## APPENDIX

In this appendix, we give the mathematical derivation of the optimum aperiodic sampling interval discussed in Section 6.1. We start with the general solution, where the objective function to be minimized is given by Equation (4):

$$\Rightarrow \mathcal{E} = \alpha\mathbb{E}[\mathcal{S}] + \beta\mathbb{E}[\mathcal{W}] + C$$

We first borrow the idea of checkpointing density from [35] and define an instantaneous sampling rate function $r(t)$ which is related to $\{t_n\}$ such that,

$$\int_{t_{n-1}}^{t_n} r(t)\,\mathrm{d}t = 1,\ \forall n \geq 1 \tag{9}$$

Note that, for periodic sampling, this function is a constant, equal to the sampling frequency. In the aperiodic case, we find $r^*(t)$, the $r(t)$ that minimizes $\mathcal{E}$. By construction, the number of samples taken up to any time instant can be computed directly by computing the area under $r(t)$. Thus, we obtain the expected number of samples $\mathbb{E}[\mathcal{S}]$ as

$$\mathbb{E}[\mathcal{S}] = \int_{t=0}^{\infty}\left(\int_{x=0}^{t} r(x)\,\mathrm{d}x\right) f_{\mathcal{T}}(t)\,\mathrm{d}t. \tag{10}$$

To find $\mathbb{E}[\mathcal{W}]$, we use the conditional CDF of the execution time.

$$\begin{aligned}&\mathbb{P}(\mathcal{W} = t_n - \mathcal{T} \leq t \,\big|\, t_{n-1} < \mathcal{T} \leq t_n) \\ &\qquad = \frac{\mathbb{P}(\mathcal{T} \geq t_n - t\,,\, t_{n-1} < \mathcal{T} \leq t_n)}{\mathbb{P}(t_{n-1} < \mathcal{T} \leq t_n)}.\end{aligned}$$

The numerator is degenerate when $t < 0$ or $t > (t_n - t_{n-1})$. Thus, we are only interested in $0 \leq t \leq (t_n - t_{n-1})$. Let $F_{\mathcal{T}}$, $\bar{F}_{\mathcal{T}}$ and $f_{\mathcal{T}}$ correspond to the CDF, Complementary CDF (CCDF) and Probability Density Function (PDF) of the execution time distribution.

$$\begin{aligned}\Rightarrow \mathbb{P}(\mathcal{W} \leq t \,\big|\, t_{n-1} < \mathcal{T} \leq t_n) &= \frac{\mathbb{P}(t_n - t \leq \mathcal{T} \leq t_n)}{F_{\mathcal{T}}(t_n) - F_{\mathcal{T}}(t_{n-1})} \\ &\approx \frac{F_{\mathcal{T}}(t_n) - F_{\mathcal{T}}(t_n - t)}{F_{\mathcal{T}}(t_n) - F_{\mathcal{T}}(t_{n-1})} \quad (11)\end{aligned}$$

Here, Equation (11) is an approximation merely for mathematical maturity due to the slackness of the first inequality in the numerator. We expand $F_{\mathcal{T}}(t_n - t)$ and $F_{\mathcal{T}}(t_{n-1})$ using Taylor series. Thus we can write the CCDF as

$$\begin{aligned}= \Big(&F_{\mathcal{T}}(t_n) - \\ &\big(F_{\mathcal{T}}(t_n) + f_{\mathcal{T}}(t_n)(-t) + f'_{\mathcal{T}}(t_n)(-t)^2/2! + \dots\big)\Big) \\ &\div \Big(F_{\mathcal{T}}(t_n) - \big(F_{\mathcal{T}}(t_n) + f_{\mathcal{T}}(t_n)(t_{n-1} - t_n) \\ &\qquad + f'_{\mathcal{T}}(t_n)(t_{n-1} - t_n)^2/2! + \dots\big)\Big).\end{aligned}$$

Simplifying and approximating by ignoring the higher order terms, we arrive at

$$\mathbb{P}(\mathcal{W} \leq t \,\big|\, t_{n-1} < \mathcal{T} \leq t_n) \approx \frac{t f_{\mathcal{T}}(t_n)}{(t_n - t_{n-1}) f_{\mathcal{T}}(t_n)} \tag{12}$$

$$\Rightarrow \mathbb{P}(\mathcal{W} > t \,|\, t_{n-1} < \mathcal{T} \leq t_n) = 1 - \frac{t}{(t_n - t_{n-1})}$$

Next, using the above CCDF, we find the conditional expectation of $\mathcal{W}$.

$$\begin{aligned}\Rightarrow \mathbb{E}[\mathcal{W} \,|\, t_{n-1} < \mathcal{T} \leq t_n] &= \int_0^{t_n - t_{n-1}} \Big(1 - \frac{t}{(t_n - t_{n-1})}\Big)\,\mathrm{d}t \\ &= \frac{(t_n - t_{n-1})}{2}.\end{aligned}$$

If $r(t)$ is varying slowly between two consecutive sampling instants due to the closeness of two sampling intervals, we can approximate the sampling interval $(t_n - t_{n-1})$ as

$$(t_n - t_{n-1}) \approx \frac{1}{r(t)},\ \forall t, n : t_{n-1} < t \leq t_n. \tag{13}$$

$$\begin{aligned}\Rightarrow \mathbb{E}[\mathcal{W}] &= \int_0^{\infty} \mathbb{E}[\mathcal{W} \,|\, t_{n-1} < \mathcal{T} \leq t_n] f_{\mathcal{T}}(t)\,\mathrm{d}t \\ &= \int_0^{\infty} \frac{1}{2r(t)} f_{\mathcal{T}}(t)\,\mathrm{d}t. \quad (14)\end{aligned}$$

We can thus find the energy penalty using Equation (10) and Equation (14) as

$$\mathcal{E} = \int_0^{\infty} \Big(\alpha \int_{x=0}^{t} r(x)\,\mathrm{d}x + \beta\frac{1}{2r(t)}\Big) f_{\mathcal{T}}(t)\,\mathrm{d}t.$$

Let $g(t) = \int_0^t r(x)\,\mathrm{d}x$. Then $g'(t) = \frac{\mathrm{d}}{\mathrm{d}t} g(t) = r(t)$. That is,

$$\mathcal{E} = \int_0^{\infty} \Big(\alpha g(t) + \frac{\beta}{2g'(t)}\Big) f_{\mathcal{T}}(t)\,\mathrm{d}t$$

As per the Euler-Lagrange equation from the calculus of variations [36, 37], the extreme value of $\mathcal{E}$ is obtained at

$$r^*(t) = \sqrt{\frac{\beta f_{\mathcal{T}}(t)}{2\alpha \bar{F}_{\mathcal{T}}(t)}}. \tag{15}$$

Thus, for a Rayleigh distributed $\mathcal{T}$ with parameter $\sigma$,

$$r^*(t) = \sqrt{\frac{\beta t}{2\alpha\sigma^2}} \tag{16}$$

$$\Rightarrow \int_{t_n}^{t_{n+1}} \sqrt{\frac{\beta t}{2\alpha\sigma^2}}\,\mathrm{d}t = 1,\ \forall n \geq 1 \qquad \text{(from (9))}$$

$$\Rightarrow t_{n+1}^{\frac{3}{2}} - t_n^{\frac{3}{2}} = 3\sigma\sqrt{\tfrac{\alpha}{2\beta}}$$

We have $t_0 = 0$. Substituting $n = 1, 2, \dots$, in order, in the above equation provides us our final result

$$t_n = \Big(3\sigma\sqrt{\tfrac{\alpha}{2\beta}}\Big)^{\frac{2}{3}} n^{\frac{2}{3}}.x \tag{17}$$

Note that, due to the close similarity in their density functions, the task times can be approximated fairly equally to an Exponentially Modified Gaussian distribution as well as a Rayleigh distribution, with the former attracting more attention from works like [32–34]. This is the reason why the above results are applicable in this work where we have predominantly considered Ex-Gaussian distribution. Furthermore, we have also verified the closeness of the results as well as the validity of the approximations made in the proofs using distribution fitting and simulations.

By making use of this general solution, we can also prove the results given in Section 6.2, where we modify the problem and find the optimum sampling instants that minimize the expected number of samples for a given upper bound

$w_0$ for the expected wait time. First note that the general solution has constants $\alpha$ and $\beta$ corresponding to the weights given to the cost of sampling and waiting, respectively. The optimization criteria changes from minimizing wait time to minimizing the number of samples when the ratio $\frac{\alpha}{\beta}$ goes from zero to infinity. Since any positive real value is valid for this ratio, one can achieve any valid point $(\mathbb{E}[\mathcal{S}], \mathbb{E}[\mathcal{W}])$ via simply by varying the ratio $\frac{\alpha}{\beta}$.

Furthermore, since the sampling instants are aperiodic and can take any positive real values, the bound will be tight at the optimum. Hence, to solve for the modified optimization problem explained in Section 6.2 that minimizes $\mathbb{E}[\mathcal{S}]$ with an upper bound $w_0$ on $\mathbb{E}[\mathcal{W}]$, we equate Equation (14) to $w_0$, find the corresponding $\frac{\alpha}{\beta}$, and find the optimum set of sampling instants by plugging this ratio into Equation (17).

$$\begin{aligned}
\mathbb{E}[\mathcal{W}] &= \int_0^\infty \frac{1}{2r(t)} f_{\mathcal{T}}(t)\,\mathrm{d}t. \\
&= \int_0^\infty \frac{1}{2}\sqrt{\frac{2\alpha\sigma^2}{\beta t}} \cdot \frac{t}{\sigma^2} e^{-t^2/2\sigma^2}\,\mathrm{d}t \\
&= \sqrt{\frac{\alpha\sigma^2}{2\beta}} \int_0^\infty \frac{\sqrt{t}}{\sigma^2} e^{-t^2/2\sigma^2}\,\mathrm{d}t \\
&= \sqrt{\frac{\alpha\sigma^2}{2\beta}} \left(\frac{1}{2\sigma^2}\right)^{1/4} \cdot \int_0^\infty y^{-1/4} e^{-y}\,\mathrm{d}y \\
&= \sqrt{\frac{\alpha\sigma^2}{2\beta}} \left(\frac{1}{2\sigma^2}\right)^{1/4} \cdot \Gamma(\tfrac{3}{4}),
\end{aligned}$$

where $\Gamma(x)$ is the gamma function. Thus, when the upper bound $w_0$ is tight,

$$\begin{aligned}
\mathbb{E}[\mathcal{W}] = w_0 &= \sqrt{\frac{\alpha\sigma}{2\sqrt{2}\beta}}\Gamma(\tfrac{3}{4}) \\
\Rightarrow \frac{\alpha}{\beta} &= \frac{w_0^2}{\sigma} \frac{2\sqrt{2}}{(\Gamma(\frac{3}{4}))^2} \\
&\approx 1.9\frac{w_0^2}{\sigma}.
\end{aligned}$$