1 Timeline

Entrants will have three times in which they may (re)submit strategies. Submissions will be spaced three weeks apart, during which entrants will be able to view the performance of their strategies and prepare modifications for the next submission. The details for this are in Section 4. The dates are as follows:

- *Monday 2/27*: Deadline to submit a strategy. Please submit your Matlab functions to the TA via email by midnight.
- *Monday 3/27*: Second submission date (midnight, via email).
- *Wednesday 4/14*: Third and final submission date (midnight, via email).

2 Game Description

To begin, let's establish some notation:

- N: Number of players
- B: Initial budget of tokens/period
- M: Total public pot
- K: Inflation factor
- T: Number of periods
- x_i^t: ***Integer*** contribution of player i in period t
- Π_i: Payoff to player i in a given period

We will be playing a modified version of the public goods game, a standard game in experimental economics. Typically, the public goods game is studied as a static game in which players are given a number of tokens B and need to privately choose an integer amount $x_i \in \{0, 1, \ldots, B\}$ to contribute to a public pot. After all players contribute what they wish, the amount in the pot is increased by some known multiple $K > 1$ and the resulting quantity is evenly distributed among all the players, regardless of their actual contribution. Interestingly, it may be shown that while under typical parameter values the overall social welfare is maximized if all agents fully contribute to the pot ($x_i = B\ \forall i$), that is not a stable equilibrium. Rather, the only Nash equilibrium is for all agents to contribute nothing, $x_i = 0\ \forall i$. The overall optimal social welfare is not achieved as each agent attempts to free ride off the others, regardless of their contributions.

Our class game will be an alteration of this basic format in several ways:

- The payoff players derive from the public pot will have two components. The first component is the same as the base public goods game, i.e. an equal portion of the inflated pot amount. In addition, a second component will be added that is a portion of M, the portion being equal to the ratio of that player's contribution relative to all contributions. Of course, uncontributed tokens are kept by the player. In other words, player i's payoff in period t is given by
$$\Pi_i(x_i^t, \vec{x}_{-i}^t) = K\left(\frac{\sum_{i=1}^N x_i^t}{N}\right) + M\left(\frac{x_i^t}{\sum_{j=1}^N x_j^t}\right) + (B - x_i^t).$$

- The game will be a repeated game, with the stage game repeated T times. The mechanics and stage game payoffs are as described, with an endowment of B tokens per period.

- Each player begins the game with an empty bank account, into which each stage game payoff $\Pi_i(\vec{x}^t)$ will be deposited. Note that during each period players may only contribute tokens to the public pot from their budget B that period; they may not contribute using tokens from earlier payoffs.

- The player with the largest balance in their bank account at the end of the T periods is the winner of that round.

With these modifications, the game can be viewed as loosely modeling a state run lottery. Each pay period, agents decide to use some portion of their salary to participate in the lottery by buying tickets, the proceeds from which are allocated to pay for public works from which all may derive some basic benefit in equal proportion. Those who buy lottery tickets, however, also on average will receive additional benefit in proportion to the number of tickets purchased.

3 Computing Format

We will be running the game in Matlab. Each entrant will be assigned an identification number xx and will submit their strategy in the form of a Matlab function entitled Strategyxx. In order to ensure a common interface, a Matlab function template will be provided, into which entrants may insert the code to implement their strategies. Some comments:

- In the interests of transparency, all source code will be made available for the master game.

- Each entrant's function will be called once for each stage. The input will be the state, composed of

  - t: The number of the current stage game being played (starting from 1)

– X: A $6 \times (t-1)$ matrix, each column of which are the values $\begin{bmatrix} x_i^t \\ \sum_{j \neq i} x_j^t \end{bmatrix}$ for each previous period.

- Strategies are free to incorporate randomization. However, different players' submitted functions may not incorporate correlated randomization. To this end, players' functions may *not* include any code that alters the state (i.e. the seeding) of the random number generator.

4 Competition Organization

As mentioned in Section 1, the competition will be organized into two week-long periods of "testing", followed by the final competition. The organization of these will be slightly different.

The parameter values will be as follows:

- $N = 6$
- $B = 20$
- $M = NB/4 = 30$
- $K = N/2 = 3$
- $T = 300$

In the testing periods, several times (possibly daily) each week entrants will be randomly organized into groups of 6 entrants/group. Each of these groups will then play 20 rounds against each other. For *each of these rounds*, the following data will be provided to each entrant:

- A 6×300 state matrix.
- That specific entrant's overall score for that round.
- The 5 overall scores in that round for each of the other players in that group.

(Note that you do not learn the identity of the players you played against.)

In each tournament stage, an entrant's performance is determined by the average payoff per (independent) round, i.e. the value Π_i^*, where

$$\Pi_i^* = \frac{1}{20} \sum_{r=1}^{20} \sum_{t=1}^{300} \Pi_i((x_i^t(r), \vec{x}_{-i}^t(r)).$$

5 Award

The winner of the competition will be the entry whose strategy results in the largest amount Π_i^* in the last stage of the tournament described above. Teams will get a extra credit of at most 5% (normalized by the maximimum winnings) in proportion to their winnings.