

STAT 431 Final Project Report

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Abstract

In this project, we investigate how serious crime rates vary across 21 districts of Los Angeles Police Department (LAPD). We apply a Bayesian hierarchical modeling framework and implement Markov Chain Monte Carlo methods using an open-access dataset from the Data.gov ¹ platform. Specifically, we estimate posterior means and confidence intervals for district-level serious crime rates, conduct posterior predictive checks to assess model fit, and perform prior sensitivity analyses to evaluate the effectiveness of our results. Our findings quantify uncertainty in serious-crime rate estimates within Los Angeles and offer a modeling template for analyzing crime patterns in other metropolitan areas around the world.

1 Introduction

1.1 Motivation

Urban crime rarely occurs uniformly across space. Even within a single city, some districts experience a much larger share of serious crimes than others. These severe offenses include homicide, robbery, and aggravated assault. Successfully identifying these spatial differences in serious crime rates is essential for local governments and police departments in that it can generate promising effects. These include more efficient resource allocation, targeted prevention strategies, and evaluating whether particular communities face disproportionate exposure to violence.

1.2 Goal

The Los Angeles Police Department (LAPD) divides the city into 21 geographic areas, each with its own demographic composition and policing needs, creating a natural setting for our analysis. The primary goal of this project is to reflect and compare serious crime rates across the 21 geographic areas in Los Angeles.

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¹<https://catalog.data.gov/dataset/crime-data-from-2020-to-present>

However, since different geographic areas are exposed to different total numbers of incidents, comparing raw proportions of serious crimes between districts can produce misleading results. Under the assumption that district-level probability parameters arise from a common population distribution, a hierarchical model achieves a bias–variance tradeoff through partial pooling, allowing each district’s estimate to be informed by both its local data and information shared across districts.

Hence, rather than focusing solely on raw sample proportions, we aim to:

- construct a probabilistic model for the number of serious crimes in each area, using the total number of reported crimes as the binomial “exposure”;
- estimate area-specific serious-crime rates under a Bayesian hierarchical model that allows areas to share information while retaining their own local patterns; and
- assess the extent of spatial heterogeneity in serious-crime risk, including identifying areas that are credibly higher or lower than the city-wide average after accounting for uncertainty.

2 Data

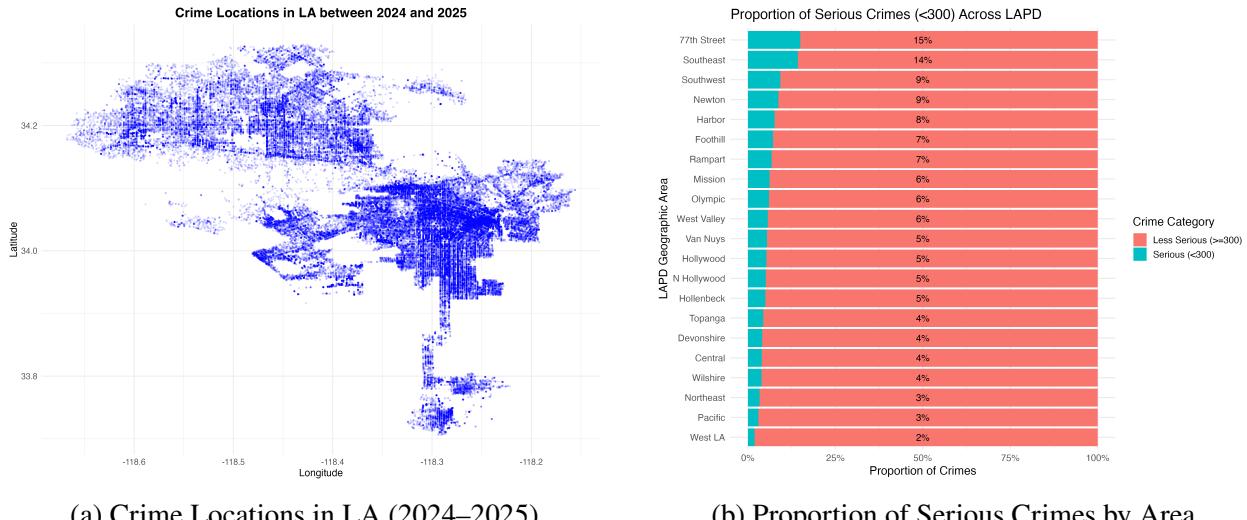
We use the public Crime Data from 2020 to Present dataset maintained by the LAPD and distributed through Data.gov¹. Each row is a single reported crime incident. For this project, we extract all incidents with a valid date and police area code that occurred between January 1, 2024 and October 31, 2025. After cleaning, we retain crimes from the 21 LAPD Geographic Areas.

From the incident-level data, we keep only variables needed to study crime severity. In particular, Primary Crime Code 1 is an official LAPD offense code indicating the seriousness of the crime in a particular incident. Following LAPD coding conventions, we classify an incident as a serious crime if its corresponding Primary Crime Code 1 is less than 300. For each area, we then compute (i) the total number of reported crimes and (ii) the number of serious crimes in the study period; the proportion of serious crimes is the main response in our hierarchical models. We also use the latitude/longitude of each incident only for mapping and exploratory plots.

Table 1: Selected Variables from the LAPD Crime Dataset

Variable	Definition
Date of Occurrence	Calendar date when the crime occurred (MM/DD/YYYY), converted to a standard date-time format and restricted the study period to 2024–2025.
Police Area Code	Identifies one of the 21 LAPD Geographic Areas. Each area is a distinct administrative region and is the unit of analysis in our study.
Primary Crime Code 1	Official LAPD code for the primary and most serious offense in an incident. We define a serious crime as any incident with Primary Crime Code 1 < 300.
Location (Longitude, Latitude)	Geographic coordinates of the incident, used to map crime locations and explore spatial patterns across Los Angeles.

Figure 1 provides a brief exploratory summary of the cleaned dataset. The left panel plots all recorded crime locations in Los Angeles over the study period, regions with darker color indicates higher number of crimes. The right panel shows the proportion of serious crimes in each LAPD area, illustrating clear spatial variation in crime severity.



(a) Crime Locations in LA (2024–2025)

(b) Proportion of Serious Crimes by Area

Figure 1: Exploratory Data Analysis of LAPD Crime Data

3 Model

Data and question: Let the LAPD Geographic Areas be indexed by $i = 1, \dots, K$, where $K = 21$ is the total number of areas observed. We are interested in the number of “serious” crimes (defined as crimes with Crime Code 1 < 300), denoted by y_i , out of the total number of reported crimes N_i for each area during the fixed period (2024-2025). Assuming independent events, the counts can be modeled as:

$$y_i \mid p_i \sim \text{Binomial}(N_i, p_i),$$

where p_i is the underlying serious crime rate for area i .

Hierarchical prior: To borrow strength across areas and enable shrinkage, we place a shared conjugate prior on the proportions:

$$p_i \mid \alpha, \beta \stackrel{\text{iid}}{\sim} \text{Beta}(\alpha, \beta), \quad \text{for } i = 1, \dots, K.$$

To allow the data to inform the overall mean and variance of these rates, we assign independent hyperpriors to the shape parameters α and β :

$$\alpha \sim \text{Gamma}(a_1, b_1), \quad \beta \sim \text{Gamma}(a_2, b_2).$$

In our analysis, we set weakly informative fixed hyperparameters $a_1 = a_2 = 2$ and $b_1 = b_2 = 0.5$ which provide mild regularization while allowing the data to dominate posterior inference.

Posterior and computation: By Bayes’ rule, the joint posterior distribution is proportional to the product of the Binomial likelihood, the Beta priors, and the Gamma hyperpriors:

$$\pi(\mathbf{p}, \alpha, \beta \mid \mathbf{y}) \propto \left[\prod_{i=1}^K \text{Binomial}(y_i \mid N_i, p_i) \times \text{Beta}(p_i \mid \alpha, \beta) \right] \times \text{Gamma}(\alpha \mid a_1, b_1) \times \text{Gamma}(\beta \mid a_2, b_2).$$

Since the full posterior is analytically intractable, we use Markov Chain Monte Carlo (MCMC) methods implemented in JAGS to approximate the posterior distributions of the parameters $\mathbf{p} = (p_1, \dots, p_K)$, α , and β .

4 Computational Strategy

We implemented the Bayesian hierarchical Beta–Binomial model in JAGS, whose Gibbs sampling framework efficiently handles models with conjugate full conditional distributions. The hierarchical structure enables partial pooling across the 21 LAPD areas, yielding stabilized esti-

mates for districts with limited data. To assess convergence and avoid dependence on initial values, we ran three parallel chains with deliberately over-dispersed starting points for (α, β) : $(0.5, 0.5)$, $(2.0, 2.0)$, and $(5.0, 1.0)$.

For each chain, the first 1,500 iterations were discarded as burn-in. We then obtained 100,000 additional iterations per chain, resulting in 300,000 posterior draws in total. No thinning was applied, as autocorrelation remained moderate and storage constraints were minimal. The large effective sample size supports precise estimation of posterior means, standard deviations, and credible intervals for all monitored parameters.

5 Diagnostic and Robustness Check

5.1 Traceplot

Figure 2 provides visual evidence of satisfactory MCMC performance. Good mixing is indicated by the rapid fluctuation of the chains around a stable level and the substantial overlap across chains, with no visible trends or drift. These visual patterns are characteristic of samples drawn from the stationary posterior distribution and therefore suggest that the MCMC algorithm has successfully converged.

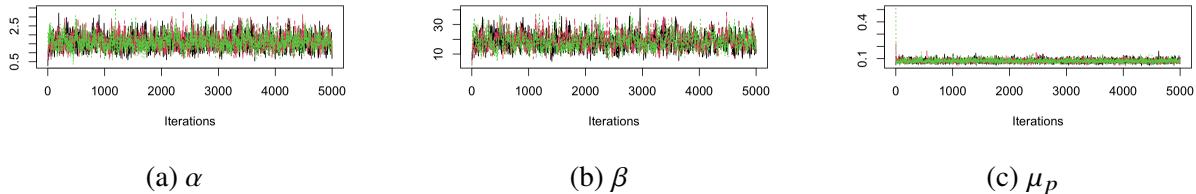


Figure 2: Trace plots for the monitored parameters.

Note: Each panel shows posterior samples from three parallel MCMC chains after burn-in.

In addition to the visual diagnostics discussed above, we also include the Gelman–Rubin statistic (PSRF) as a supplementary convergence check. The PSRF compares the variability within each MCMC chain to the variability between chains; when all chains have converged to the same posterior distribution, these quantities should be nearly identical, producing values very close to 1. Our results in 2 show PSRF values essentially equal to 1, providing additional confirmation of convergence.

5.2 Posterior Predictive Check

We performed posterior predictive check (PPC) to validate that our proposed model provides an adequate fit.

Figure 3 displays the posterior predictive distribution of the total serious crime count across all areas, yielding a Bayesian p-value of 0.52. This value indicates that approximately 52% of the replicated datasets produced totals greater than or equal to the observed count. A p-value close to 0.5 suggests the model adequately reproduces the key summary statistic of the data, providing evidence that our hierarchical Beta-Binomial model captures the essential features of the crime distribution across LAPD areas.

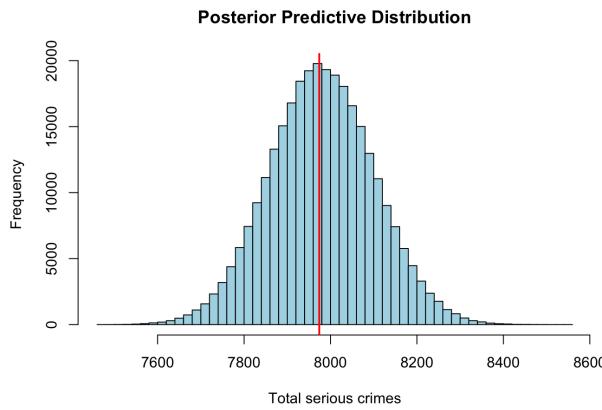


Figure 3: Posterior Predictive Distribution of Total Serious Crimes.

Note: The distribution of total serious crimes predicted by the model (histogram) compared to the observed value (red line). The observed data falls within the central region of the predictive distribution, indicating adequate model fit.

5.3 Sensitivity Analysis

To validate the stability of our posterior inferences against prior specification, we conducted a sensitivity analysis by comparing the baseline model ($\alpha, \beta \sim \text{Gamma}(2, 0.5)$) with a more diffuse alternative ($\alpha, \beta \sim \text{Gamma}(1, 0.25)$). Figure 4 displays the correspondence between the posterior mean estimates of the area-level serious crime rates (p_i) under these two regimes. The estimates align almost perfectly along the 45-degree identity line, indicating that the results are virtually invariant to the choice of hyperpriors. This confirms that the inference is primarily data-driven, with the likelihood of the observed data dominating the prior information.

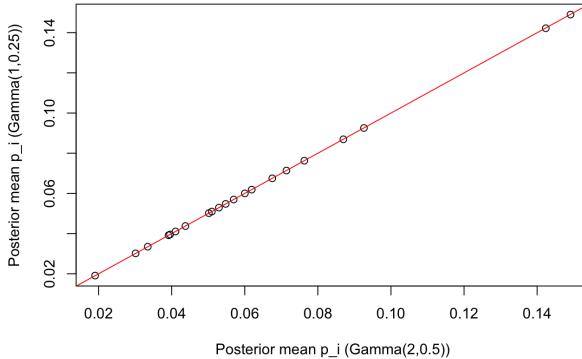


Figure 4: Sensitivity of posterior means to hyperprior choice.

Note: Each point compares the posterior mean estimate of p_i under the baseline hyperpriors (horizontal axis) versus diffuse hyperpriors (vertical axis). The red line denotes the identity line. Both models were fit using identical MCMC settings (300,000 post-burn-in draws across three chains).

6 Results and Discussions

The hierarchical model provides estimated serious crime rates (p_i) for all 21 LAPD geographic areas.

Table 2 presents the posterior summaries for the hyperparameters and the global serious crime rate. The convergence diagnostics provide strong evidence of model stability: all monitored parameters exhibit negligible Monte Carlo Standard Errors and Gelman-Rubin statistics of approximately 1.01, confirming that the MCMC sampler has successfully converged to the stationary distribution. Statistically, the posterior estimates for the hyperparameters yield a global mean crime rate of approximately 0.08, which serves as the population-level baseline toward which individual area estimates are shrunk.

Table 2: Posterior summaries for hyperparameters and the global serious crime rate.

Parameter	Mean	SD	2.5%	97.5%	MCSE	\hat{R}
α (shape 1)	1.61	0.40	0.93	2.50	0.0021	1
β (shape 2)	18.79	5.38	9.71	30.68	0.028	1
μ_p (serious crime rate)	0.08	0.01	0.06	0.11	0.000026	1

Note: Each row reports posterior summaries based on 300,000 MCMC draws from three parallel chains. Mean and SD denote the posterior mean and standard deviation. Columns 4 and 5 give the bounds of the central 95% credible interval. MCSE is the time-series Monte Carlo standard error accounting for autocorrelation. \hat{R} is the Gelman–Rubin convergence diagnostic.

Complementing the Bayesian analysis, Table 3 summarizes the raw empirical proportions for the 21 LAPD geographic areas. These represent the frequentist maximum likelihood estimates calculated under the assumption that each area is independent. The data reveal substantial spatial heterogeneity in crime distribution. The 77th Street exhibit the highest frequentist rates, but ar-

eas such as West LA and Pacific show the lowest proportions, clustering between 0.02 and 0.03. The majority of districts fall within an intermediate band of 0.04 to 0.08. This observable variation—combined with the differing sample sizes across districts—provides the empirical motivation for adopting a hierarchical Bayesian approach to stabilize estimates in areas with higher statistical noise.

Table 3: Frequentist estimates of serious crime rates by LAPD area.

Area	y_i	N_i	\hat{p}_i	Area	y_i	N_i	\hat{p}_i	Area	y_i	N_i	\hat{p}_i
77th Street	1012	6777	0.15	Mission	332	5369	0.06	Southeast	800	5611	0.14
Central	403	10214	0.04	N Hollywood	369	7249	0.05	Southwest	770	8314	0.09
Devonshire	241	5892	0.04	Newton	469	5390	0.09	Topanga	242	5548	0.04
Foothill	282	3951	0.07	Northeast	175	5260	0.03	Van Nuys	315	5761	0.06
Harbor	405	5309	0.08	Olympic	346	5771	0.06	West LA	102	5416	0.02
Hollenbeck	216	4317	0.05	Pacific	245	8163	0.03	West Valley	301	5292	0.06
Hollywood	325	6152	0.05	Rampart	378	5602	0.07	Wilshire	246	6306	0.04

Note: Each row corresponds to one LAPD geographic area. y_i is the number of serious crimes in 2024–2025, N_i is the total number of reported crimes, and \hat{p}_i is the frequentist estimate of the serious crime rate obtained as the sample proportion y_i/N_i .

To investigate the spatial pattern of serious crime rates, we obtained the official LAPD division boundaries from the department’s public mapping resources and overlaid our posterior estimates onto the corresponding geographic regions.

Figure 5 presents the resulting spatial summaries. Panel (a) shows the posterior mean estimates of the serious-crime rates across the 21 areas, revealing clear geographic variation in crime intensity. Panel (b) displays the posterior standard deviations, which identify regions with greater estimation uncertainty and therefore less stable local information.

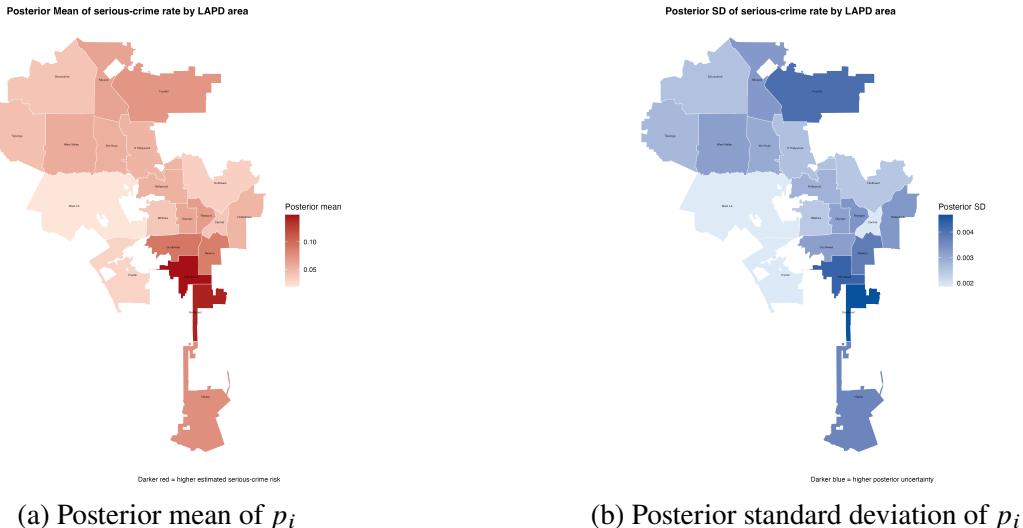


Figure 5: Spatial posterior estimates of serious crime rates across LAPD areas.

Note: The left panel displays the posterior mean serious crime rate for each area, where darker shades indicate higher estimated risk. The right panel shows the posterior standard deviation, reflecting uncertainty; darker regions correspond to greater uncertainty due to smaller sample sizes or greater variability.

To further quantify these differences and visualize estimation uncertainty, Figure 6 presents the posterior distributions for each area, ranked by their posterior mean. The red dots represent the raw sample proportions (y_i/N_i), while the boxplots depict the 95% credible intervals. Notably, the precision of these estimates varies across the spectrum: areas with lower crime rates (bottom-left) exhibit markedly narrower credible intervals compared to those with higher rates (top-right). This pattern reflects both the theoretical mean-variance relationship of the Binomial distribution—where variance increases with the proportion p —and the variation in sample sizes, where districts with larger total counts yield more precise posterior inference.

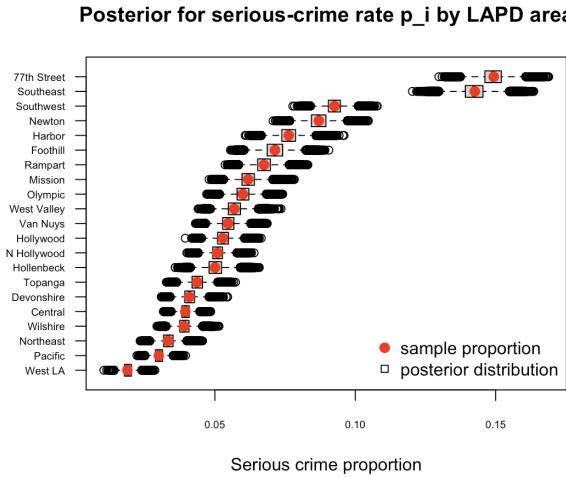


Figure 6: Posterior distributions of the area-specific serious-crime rates.

Note: Each boxplot summarizes the posterior distribution of p_i for a given LAPD area, sorted by the posterior mean. The red dots denote the empirical proportions y_i/N_i . The horizontal extent of each boxplot represents the width of the central 95% credible interval for p_i .

These posterior findings corroborate the preliminary patterns observed in our exploratory analysis in Figure 1. The distinct spatial gradient—characterized by elevated risk in southern divisions versus lower risk in western districts—persists even after hierarchical regularization. This consistency confirms that the geographic disparities identified in the raw data are driven by a robust underlying signal rather than stochastic fluctuations in local crime counts.

7 Conclusion

To sum up, this study applies a Bayesian hierarchical Beta–Binomial model to estimate serious-crime rates across the 21 LAPD geographic areas. The model shows strong MCMC convergence and produces stabilized posterior estimates that appropriately shrink extreme sample proportions, especially in regions with smaller crime counts.

The results reveal clear geographic disparities: southern districts exhibit the highest underly-

ing crime risk, while western areas remain comparatively low. By quantifying uncertainty and borrowing strength across areas, the hierarchical approach provides more reliable estimates than raw proportions and offers a statistically principled foundation for resource allocation and policy decisions.

8 Group Members Contributions

The contributions of each group member to this project are listed below:

- **Yiming Gao:** Performed MCMC convergence diagnostics (including trace plots and Gelman-Rubin statistics) and wrote the Model section.
- **Yuran Wang:** Conducted the sensitivity analysis, created the posterior spatial maps, and drafted the Results section.
- **Molin Yang:** Responsible for determining the priors and hyperpriors for the final model, performed PPC and its interpretations, and drafted the Introduction and Data sections.
- **Rongsheng Zhang:** Handled data cleaning and visualization, developed the Bayesian hierarchical model in JAGS, and compiled the Appendix and code.

All members participated in the interpretation of the results and reviewed the final manuscript.

9 AI Usage Statement

We use generative AI tools (ChatGPT and Gemini) to assist with debugging R code and refining the academic English and formatting of this report. No AI tools were used to conduct the statistical inference or interpret the findings. All final content, code, and results were verified by the group members.

References

- City of Los Angeles (2025). *Crime Data from 2020 to Present*. <https://catalog.data.gov/dataset/crime-data-from-2020-to-present>. Accessed November 2025.
- R Core Team (2025). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. URL: <https://www.R-project.org/>.
- Reich, Brian J. and Sujit K. Ghosh (2019). *Bayesian Statistical Methods*. Boca Raton, FL: CRC Press.

Appendix

Figures

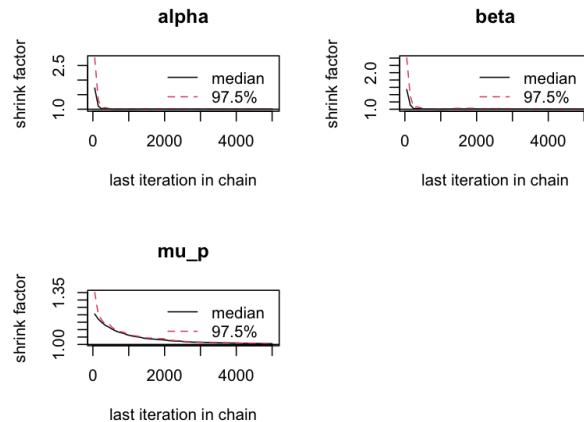


Figure 7: Gelman-Rubin Convergence Diagnostic.

Note: The shrink factor approaches 1 as the number of iterations increases, indicating that the chains have converged to the stationary distribution.

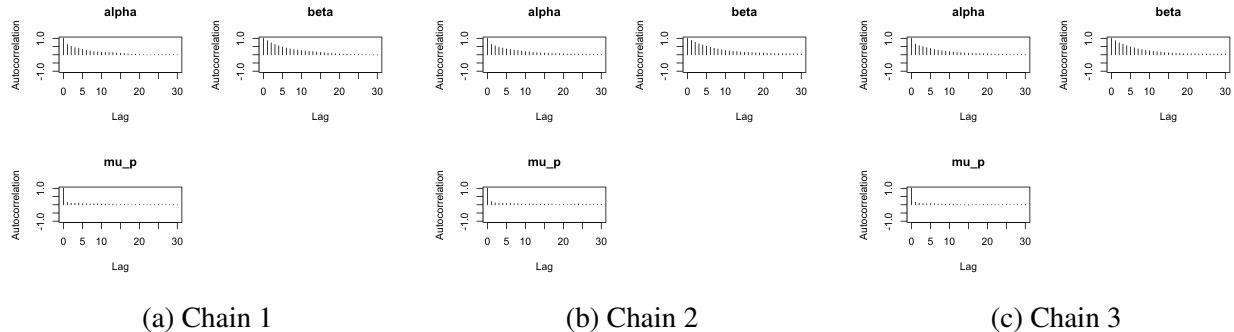


Figure 8: Autocorrelation Plots for MCMC Chains.

Note: The rapid decay of autocorrelation across all three chains confirms that the samples are effectively independent and mixing well.

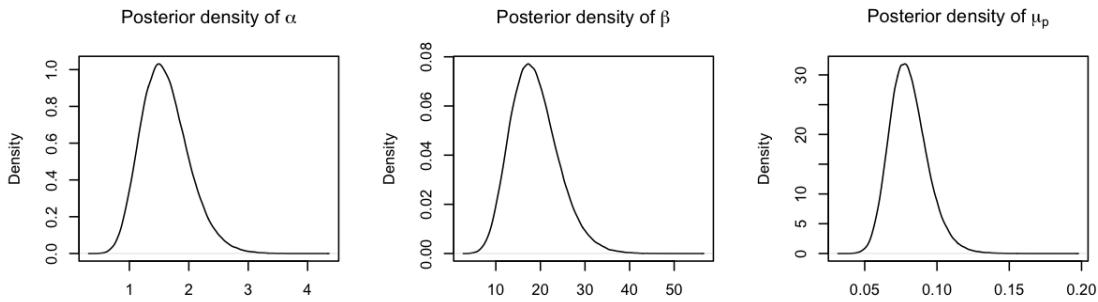


Figure 9: Posterior Density Estimates.

Note: Density plots for the hyperparameters α and β , and μ_p . The smooth, unimodal distributions further support model stability.

Code

```

1 ## -----
2
2 library(dplyr)
3 library(lubridate)
4 library(janitor)
5
6 crimes = read_csv("~/Downloads/Crime_Data_from_2020_to_Present.csv",
7   show_col_types = FALSE) %>%
8   clean_names() %>%
9   mutate(date_occ = mdy_hms(date_occ)) %>%
10  filter(year(date_occ) %in% c(2024, 2025))
11
11 head(crimes)
12 print(dim(crimes))
13
14
15
16
17 ## -----
18
18 library(dplyr)
19 library(ggplot2)
20 library(scales)

```

```
21  
22 area_serious_counts <- crimes %>%  
23   mutate(  
24     serious = if_else(crm_cd_1 < 300,  
25                           "Serious (<300)",  
26                           "Less Serious (>=300)")  
27   ) %>%  
28 group_by(area_name, serious) %>%  
29 summarise(count = n(), .groups = "drop")  
30  
31 area_serious_rate <- area_serious_counts %>%  
32   group_by(area_name) %>%  
33   mutate(rate = count / sum(count))  
34  
35 ordered_areas <- area_serious_rate %>%  
36   filter(serious == "Serious (<300)") %>%  
37   arrange(rate) %>%  
38   pull(area_name)  
39  
40 area_serious_rate$area_name <- factor(area_serious_rate$area_name,  
41                                         levels = ordered_areas)  
42  
42 ggplot(area_serious_rate, aes(x = area_name, y = rate, fill = serious))  
43   +  
44   geom_col(position = "fill") +  
44   geom_text(  
45     data = subset(area_serious_rate, serious == "Serious (<300)",  
46                   aes(label = percent(rate, accuracy = 1)),  
47                   position = position_fill(vjust = 0.5),  
48                   color = "black",  
49                   size = 4  
50   ) +  
51   coord_flip() +  
52   labs(  
53     title = "Proportion of Serious Crimes (<300) Across LAPD",  
54     x = "LAPD Geographic Area",  
55     y = "Proportion of Crimes",  
56     fill = "Crime Category"
```

```
57 ) +
58   scale_y_continuous(labels = percent_format()) +
59   theme_minimal(base_size = 14)
60
61
62
63 ## -----
64
65
66
67
68 ggplot(crimes, aes(x = lon, y = lat)) +
69   geom_point(alpha = 0.1, color = "blue", size = 0.5) +
70   labs(title = "Crime Locations in LA between 2024 and 2025", x =
71     "Longitude", y = "Latitude") +
72   theme_minimal() +
73   theme(
74     plot.title = element_text(hjust = 0.5, size = 16, face = "bold"),
75     axis.title = element_text(size = 12),
76     axis.text = element_text(size = 10)
77   )
78
79 ## -----
80
81
82 # By area_name
83 serious_by_area <- crimes %>%
84   mutate(serious = crm_cd_1 < 300) %>%    # TRUE = serious crime
85   group_by(area_name) %>%
86   summarise(
87     y = sum(serious, na.rm = TRUE),          # serious crimes count
88     N = n(),                                # total crimes in area
89     prop_serious = y / N,
```

```

90     .groups = "drop"
91 ) %>%
92 arrange(area_name)
93
94 serious_by_area
95 n_areas <- nrow(serious_by_area)
96
97 y <- serious_by_area$y
98 N <- serious_by_area$N
99
100
101 ## -----
102 # hyperprior
103 a1 <- 2; b1 <- 0.5    # alpha ~ Gamma(2, 0.5)
104 a2 <- 2; b2 <- 0.5    # beta ~ Gamma(2, 0.5)
105
106 jags_data_serious <- list(
107   y = y,
108   N = N,
109   K = n_areas,
110   a1 = a1, b1 = b1,
111   a2 = a2, b2 = b2
112 )
113
114
115 ## -----
116 serious_model_string <- "
117 model {
118   # likelihood: Binomial for serious crimes in each LAPD area
119   for (i in 1:K) {
120     y[i] ~ dbin(p[i], N[i])      # Y_i | p_i ~ Binomial(N_i, p_i)
121     p[i] ~ dbeta(alpha, beta)    # area-level serious crime rate
122   }
123"

```

```
124 # hyperpriors for Beta shape parameters (partially informative)
125 alpha ~ dgamma(a1, b1)
126 beta ~ dgamma(a2, b2)
127
128 # derived quantity: overall mean serious-crime rate across areas
129 mu_p <- alpha / (alpha + beta)
130 }
131 "
132
133 # write as bug
134 writeLines(serious_model_string, con = "serious_model.bug")
135
136
137
138 ##
```

```
139 library(rjags)
140 library(coda)
141
142 # three chain
143 inits_list_serious <- list(
144   list(alpha = 0.5, beta = 0.5),
145   list(alpha = 2.0, beta = 2.0),
146   list(alpha = 5.0, beta = 1.0)
147 )
148
149 params_pre <- c("alpha", "beta", "mu_p")
150
151 mod_serious <- jags.model(
152   file      = "serious_model.bug",
153   data      = jags_data_serious,
154   inits    = inits_list_serious,
155   n.chains = 3,
156   n.adapt  = 0
157 )
158
159 pre_samp_serious <- coda.samples(
```

```
160 model = mod_serious,
161 variable.names = params_pre,
162 n.iter = 5000
163 )
164
165 # traceplot
166 traceplot(pre_samp_serious[, "alpha"])
167 traceplot(pre_samp_serious[, "beta"])
168 traceplot(pre_samp_serious[, "mu_p"])
169
170 # Gelman-Rubin PSRF
171 gelman.diag(pre_samp_serious, autoburnin = FALSE)
172
173 gelman.plot(pre_samp_serious, autoburnin = FALSE)
174
175 # Autocorrelation plots for one chain
176 autocorr.plot(window(pre_samp_serious, 1500)[[1]])
177 autocorr.plot(window(pre_samp_serious, 1500)[[2]])
178 autocorr.plot(window(pre_samp_serious, 1500)[[3]])
179
180
181 ##
182 ##
-----
```



```
183 # rebuild the model + burn-in
184 mod_serious <- jags.model(
185   file      = "serious_model.bug",
186   data      = jags_data_serious,
187   inits    = inits_list_serious,
188   n.chains = 3,
189   n.adapt   = 0
190 )
191
192 # drop first 1500 as burn-in
193 update(mod_serious, 1500)
194
195 params_post <- c("alpha", "beta", "mu_p", "p")
```

```
196
197 post_samp_serious <- coda.samples(
198   model = mod_serious,
199   variable.names = params_post,
200   n.iter = 100000
201 )
202
203 summary(post_samp_serious)
204
205
206
207 ##
```

```
208 post_mat <- as.matrix(post_samp_serious)
209
210 alpha_draws <- post_mat[, "alpha"]
211 beta_draws <- post_mat[, "beta"]
212 mu_p_draws <- post_mat[, "mu_p"]
213
214 # Some summary
215 c(
216   mean_alpha = mean(alpha_draws),
217   sd_alpha   = sd(alpha_draws)
218 )
219
220 c(
221   mean_beta = mean(beta_draws),
222   sd_beta   = sd(beta_draws)
223 )
224
225 quantile(mu_p_draws, c(0.025, 0.5, 0.975)) # overall serious crime
226   rate 95% CI
227
228 # plot posterior density
229 par(mfrow = c(1, 3))
230 plot(density(alpha_draws),
231       main = expression(paste("Posterior density of ", alpha)),
```

```
231     xlab = "")  
232 plot(density(beta_draws),  
233       main = expression(paste("Posterior density of ", beta)),  
234       xlab = "")  
235 plot(density(mu_p_draws),  
236       main = expression(paste("Posterior density of ", mu[p])),  
237       xlab = "")  
238  
239  
240 ##  
-----  
241 # find all p[i]  
242 p_cols <- grep("^p\\\[", colnames(post_mat))  
243 p_draws <- post_mat[, p_cols]  
244  
245 p_summary <- apply(  
246   p_draws,  
247   2,  
248   function(z) c(  
249     mean = mean(z),  
250     sd = sd(z),  
251     q2.5 = quantile(z, 0.025),  
252     q50 = quantile(z, 0.50),  
253     q97.5 = quantile(z, 0.975)  
254   )  
255 )  
256  
257 p_summary <- as.data.frame(t(p_summary))  
258 p_summary$area_name <- serious_by_area$area_name  
259  
260 # Sort by posterior mean  
261 p_summary <- p_summary %>%  
262   arrange(desc(mean))  
263  
264 p_summary  
265
```

```
267 ##  
268 # Box plot with posterior mean and CI  
269 ord <- order(colMeans(p_draws), decreasing = FALSE)  
270  
271 par(mar = c(5, 10, 4, 2) + 0.1)  
272  
273 boxplot(  
274   p_draws[, ord],  
275   horizontal = TRUE,  
276   names = serious_by_area$area_name[ord],  
277   las = 1,  
278   cex.axis = 0.60,  
279   xlab = "Serious crime proportion",  
280   main = "Posterior for serious-crime rate p_i by LAPD area"  
281 )  
282  
283 points(  
284   serious_by_area$prop_serious[ord],  
285   1:ncol(p_draws),  
286   col = "red", pch = 19, cex = 1.2  
287 )  
288  
289 legend("bottomright", legend = c("sample proportion", "posterior  
distribution"),  
290   pch = c(19, 22), pt.cex = c(1.2, 1), col = c("red", "black"),  
291   bty = "n")  
292  
293  
294 ##  
295 library(sf)  
296 library(dplyr)  
297 library(ggplot2)
```

```
299 lapd_sf <- st_read("~/Downloads/LAPD_Divisions.geojson", quiet = TRUE)
300
301 area_lookup <- crimes %>%
302   distinct(area, area_name) %>%
303   arrange(area)
304
305 area_lookup
306
307 name_match <- tibble::tibble(
308   area_name = c(
309     "Central",
310     "Rampart",
311     "Southwest",
312     "Hollenbeck",
313     "Harbor",
314     "Hollywood",
315     "Wilshire",
316     "West LA",
317     "Van Nuys",
318     "West Valley",
319     "Northeast",
320     "77th Street",
321     "Newton",
322     "Pacific",
323     "N Hollywood",
324     "Foothill",
325     "Devonshire",
326     "Topanga",
327     "Olympic",
328     "Mission",
329     "Southeast"
330   ),
331   APREC = c(
332     "CENTRAL",
333     "RAMPART",
334     "SOUTHWEST",
335     "HOLLENBECK",
336     "HARBOR",
```

```
337 "HOLLYWOOD",
338 "WILSHIRE",
339 "WEST LOS ANGELES",
340 "VAN NUYS",
341 "WEST VALLEY",
342 "NORTHEAST",
343 "77TH STREET",
344 "NEWTON",
345 "PACIFIC",
346 "NORTH HOLLYWOOD",
347 "FOOTHILL",
348 "DEVONSHIRE",
349 "TOPANGA",
350 "OLYMPIC",
351 "MISSION",
352 "SOUTHEAST"
353 )
354 )
355
356 p_map <- p_summary %>%
357   select(area_name, mean) %>%
358   left_join(name_match, by = "area_name")
359
360 lapd_map <- lapd_sf %>%
361   left_join(p_map, by = "APREC")
362
363 ggplot(lapd_map) +
364   geom_sf(aes(fill = mean), color = "white", size = 0.2) +
365   geom_sf_text(aes(label = area_name), size = 1.2, color = "black") +
366   scale_fill_gradient(
367     low = "#fee5d9",
368     high = "#a50f15",
369     name = "Posterior mean"
370   ) +
371   labs(
372     title = "Posterior Mean of serious-crime rate by LAPD area",
373     caption = "Darker red = higher estimated serious-crime risk"
374   ) +
```

```
375 theme_void(base_size = 10) +
376 theme(
377   plot.title = element_text(hjust = 0.5, face = "bold"),
378   legend.position = "right"
379 )
380
381
382
383 ##
```

```
384 library(dplyr)
385 library(ggplot2)
386 library(sf)
387
388 p_sd_map <- p_summary %>%
389   select(area_name, sd) %>%
390   left_join(name_match, by = "area_name")
391
392 lapd_sd_map <- lapd_sf %>%
393   left_join(p_sd_map, by = "APREC")
394
395 ggplot(lapd_sd_map) +
396   geom_sf(aes(fill = sd), color = "white", size = 0.2) +
397   geom_sf_text(aes(label = area_name), size = 1.2, color = "black") +
398   scale_fill_gradient(
399     low = "#deebf7",
400     high = "#08519c",
401     name = "Posterior SD"
402   ) +
403   labs(
404     title = "Posterior SD of serious-crime rate by LAPD area",
405     caption = "Darker blue = higher posterior uncertainty"
406   ) +
407   theme_void(base_size = 10) +
408   theme(
409     plot.title = element_text(hjust = 0.5, face = "bold"),
410     legend.position = "right"
```

```
411 )
412
413
414 ##
```

```
415 library(rjags)
416 library(coda)
417 library(dplyr)
418
419 run_hyper_model <- function(a1, b1, a2, b2, label) {
420   data_list <- list(
421     y = y,
422     N = N,
423     K = n_areas,
424     a1 = a1, b1 = b1,
425     a2 = a2, b2 = b2
426   )
427
428   mod <- jags.model(
429     file      = "serious_model.bug",
430     data      = data_list,
431     n.chains = 3,
432     n.adapt  = 0
433   )
434
435   update(mod, 1500)
436
437   samp <- coda.samples(
438     model = mod,
439     variable.names = c("alpha", "beta", "p"),
440     n.iter = 40000
441   )
442
443   mat <- as.matrix(samp)
444   p_cols <- grep("^p\\\[", colnames(mat))
445
446   list(
```

```
447     label      = label ,
448     alpha_mean = mean(mat[, "alpha"]),
449     beta_mean  = mean(mat[, "beta"]),
450     p_means    = colMeans(mat[, p_cols])
451   )
452 }
453
454 # baseline
455 fit_base <- run_hyper_model(2, 0.5, 2, 0.5, "Gamma(2,0.5)")
456
457 fit_diffuse <- run_hyper_model(1, 0.25, 1, 0.25, "Gamma(1,0.25)")
458
459 fit_conc <- run_hyper_model(4, 1, 4, 1, "Gamma(4,1)")
460
461 # put all into a dataframe
462 sens_df <- data.frame(
463   area_name = serious_by_area$area_name,
464   base       = fit_base$p_means,
465   diffuse    = fit_diffuse$p_means,
466   conc       = fit_conc$p_means
467 )
468
469 head(sens_df)
470
471
472 cor(sens_df$base, sens_df$diffuse)
473 cor(sens_df$base, sens_df$conc)
474
475
476 plot(
477   sens_df$base, sens_df$diffuse,
478   xlab = "Posterior mean p_i (Gamma(2,0.5))",
479   ylab = "Posterior mean p_i (Gamma(1,0.25))"
480 )
481 abline(0, 1, col = "red")
482
483 # Posterior Predictive Distribution
484 y_rep <- matrix(NA, nrow = nrow(post_mat), ncol = n_areas)
```

```
485 for (i in 1:nrow(post_mat)) {  
486   for (j in 1:n_areas) {  
487     y_rep[i, j] <- rbinom(1, N[j], post_mat[i, paste0("p[", j, "]")])  
488   }  
489 }  
490  
491 T_obs <- sum(y)  
492 T_rep <- apply(y_rep, 1, sum)  
493  
494 # Bayesian p-value  
495 mean(T_rep >= T_obs)  
496  
497 hist(T_rep, breaks = 50, col = "lightblue",  
498       main = "Posterior Predictive Distribution",  
499       xlab = "Total serious crimes")  
500 abline(v = T_obs, col = "red", lwd = 2)
```