# Fetal ECG Extraction from Single-Channel Maternal ECG Using Singular Value Decomposition

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*Abstract*— **The extraction of fetal electrocardiogram (ECG) from the composite maternal ECG signal obtained from the abdominal lead is discussed. The proposed method employs singular value decomposition (SVD) and analysis based on the singular value ratio (SVR) spectrum. The maternal ECG (M-ECG) and the fetal ECG (F-ECG) components are identified in terms of the SV-decomposed modes of the appropriately configured data matrices, and elimination of the M-ECG and determination of F-ECG are achieved through selective separation of the SV-decomposed components. The unique feature of the method is that only one composite maternal ECG signal is required to determine the F-ECG component. The method is numerically robust and computationally efficient.**

*Index Terms*—**Cardiography, fetal ECG, orthogonal transformation, pattern estimation, periodic processes, singular value decomposition, spectral analysis.**

## I. Introduction

In physiological processes, the desired signal may not be directly measurable, and the investigator may have to determine the signal from measurable composite signals. The fetal electrocardiogram (ECG) [(F-ECG)] is one such case. The basic problem is to extract the F-ECG signal from the composite maternal ECG signal obtained from the abdominal lead, where the interfering maternal ECG (M-ECG) is a stronger signal. Different approaches have been proposed for detection of the F-ECG. Widrow *et al.* [1] proposed an adaptive filtering and adaptive noise cancellation method to extract the F-ECG from the composite maternal ECG signal; multiple M-ECG signals obtained from chest leads were used to cancel the M-ECG component identified as noise in the composite maternal ECG signal. A variant of the same approach was used by Longini *et al.* [2], where the F-ECG was obtained through a direct scaled subtraction of the thoracic ECG from the abdominal ECG. Among the other methods, auto-correlation and cross-correlation techniques were used by Van Bemmel [3]; methods termed as "spatial filtering" were used by Bergveld and Meijer [4] and Van Oosterom [5], where the F-ECG signal was produced through a weighted combination of signals from multiple electrodes. These methods were further developed using Singular Value Decomposition (SVD) by Vanderschoot *et al.* [6], and with adaptive on-line implementation [7]. A comparison between these methods is discussed in Callaerts *et al.* [8]. All these methods have a common requirement of multiple maternal thoracic ECG signals together producing an estimate of the M-ECG component, which is eliminated from the composite maternal ECG signal to obtain the F-ECG component. None of these methods utilizes the characteristic that the ECG signals are nearly periodic with repetitive patterns over the normalized period length. Our proposed method exploits this feature for selective separation of M-ECG and F-ECG components by formulating the problem in the SVD framework.

Two unique features of the proposed method of F-ECG extraction are: 1) only a single composite maternal ECG signal is required, and 2) the detection of the F-ECG component is achieved through successive extraction (or filtering) in an algebraically orthogonal transform domain, instead of the usual frequency domain. The tool used is SVD; the principle used is that SVD can be used to decompose any information matrix into orthogonal component dyads or modes. The basic idea is to identify the M-ECG component and the F-ECG component as the prime modes in appropriately configured data matrices; the components are separated through a successive procedure of configuring the data matrix, SVD and separation of the most dominant mode.

In the present work, the *singular value ratio (SVR) spectrum* has been used to define the dimension of the data matrix; the SVR spectrum [9], [10] can provide an estimate of the period length of the most dominant periodic component present in any signal.

The organization of this paper is as follows. In Section II, the theoretical background is provided. SVD and the SVR spectrum are introduced. Signal characterization using SVD is discussed, and the concept of the extraction of the principal periodic component from the appropriately configured data matrix is presented. Section III presents the proposed concept of F-ECG extraction from the single composite maternal ECG signal. Section IV presents examples and the procedure for the extraction of M-ECG and F-ECG components. Two sets of real-life composite maternal ECG data are considered; the relevant practical aspects are also addressed in Section IV. One prime concern in the proposed scheme is the effect of additive noise on the extraction of the principal periodic component in the signal, which is analyzed in Section V through Monte

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Carlo runs on a simulated composite maternal ECG signal. The computational aspects are discussed in Section VI. A comparative analysis of the proposed method against a few conventional methods is presented in Section VII. Section VIII discusses the concept of an average energy (periodic) pattern for the ECG signal, which can be useful for data compression as well as to produce filtered M-ECG and F-ECG estimates.

## II. Theoretical Background

### A. Basics of SVD

The SVD of an $m \times n$ matrix $\mathbf{A}$ is given by [11] $\mathbf{A} = \mathbf{U\Sigma V^T}$ where $\mathbf{U} \in \mathbb{R}^{m\times m}, \mathbf{V} \in \mathbb{R}^{n\times n}; \mathbf{U}^T\mathbf{U} = \mathbf{I}, \mathbf{V}^T\mathbf{V} = \mathbf{I}$, the $m \times n$ matrix $\mathbf{\Sigma} = [\text{diag}\{\sigma_1, \cdots, \sigma_p\} : 0], \mathbf{p} = \min(\mathbf{m}, \mathbf{n}), \sigma_1 \geq \cdots \geq \sigma_\mathbf{p} \geq \mathbf{0}$, and $\sigma_1, \cdots, \sigma_p$ are the singular values. $\mathbf{U}$ and $\mathbf{V}$ are the left and the right singular vector matrices, respectively. The left and the right singular vectors form a basis for the column-space and the row-space of $\mathbf{A}$, respectively. The energy content of the system represented by $\mathbf{A}$ is given by $Q_\mathbf{A} = \|\mathbf{A}\|_\mathrm{F}^2 = \sigma_1^2 + \sigma_2^2 + \cdots + \sigma_\mathbf{p}^2$; the subscript $F$ stands for the Frobenius norm. If $q$ of the $p$ singular values are predominant, the prime information of $\mathbf{A}$ will be contained in

$$\hat{\mathbf{A}} = \sum_{i=1}^{q} \mathbf{u}_i \sigma_i \mathbf{v}_i^T \tag{1}$$

where $\mathbf{u}_i$ and $\mathbf{v}_i$ are columns of $\mathbf{U}$ and $\mathbf{V}$, respectively. It can be shown that $\|\mathbf{A} - \hat{\mathbf{A}}\|_2 = \sigma_{q+1}$. Further, if $\sigma_1^2/\sigma_2^2 \gg 1$, the information energy will be mostly concentrated in the most dominant dyad $\mathbf{u}_1\sigma_1\mathbf{v}_1^T$ associated with $\sigma_1$, which is the fundamental concept behind the method of selective filtering used in the present work.

SVD is remarkably robust from the numerical point of view [12], as compared to other eigen decompositions.

### B. Signal Characterization Using SVD

Consider a series or signal $x(k) = \{\cdots, x(-1), x(0), x(1), x(2), \cdots\}$. If the signal $\{x(k)\}$ is periodic, with period length $n$, a typical set of $m$ consecutive periods of length $n$ can be expressed by the matrix $\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} x(1) & x(2) & \cdots & x(n) \\ x(n+1) & x(n+2) & \cdots & x(2n) \\ \vdots & \vdots & & \vdots \\ x((m-1)n+1) & x((m-1)n+2) & \cdots & x(mn) \end{bmatrix}. \tag{2}$$

*Remark:* The three characteristic features of a *periodic* series are 1) the period length, 2) the pattern over the periodic segment, and 3) the scaling factor, multiplicatively associated with the periodic segment.

For periodic or nearly periodic $\{x(k)\}$, the following features may be noted.

1) If $\{x(k)\}$ is *strictly periodic* with period length $n$, i.e., $x(k) = x(k+n)$, $\mathbf{A}$ $(= \mathbf{U\Sigma V^T})$ will be a strictly rank-one matrix; all singular values except $\sigma_1$ will be zero, and $\mathbf{v}_1$, the first column of $\mathbf{V}$, will represent the pattern or the normalized distribution of the process over one period; $\sigma_1/\sigma_2 = \infty$, and the elements of $\mathbf{u}_1\sigma_1$ will all be the same.
2) If $\{x(k)\}$ is *nearly periodic* with fixed period length $n$ but $x(k) \neq x(k+n)$, there are two main possibilities: 1) $\{x(k)\}$ has the same repeating pattern which are scaled differently over different periods. Still, Rank($\mathbf{A}$) = 1; $\mathbf{v}_1$ will represent the pattern, but the elements of $\mathbf{u}_1\sigma_1$ will be different. 2) $\{x(k)\}$ has nearly repeating patterns with different amplitudes over different periods. $\mathbf{A}$ can be a full rank matrix; $\sigma_1$ will still be dominant with the other singular values being insignificantly small but nonzero.

*Remark:* A predominant first singular value for any $m \times n$ matrix $\mathbf{A}$ is indicative of the presence of a strong periodic component (of period length $n$) in $\{x(k)\}$ given by the rows of $\mathbf{u}_1\sigma_1\mathbf{v}_1^T$ but the converse is not necessarily true.

### C. Analysis Using the SVR Spectrum

The SVR spectrum [9], [10] is a spectrum of $\sigma_1/\sigma_2$ values against varying row length of the data matrix $\mathbf{A}$, where the structure of $\mathbf{A}$ is as in (2). It can be used to determine the presence of a dominant periodic component (which need not be sinusoidal) in the given signal. The concept of the SVR sectrum can be briefly stated as follows.

If $\{x(k)\}$ is strictly periodic with period length $N, \sigma_1/\sigma_2$ of $\mathbf{A} = \infty$, if $n = iN$, $i$ being a positive integer. If $i$ is a noninteger or if $\{x(k)\}$ deviates from periodicity, $\sigma_1/\sigma_2$ will decrease. For a random series $\sigma_1/\sigma_2$ can be as low as 1. Hence, if the data matrices $\mathbf{A}(n)$ are formed with varying row length $n$, the corresponding pattern of $\sigma_1/\sigma_2$ of $\mathbf{A}(n)$ will peak at the values of $n$ for which there is a dominant periodic component of period length $n$ or any of its submultiples present in $\{x(k)\}$.

If the period length of the most dominant component varies with time or if the prime component is not very dominant with respect to the other components and noise, the SVR spectrum can be developed as follows. For a certain row length $n$, instead of handling the whole $m \times n$ matrix $\mathbf{A}(n)$, an $m_1 \times n$ moving window $\tilde{\mathbf{A}}(n)$ comprising $m_1$ successive rows of $\mathbf{A}(n), m_1 < m$, may be considered; for successive positions of $\tilde{\mathbf{A}}(n), \sigma_1/\sigma_2$ can be computed, and the median may be considerd as the value to be used for the SVR spectrum.

In this paper, the SVR spectrum has been used to determine the prime period lengths of the M-ECG and the F-ECG components.

### D. Estimation of the Strongest Periodic Component

In case of apparently periodic physiological signals (like the ECG), both the period length and the periodic pattern may vary to a certain extent. Usually there are one or more periodic or aperiodic signals mixed with the prime periodic or nearly periodic signal. If the data periods are arranged into the rows of a matrix, the deviation from repetitiveness between rows shows up in the values of one or more of the nonprime singular values $\sigma_2$ to $\sigma_p$ being large. The estimated dominant periodic pattern primarily belongs to two categories: the principal SV-decomposed periodic pattern and the average energy pattern; the problem of extraction of the most dominant periodic signal from a composite signal (which is treated next) belongs to the former category; the latter is discussed in Section VIII.

*Principal SV-Decomposed Periodic Pattern:* The most dominant periodic component present in the signal $\{x(k)\}$ configured into a matrix $\mathbf{A}$ is given by $\mathbf{A}_{p1} = \mathbf{u}_1\sigma_1\mathbf{v}_1^T$. The time series given by $\mathbf{A}_{p1}$ will have the *same repeating pattern* given by $\mathbf{v}_1^T$, which will be weighted by the factors $\{u_{1j}\sigma_1\}$ where $u_{1j}$ is the $j$th element of $\mathbf{u}_1$, scaling the $j$th row of $\mathbf{A}_{p1}$. The time series given by the residual matrix $(\mathbf{A} - \mathbf{A}_{p1})$ will contain additional information (if any) along with noise.

*Remark 1:* In (1), each component dyad $\mathbf{u}_i\sigma_i\mathbf{v}_i^T$ is a matrix representation of a time series with periodic pattern $\mathbf{v}_i^T$ of period length $n$; the energy of the series is given by $\sigma_i^2$.

*Remark 2:* In a real-life noisy environment, the validity of the extraction of the prime periodic component in terms of the bounds on the shifts in the values of $\sigma_1, \mathbf{u}_1$, and $\mathbf{v}_1$ due to perturbations is an important question, which has been discussed in the Appendix.

## III. Fetal ECG Extraction Scheme from Composite Maternal ECG

In the present case, the F-ECG signal is contained in the composite maternal ECG signal obtained from the abdominal lead; the signal available contains a strong M-ECG component, the F-ECG component and a large amount of noise, which may be due to maternal muscle contractions, motion artifacts, etc. Due to physiological reasons the periods of both the M-ECG as well as the F-ECG may vary to a certain extent, and these components are mutually asynchronous; further, it is likely that these two signals contain overlapping bands of frequencies.

### A. Outline of the Extraction Procedure

1) Separation of the M-ECG component from the composite signal: The data are first arranged in the form of a matrix $\mathbf{A}$ such that the consecutive maternal ECG cycles occupy the consecutive rows, and the peak maternal component lies in the same column. SVD is performed on $\mathbf{A}$, and $\mathbf{A}_M = \mathbf{u}_1\sigma_1\mathbf{v}_1^T$ is separated from $\mathbf{A}$, forming $\mathbf{A}_{R1} = \mathbf{A} - \mathbf{A}_M$.
2) Extraction of the F-ECG component: The time series formed from the successive rows of $\mathbf{A}_{R1}$ will contain the F-ECG component along with noise; this series is rearranged into a matrix $\mathbf{B}$ such that each row contains one fetal ECG cycle, with the peak value lying in the same column. SVD is performed on $\mathbf{B}$, from which the most dominant component $\mathbf{u}_1\sigma_1\mathbf{v}_1^T$ is extracted, which will give the desired F-ECG component.

*Remark 1:* The variations in the period lengths of the M-ECG and the F-ECG components can be taken into account in the formation of the data matrices $\mathbf{A}$ and $\mathbf{B}$. Two different approaches are used in the present work as discussed in Section III-B.

*Remark 2:* The composite maternal ECG signal may be riding on a slowly varying low-frequency signal (for example due to the respiratory process). Before the M-ECG component is separated, such low-frequency interference should be eliminated.

### B. Mechanism of Fetal ECG Extraction

Three different approaches for fetal ECG extraction are proposed; the first two concern the way the available data are arranged into matrices $\mathbf{A}$ and $\mathbf{B}$, and the third approach concerns consideration of a moving data window over $\mathbf{A}$ and $\mathbf{B}$ while extracting the M-ECG and F-ECG components, instead of extracting the same directly from $\mathbf{A}$ and $\mathbf{B}$.

*Method 1:* The *most commonly occurring period length* for the M-ECG signal is considered to be the row length $(n)$ of the data matrix $\mathbf{A}$. The consecutive periods with respect to M-ECG cycles are aligned into the consecutive rows of $\mathbf{A}$, and linearly interpolated data are used for the periods shorter than $n$. The row length of Matrix $\mathbf{B}$ is also decided similarly with respect to the F-ECG cycles. The part of the data in $\mathbf{u}_1\sigma_1\mathbf{v}_1^T$ that corresponds to the interpolated data in $\mathbf{A}$ or $\mathbf{B}$ is eliminated while forming the extracted M-ECG and the F-ECG series.

*Remark:* The incorporation of interpolated data is an algebraic necessity; it does not affect the proposed extraction scheme, as the error due to the interpolation remains largely confined to the nonprime singular values; see also Appendix.

*Method 2:* Here, the *most likely period length* (in terms of energy) *of the most dominant periodic component* present in the signal is considered to be the row lengths of the data matrices $\mathbf{A}$ and $\mathbf{B}$. The SVR spectrum of the composite maternal ECG signal is used to determine the period length (say $n$) of the M-ECG component in the composite signal. The periodic segments which are not equal to $n$ in length, are compressed or expanded to the length $n$ as follows.

The data segment, $y(1), y(2), \cdots, y(n^*)$, can be replaced by the set $x(1), x(2), \cdots, x(n)$ where $n \neq n^*$, using the transformation

$$x(j) = y(j^*) + (y(j^*+1) - y(j^*))(r_j - j^*) \qquad (3)$$

where $r_j = (j-1)(n^*-1)/(n-1) + 1$, and $j^*$ is the integral part of $r_j$. Thus the successive (pseudo-)periods of the composite ECG series $\{y(\cdot)\}$ are converted into data lengths having the same period length $n$, which are now aligned in the rows of the matrix $\mathbf{A}$. Appropriate reverse transformation is performed on the data at the time of reconstruction of the extracted M-ECG series.

The period length of the F-ECG component is determined from the SVR spectrum of the residual series. The subsequent steps for arranging data into $\mathbf{B}$ are the same as above.

The M-ECG and F-ECG components, are extracted as discussed in Section III-A or as discussed in Method 3.

*Remark:* The advantage of the interpolation or extrapolation scheme, within the defined period length $(n)$, is that the patterns of individual periods remain unaltered.

*Method 3:* Once the $m \times n$ data matrix $\mathbf{A}$ is formed, an $m_1 \times n$ $(m_1 < m)$ matrix $\tilde{\mathbf{A}}$ is assumed to move over $\mathbf{A}$ such that the first row is dropped and a new row is appended for successive locations of the window. $\tilde{\mathbf{A}}$ is SV-decomposed, and the last row of $\mathbf{u}_1\sigma_1\mathbf{v}_1^T$ is assumed to be the extracted M-ECG component corresponding to the last row of $\tilde{\mathbf{A}}$, which is subtracted from the original composite ECG series. The residual series is arranged into $\mathbf{B}$, from which

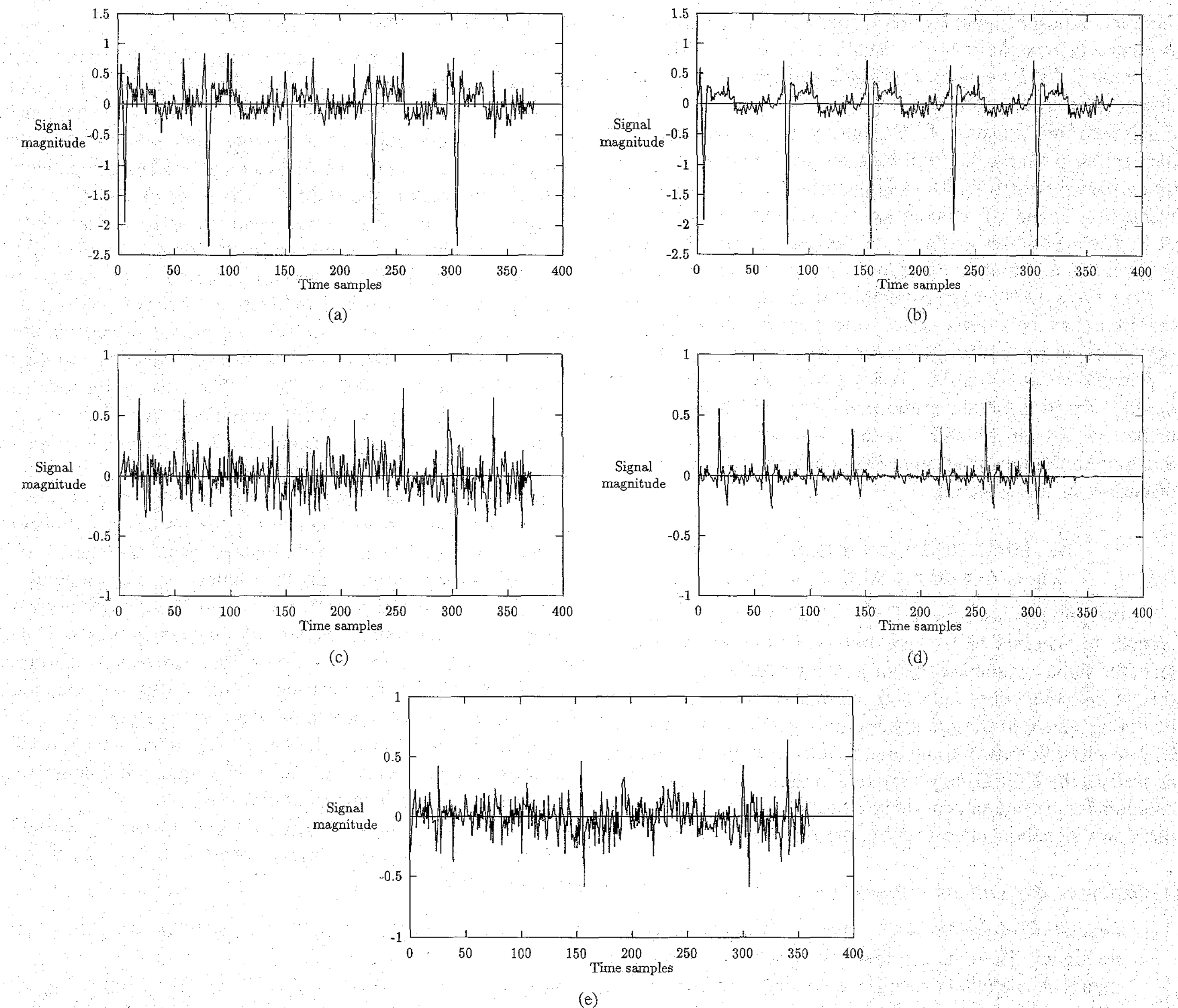


Fig. 1. (a) The composite maternal ECG signal: Data set 1, (b) the M-ECG component extracted from Data set 1, (c) the first residual series, with M-ECG series subtracted from Data set 1, (d) the F-ECG component extracted from the first residual series [(c)], and (e) the final residual series from Data set 1 (after extraction of the M-ECG and F-ECG components).

the F-ECG component is extracted the same way as the M-ECG component. This scheme is particularly applicable when sufficient amount of data are available.

## IV. Implementation and Results

### A. The Data

Results with two categories of composite maternal (abdominal lead) ECG data are reported here.

The first data set was digitized from the tracing of the composite maternal ECG signal shown in Fig. 1(a). Although there is no slowly varying low-frequency component present, the recorded signal shows the noise level to be considerably high compared to the fetal ECG component.

The second data set [13] was obtained from a woman with a gestation period of 37 weeeks. Here, the series shows [see Fig. 2(a)] the fetal ECG signal mixed with the maternal ECG signal and riding on an interfering low- frequency trend component. These data were recorded with an amplifier gain of 10 000 and 3-dB bandwidth of 0.05 to 250 Hz. The data were digitized at a sampling rate of 500 Hz; a further down-sampling of the data by a factor of four was used to retain reasonable information with adequate resolution.

### B. Data Preprocessing

The second data set required preprocessing to eliminate the low-frequency drift. Bidirectional low-pass filtering [14] was used for this purpose.

In bidirectional filtering, the data series is passed through a low-pass filter, and the filtered series is time-reversed and passed through the same filter again. Another step of time-reversal gives the bidirectionally filtered signal. The result is phase-lag-free filtering. In the present context, the filtered signal is the estimate of the low-frequency drift component,

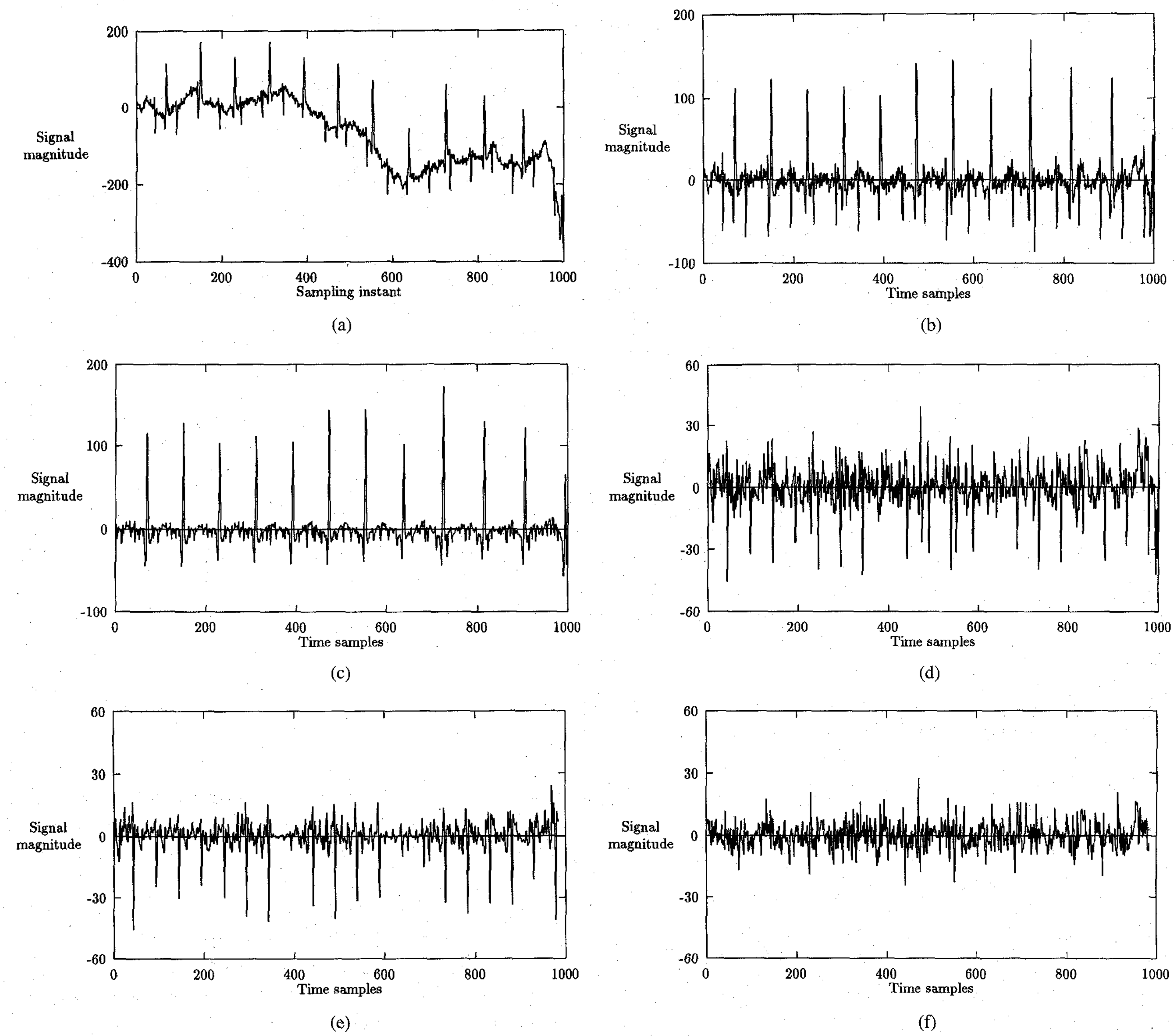


Fig. 2. (a) The composite ECG signal from a woman with 37-weeks gestation period (Data set 2), (b) the composite maternal ECG signal after extraction of low-frequency trend following bidirectional filtering, (c) the M-ECG component extracted from the filtered series shown in Fig. 2(b), (d) the first residual series after M-ECG component extraction from the filtered series [(b)], (e) the F-ECG component extracted from the residual series [(d)], and (f) the final residual series from Data set 2.

which is subtracted from the original signal to eliminate the low-frequency drift. Here, the filter used was $F(z^{-1}) = (1-\alpha)/(1-\alpha z^{-1}), 0 < \alpha < 1$; a pole at 0.88 was found to be adequate in eliminating the low-frequency drift, as shown in Fig. 2(b).

### C. M-ECG and F-ECG Extraction Using Method 1

In the first set of data, the M-ECG and the F-ECG components were found to have a period lengths varying between 73 and 76, and 37 and 44 samples, respectively. The most commonly occuring M-ECG and F-ECG period lengths are 75 and 40 samples, respectively. The data matrix $\mathbf{A}$ worked out to be $5 \times 75$ in size. SVD of $\mathbf{A}$ produced $\sigma_1/\sigma_2 > 27$. The extracted maternal ECG component is shown in Fig. 1(b). The residual series shown in Fig. 1(c), was configured into matrix $\mathbf{B}$ of size $8 \times 40$. $\mathbf{B}$ was SV-decomposed and the F-ECG component was obtained, which is shown in Fig. 1(d). The subsequent residual series is shown in Fig. 1(e). The ratio of the energy in the fetal component with respect to the final residual is $-3.7083$ dB.

*Remark:* F-ECG extraction in the present case is particularly difficult, because of 1) the short data length available and 2) the low signal-to-noise ratio (SNR) of the F-ECG.

### D. M-ECG and F-ECG Extraction Using Method 2 and Method 3

The bidirectionally filtered composite ECG data [Fig. 2(b)] showed a variation of the period length from 80 to 90 samples. The SVR spectrum of the filtered data was computed, which showed a repeating peak at a period length of 81 samples, as shown in Fig. 3(a). For all the data segments where the M-ECG showed period length other than 81 samples, the

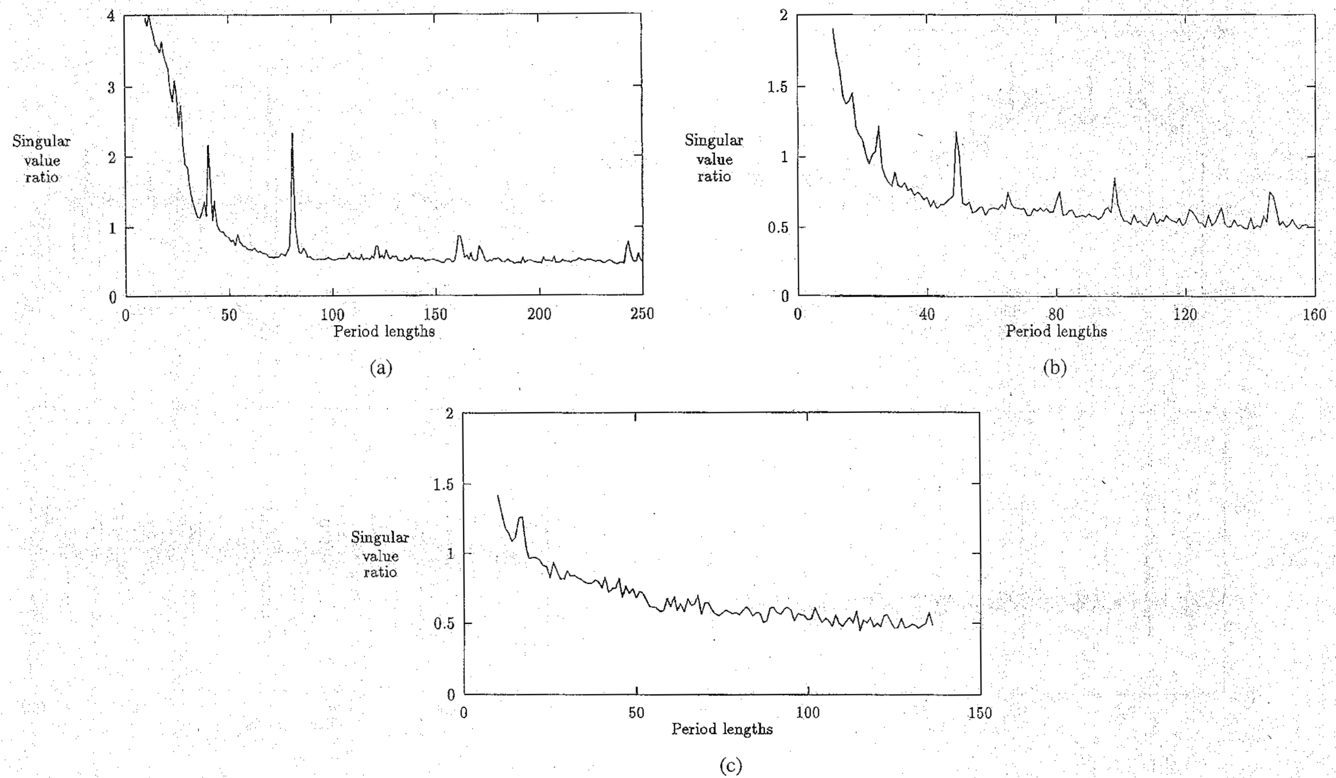


Fig. 3. (a) SVR spectrum of the filtered composite maternal ECG series [Fig. 2(b)], (b) SVR spectrum of the first residual series [Fig. 2(d)], and (c) SVR spectrum of the final residual series [Fig. 2(f)].

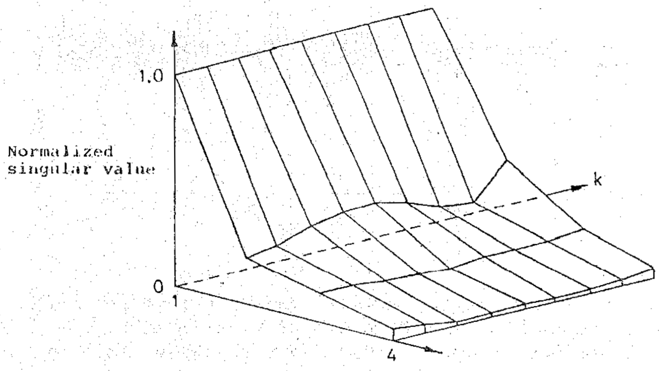


Fig. 4. Progressive distribution of the normalized singular values of $\tilde{\mathbf{A}}$.

segments were uniformly extrapolated or compressed [using (3)], and a $12 \times 81$ matrix $\mathbf{A}$ was formed. The M-ECG component [Fig. 2(c)] was extracted using Method 3 (Section III-B), with $m_1 = 4$. The distribution of the singular values of $\tilde{\mathbf{A}}$ (normalized by $\sigma_1$) for successive positions $(k)$ is shown in Fig. 4, which shows the time-varying nature of the underlying process.

The first residual time series [see Fig. 2(d)] obtained from the residual matrix $\mathbf{A}_{R1}$ was subjected to SVR spectrum analysis. Its spectrum [Fig. 3(b)] showed peaks occurring at $n = 25$, 49, 98, and 146 samples. Therefore, the period length for the F-ECG component (varying from 46 to 51) was considered to be 49 samples. A $20 \times 49$-matrix $\mathbf{B}$ was formed. The F-ECG component was extracted using Method 3 with $m_1 = 4$; the final extracted F-ECG series is shown in Fig. 2(e).

The SVR spectrum of the final residual series [Fig. 2(f)] is shown in Fig. 3(c), where no conspicuous periodicity is observed. The ratio of the F-ECG energy with respect to the final residual is 1.5956 dB.

*Remark 1:* In the SVR spectrum of the first residual series [Fig. 3(b)], the SVR magnitude corresponding to row length of 146 is marginally higher than that at 147, which is the third multiple of 49. This discrepancy is due to the associated noise.

*Remark 2:* At sampling instants 393 and 640 [Fig. 2(e)], the weakness of the F-ECG peaks is due to the merging in time with the corresponding M-ECG peaks as shown in Fig. 2(a). This is not a serious problem, as the subsequent fetal $R$-waves do appear because of the difference in the periodicity of M-ECG and F-ECG components.

*Remark 3:* The absence of any dominant periodicity in the final residual series certifies the completeness of the aimed extraction with reference to the background noise.

## V. Effects of Noise on Principal Component Extraction

The performance of the principal pattern extraction procedure in the face of additive noise is studied as follows. Twenty periods of a typical 75-point M-ECG cycle were taken from the first data set and added to a periodic series of 40-point F-ECG cycles and a series of white Gaussian noise to simulate the typical composite ECG signal obtained from the abdominal lead on the mother. For varying noise levels, the M-ECG and F-ECG components were extracted as discussed in Section IV-C. The mean squared error of the maternal and fetal ECG's, computed as the average of the sum of the squared errors

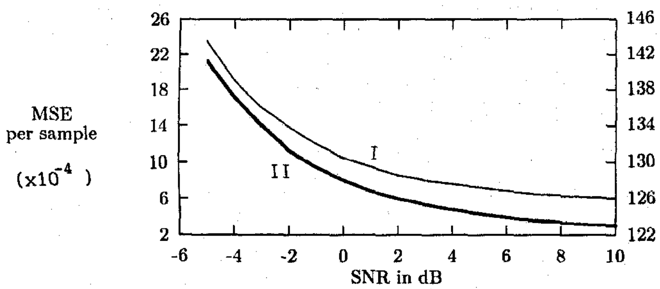


Fig. 5. Normalized mean square error (NMSE) plotted against SNR: for the (a) M-ECG component (I: between 0.0002 and 0.0026), and the (b) F-ECG component (II: between 0.0122 and 0.0146).

over 500 independent experiments, are plotted against SNR in Fig. 5(a) and (b), respectively. The SNR has been calculated with respect to the power of the fetal component; noise of the same power as the fetal component gives SNR = 0 dB. It may be noted that the M-ECG and F-ECG components can be extracted with acceptably low MSE, unless the SNR falls to abnormally low levels. The extracted M-ECG and F-ECG signals are not shown as they look similar to those in Fig. 1(b) and (d), respectively.

## VI. Computational Aspects

Computation of the SVR spectrum does not require complete SV-decomposition; only the two largest singular values are used. Estimation of the M-ECG and F-ECG components requires only the first singular value and the first left singular vector ($\mathbf{u}_1$) and the first right singular vector ($\mathbf{v}_1$).

The additional computation incurred in Method 2 is due to the computation of the SVR spectrum, and the normalization of the signal over the period lengths; the gain is in the retention of the periodic pattern, leading to improved F-ECG extraction. For all practical purposes, computation of the SVR spectrum at a predecided frequency should suffice.

In the present case, since $n \gg m$, it is advisable to perform SVD using the Golub-Reinsch implementation [15]. The flop (i.e., floating point operation) count for computing $\mathbf{\Sigma}, \mathbf{u_1}$ and $\mathbf{V}$ of an $m \times n$ matrix is given as 14 $mn^2$+ 8 $n^3$ [11, p. 239], which will be further reduced if only $\mathbf{v}_1$ is computed. Fast implementation of SVD is also possible [6], [16], [17].

For real-time implementation, configuring the data matrices $\mathbf{A}$ and $\mathbf{B}$ is another important task. The M-ECG peaks stand out prominently in the composite signal; thus the formation of the data matrix $\mathbf{A}$ is not difficult. The F-ECG peaks can also be spotted in the original composite signal by general search routines; here the additional information about the probable period length should be useful.

The above analysis leads to the conclusion that the computations can be performed fast enough for real-time implementation.

## VII. Comparative Analysis

Some of the problems of the multiple electrode methods with respect to the proposed method are as follows.

In the multiple electrode methods, there is the need to generate (either through adaptive weighting of the thoracic signals [1]–[5] or otherwise [6]–[8]) an estimate of the M-ECG component, which is close to that appearing in the composite maternal ECG signal. To achieve this, the number of thoracic as well as the abdominal ECG signals are sometimes heuristically chosen. Although some works reportedly use specific numbers of thoracic signals (for example 4 in [1] and 3 in [8]), the extraction is sensitive to the accuracy in the absolute as well as collective placement of the thoracic electrodes [6]–[8]. Further, the parametric methods [1], [4] may have estimation problems if the underlying dynamics keep changing.

Another aspect often ignored is the problem of eliminating the effects of differential interferences due to extraneous reasons (e.g., due to respiratory activity [18]) on the thoracic signals and on the composite abdominal ECG signals. All multiple electrode methods suffer from this problem.

The above-mentioned interference problems do not affect the proposed method since only one signal is handled. It is, however, necessary for the available composite signal to be representative of the fetal system, which is a common requirement. The changing dynamics are inherently taken care of through the assessment of the SVR spectrum and the concept of using a moving window for the data matrices.

One minor drawback of the present method is that it incorporates a memory of a few M-ECG or F-ECG cycles, although this has a stabilizing effect in the sense that a spurious disturbance will be largely ignored. The computational load due to SVD is not much because 1) fast implementations are possible and 2) only partial SVD is necessary. In general, SVD-based methods [6]–[8] including the proposed method are expected to be more immune to noise than others.

## VIII. Average Energy Pattern

A byproduct of SVD is the averaged energy series. For any $m \times n$ matrix $\mathbf{A}^* = \mathbf{u}_1\sigma_1\mathbf{v}_1^T$, the averaged energy series is a strictly periodic series having the periodic pattern $\mathbf{v}_1$, scaled by the factor $\sigma_1/\sqrt{m}$, as follows.

Each row of $\mathbf{A}^*$ represents a period of the signal $\{x(k)\}$. Therefore, the averaged periodic-energy is given by

$$\begin{aligned} Q_{\text{av}} &= \frac{1}{m}\sum_{i=1}^{m} u_{1i}^2(\sigma_1\mathbf{v}_1)^T(\sigma_1\mathbf{v}_1) \\ &= (\sigma_1\mathbf{v}_1/\sqrt{m})^T(\sigma_1\mathbf{v}_1/\sqrt{m}). \end{aligned}$$

Hence, if the signal strength does not vary too much, the ECG data can be compressed into $\sigma_1$ and $\mathbf{v}_1$, where $m$ is predefined; such an approach can lead to substantial reduction in the required storage space for long-term monitoring. The average energy patterns of the M-ECG and F-ECG components of Data set 2 are shown in Fig. 6.

*Remark 1:* ECG data compression is a widely studied topic [19], [20]. Instead of the averaged energy pattern, a time-averaged pattern may be produced, which can serve the same purpose. However, there is a fundamental difference between simple averaging and SVD-based averaging. Simple averaging over $m$ points is equivalent to $(m-1)$th order low-pass finite impulse response (FIR) filtering [21, p. 33], whereas SVD-based principal pattern extraction is filtering in an orthogonal

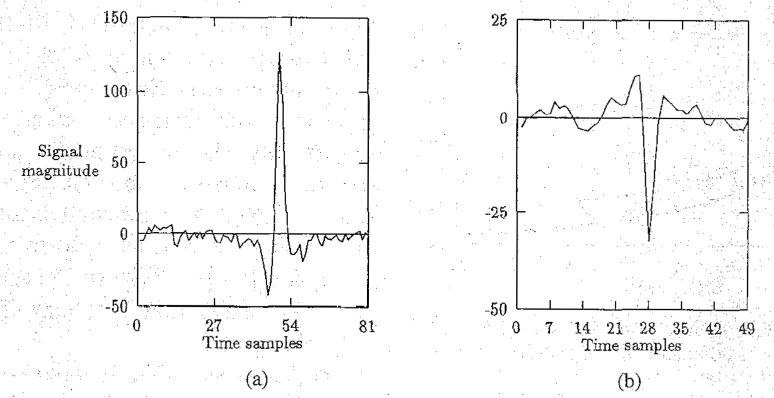


Fig. 6. Average energy patterns obtained from Data set 2: (a) M-ECG pattern ($m = 11$) and (b) F-ECG pattern ($m = 20$).

transform domain and not in the frequency domain; here, the noise, which is algebraically orthogonal to the signal is eliminated, and hence both the selected and the rejected signals may contain similar frequency components.

*Remark 2:* Either the normalized time averaged pattern (i.e., normalized to unit vector) or the averaged energy pattern may be multiplied by the elements of $\mathbf{u}_1\sigma_1$ to reconstruct the ECG data.

*Remark 3:* In line with Method 3, a *moving* average energy pattern may be produced, representing the M-ECG or F-ECG components having a memory of $m_1$ periods.

## IX. Conclusion

SVD has been applied for the extraction of the fetal ECG component from a single composite maternal ECG signal obtained from an abdominal lead. Results demonstrate that the proposed method works inspite of low SNR; drifts and low-frequency interferences can also be handled. Since the extraction procedure is direct and unambiguous, automated extraction should be possible. Use of SVD attributes high degree of numerical robustness to the proposed method, and yet the extraction can be fast through efficient implementation.

The principle of selective extraction of signal components used is generic in nature and no equivalent frequency domain method is available. The proposed scheme should be applicable for separation of components in any composite signal, where the signals can be configured to be algebraically orthogonal to each other.

## Appendix
## Error Analysis in Principal Pattern Estimation

The error in the rank-1 estimate [i.e., $q = 1$ in (1)] of the prime periodic pattern within a composite signal is analyzed here. Let this error be viewed as the deviation from the perfectly periodic signal. Let $\mathbf{u}_1\sigma\mathbf{v}_1^T$ be $m \times n$-matrix representation of a periodic series with a repeating pattern, and $\sigma$ is a positive scalar; let the $m \times n$-perturbation matrix $\mathbf{E}$ represent all the remaining signal components mixed with the periodic signal to make up the composite signal. Further, suppose the matrices $[\mathbf{u}_1\ \mathbf{U}]$ and $[\mathbf{v}_1\mathbf{V}]$ are orthogonal, and

$$[\mathbf{u}_1 \quad \mathbf{U}]^T\mathbf{E}[\mathbf{v}_1 \quad \mathbf{V}] = \begin{pmatrix} e_{11} & \mathbf{e}_{12} \\ \mathbf{e}_{21} & \mathbf{E}_{22} \end{pmatrix}$$

where $\mathbf{E}_{22} \in \mathbb{R}^{(m-1)\times(n-1)}, \mathbf{e}_{12}^T$ and $\mathbf{e}_{21}$ are $(n-1)$ and $(m-1)$ vectors, respectively; $\varepsilon = \|\mathbf{e}_{21}^T\ \mathbf{e}_{12}\|_F$, and $\mu = \sigma - |e_{11}| - \|\mathbf{E}_{22}\|_2$, $\mathbf{U} \in \mathbb{R}^{m\times(m-1)}$ and $\mathbf{V} \in \mathbb{R}^{n\times(n-1)}$. If the periodic waveform $\mathbf{u}_1\sigma\mathbf{v}_1^T$ is significantly dominant than the rest of the composite signal, $\varepsilon$ is very small and $\mu$ is of the order of $\sigma$. If $\mu > 0$ and $2\varepsilon/\mu \leq 1$, then a simplification of Theorem 8.3.5 in Golub and Van Loan [11, p. 429] ensures that there exist vectors $\mathbf{p} \in \mathbb{R}^{n-1}$ and $\mathbf{q} \in \mathbb{R}^{m-1}$ such that $(\mathbf{p}^T\mathbf{p} + \mathbf{q}^T\mathbf{q})^{1/2} \leq 2\varepsilon/\mu$, and $\mathbf{v}_1^* \propto (\mathbf{v}_1 + \mathbf{V}\mathbf{p}), \mathbf{u}_1^* \propto (\mathbf{u}_1 + \mathbf{U}\mathbf{q})$, where $\mathbf{v}_1^*$ and $\mathbf{u}_1^*$ are the perturbed $\mathbf{v}_1$ and $\mathbf{u}_1$ vectors, respectively. Since $\|\mathbf{v}_1 + \mathbf{V}\mathbf{p}\|_2 = (1 + \mathbf{p}^T\mathbf{p})^{1/2}$, and $\|\mathbf{u}_1 + \mathbf{U}\mathbf{q}\|_2 = (1 + \mathbf{q}^T\mathbf{q})^{1/2}$, the perturbations turn out to be $\Delta\mathbf{v}_1 = (\mathbf{V}\mathbf{p} + \mathbf{v}_1(1 - (1 + \mathbf{p}^T\mathbf{p})^{1/2}))/(1 + \mathbf{p}^T\mathbf{p})^{1/2}$ and $\Delta\mathbf{u}_1 = (\mathbf{U}\mathbf{q} + \mathbf{u}_1(1 - (1 + \mathbf{q}^T\mathbf{q})^{1/2}))/(1 + \mathbf{q}^T\mathbf{q})^{1/2}$. $\Delta\mathbf{v}_1$ represents the error in reconstructing the pattern of the dominant periodic waveform; the error in the amplitude is given by the elements of $((\sigma + \Delta\sigma)(\mathbf{u}_1 + \Delta\mathbf{u}_1) - \sigma\mathbf{u}_1)$, where $\Delta\sigma$ is upper-bounded by $\|\mathbf{E}\|_2$.

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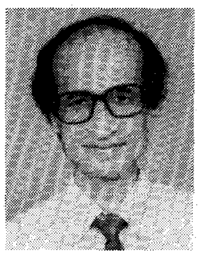

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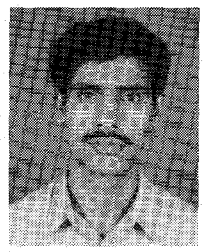

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