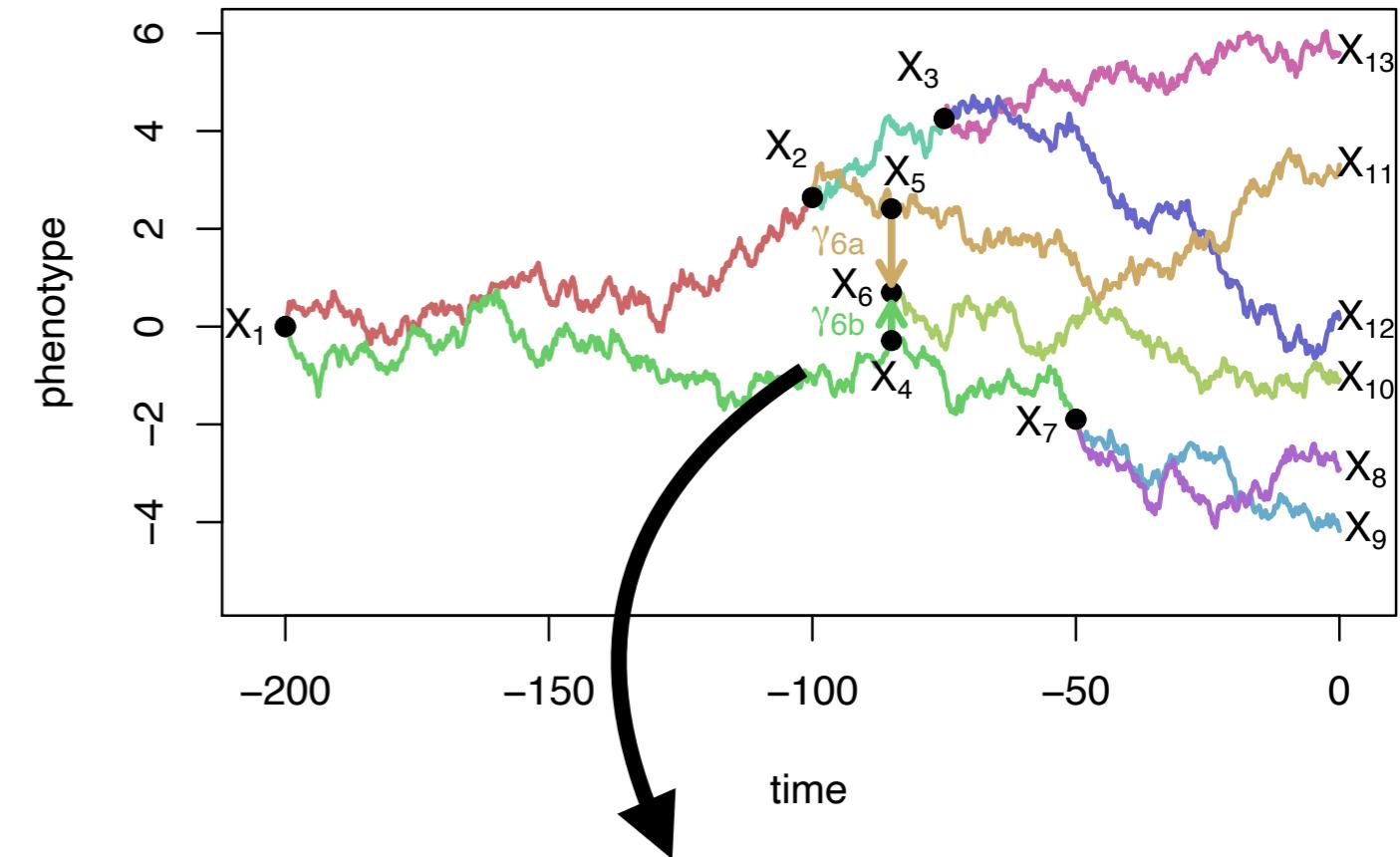
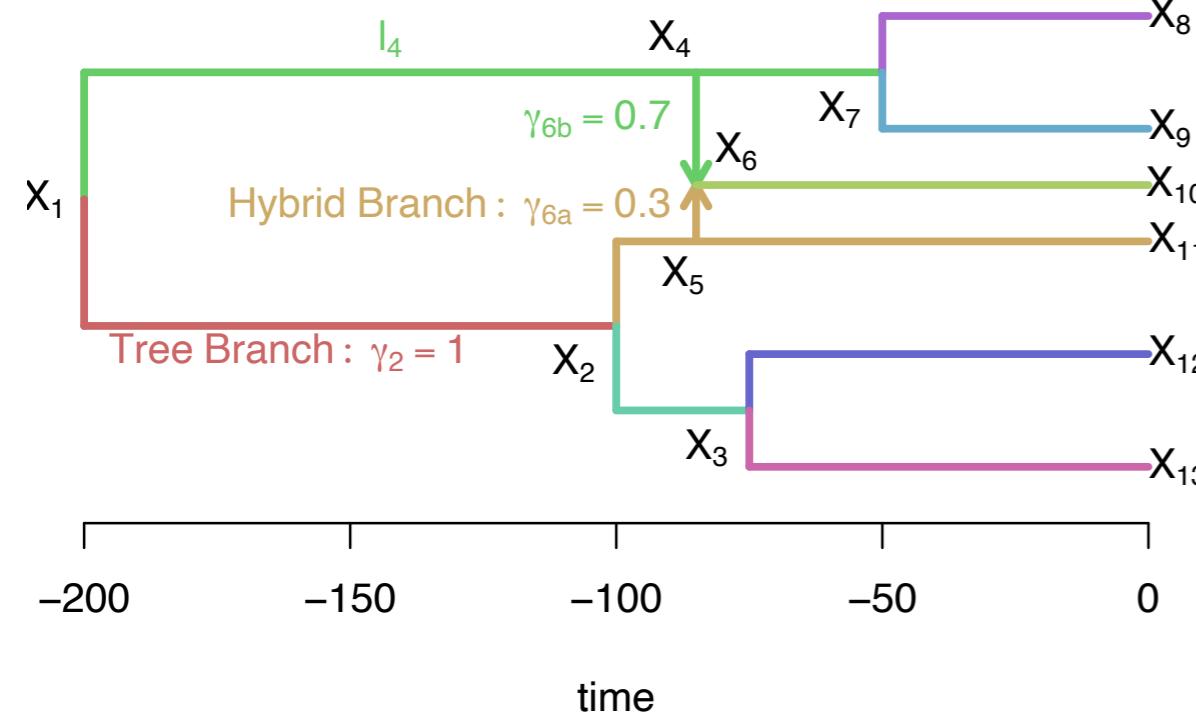
$\Delta_h = \Delta$  Single-effect transgressive evolution

$\Delta_h$  Multi-effect transgressive evolution

**F tests**

Hybrid value:  
shift from  
parents range

# Test for transgressive evolution



- Sword index:  $p=0.55$
- Female preference:  $p=0.0064$

Hybrid value:  
shift from  
parents range

# PhyloNetworks: analysis for phylogenetic networks

build passing docs stable docs dev codecov 81% coverage 67%

## Overview

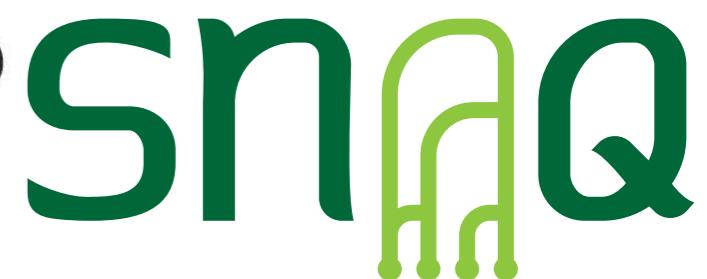


PhyloNetworks is a [Julia](#) package with utilities to:

- read / write phylogenetic trees and networks, in (extended) Newick format. Networks are considered explicit: nodes represent ancestral species. They can be rooted or unrooted.
- manipulate networks: re-root, prune taxa, remove hybrid edges, extract the major tree from a network, extract displayed networks / trees
- compare networks / trees with dissimilarity measures (Robinson-Foulds distance on trees)
- summarize samples of bootstrap networks (or trees) with edge and node support
- estimate species networks from multilocus data (see below)
- phylogenetic comparative methods for continuous trait evolution on species networks / trees



- Step-by-step tutorial
- Online documentation
- Google user group



(Solis-Lemus & Ane, 2016; Solis-Lemus. et al, 2017)



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## Phylogenomics



Rakoton-drafara  
(Plant Path)

Koch  
(Plant Path)

Lankau  
(Plant Path)

Rioux  
(Plant Path)

## Omics



Yuke Wu



Sam Ozminkowski



Yunyi Shen



Rosa Aghdam



Reed Nelson



Xudong Tang

## Microbiome

Join us: Positions available in the lab!

New collaborations welcome!



WISCONSIN  
UNIVERSITY OF WISCONSIN-MADISON

Thank you!



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