Active Label Acquisition with Personalized Incentives in Assortment Optimization

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Agenda

- **Active Label Acquisition in Customer Survey**
- Regret of the Prediction Model
- Value of Information
- Upper bound for the value of information
- Guarantees for Assortment Optimization
- Numerical Experiments

Personalized decisions based on customer features

For each customer:

Feature → Prediction model → Preferences → Maximize the revenue → OM decisions

Personalized decisions based on customer features

For each customer:

Feature → **Prediction model** → Preferences → **Maximize the revenue** → OM decisions

↑

Personalized information of customers

Personalized decisions based on customer features

For each customer:

Feature → **Prediction model** → Preferences → **Maximize the revenue** → OM decisions

↑

Preferences between different products,
e.g. utility of each product

Personalized decisions based on customer features

For each customer:

Feature → Prediction model → Preferences → Maximize the revenue → OM decisions

↑

Assortment problem, product selection…

Personalized decisions based on customer features

For each customer:

Feature → **Prediction model** → Preferences → **Maximize the revenue** → OM decisions

↑

How to build a prediction model?

Personalized decisions based on customer features

For each customer:

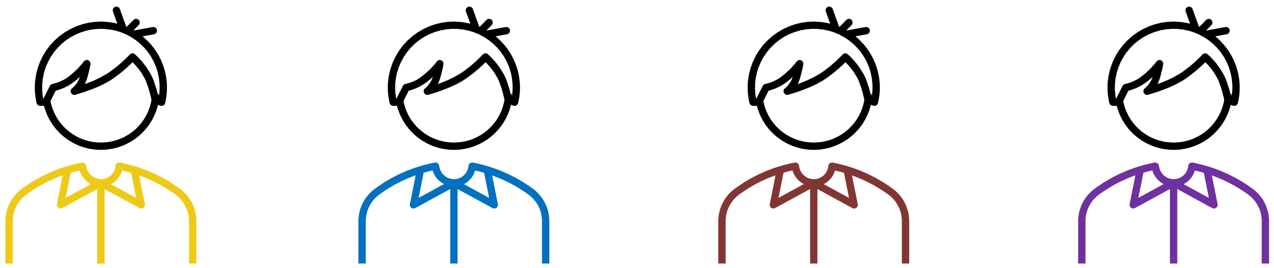
Feature → Prediction model → Preferences → Maximize the revenue → OM decisions

↑

Training set: (feature, preferences)
Preferences: label of the customer

How to obtain the true preferences of customers

- Survey customers:
  - Provide a comprehensive survey to customers
  - The response from one customer can reveal the true utility vector (with noise)



  - *“Without costly incentives, most consumers rarely provide this valuable feedback”*

---- by Maytal Saar-Tsechansky et al. (2009)

Incentives in active label acquisition

- Active label acquisition with personalized incentives:

Customer t arrives with type ξ_t →

Incentives in active label acquisition

- Active label acquisition with personalized incentives:

Customer t arrives with type ξ_t → Offer some **incentive** c_t to customer t for completing the survey →

Incentives in active label acquisition

- Active label acquisition with personalized incentives:

Customer t arrives with type ξ_t → Offer some **incentive** c_t to customer t for completing the survey → Customer t decides whether to accept the survey to get the incentive

- Probability of accepting the survey $p(c)$ depends on our offered incentives
  - ➢ More incentives we offer → Larger probability of taking the survey

- Can we provide same incentives to all customers?

Benefit of personalized incentives

Provide more incentives to representative customers

Compared to the fixed incentive policy, personalized incentives can:

- ✓ Reduce the size of the training set
- ✓ Reduce the label cost (cumulative incentives)

How to decide personalized incentives?

Personalized decisions based on customer features

Feature → **Prediction model** → Preferences → **Maximize the revenue** → OM decisions

Select customers based on the prediction errors for preferences

Smaller prediction error ? Higher revenue

Reasons:

If the prediction error is small enough to determine the true optimal decisions, then a smaller prediction error will lead to the same decision and obtain the same revenue

Personalized decisions based on customer features

Feature → Prediction model → Preferences → Maximize the revenue → OM decisions

Select customers based on the ~~prediction errors for preferences~~

Risk of OM decisions

Personalized decisions based on customer features

Feature → Prediction model → Preferences → Maximize the revenue → OM decisions

~~Select customers~~ based on the ~~prediction errors for preferences~~

Personalized Incentives

Risk of OM decisions

? Behaviors of human

? Risk is a nonlinear of decisions

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- Active Label Acquisition in Customer Survey

➢ **Regret of the Prediction Model**

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Regret of prediction

In the predict-then-optimize problems:

- Regret of the prediction:
  - Highest possible revenue – the actual revenue of our decisions based on current prediction

❑ Assortment optimization problem:

➢ Revenue of the best assortment – actual revenue of our assortment

Regret of the prediction model

- Type of customer: $\xi \in \{1, \dots, m\}$
- Utility vector: $y \in \mathbb{R}^d$
- Decision vector: $w \in \{0,1\}^d$
- Revenue function: $g(w, \mathbb{E}[y|\xi])$

  – $w^*(y)$: Best decision given the prediction. $w^*(y) = \arg\max_w g(w, y)$

Regret of prediction $\hat{y}$:

$$\ell(\hat{y}, \mathbb{E}[y|\xi]) := \boxed{g(w^*(\mathbb{E}[y|\xi]), \mathbb{E}[y|\xi])} - \boxed{g(w^*(\hat{y}), \mathbb{E}[y|\xi])}$$

Highest revenue   Actual revenue

- Given a predictor h, the expected regret of the predictor:

$$\text{Regret}(h) = \mathbb{E}[\ell(h(\xi), \mathbb{E}[y|\xi])]$$

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Tradeoff during the survey process

Comprehensive cost at time T:

$$\mathcal{C}(\boldsymbol{c}_T, h_T) := \underbrace{\sum_{t=1}^{T} c_t \mathbb{I}\{\text{Customer } t \text{ accept the offer for survey}|\, c_t\}}_{\text{Label cost}} + \underbrace{\beta \cdot \text{Regret}(h_T)}_{\text{Risk of the prediction model } h_T}$$

Market size

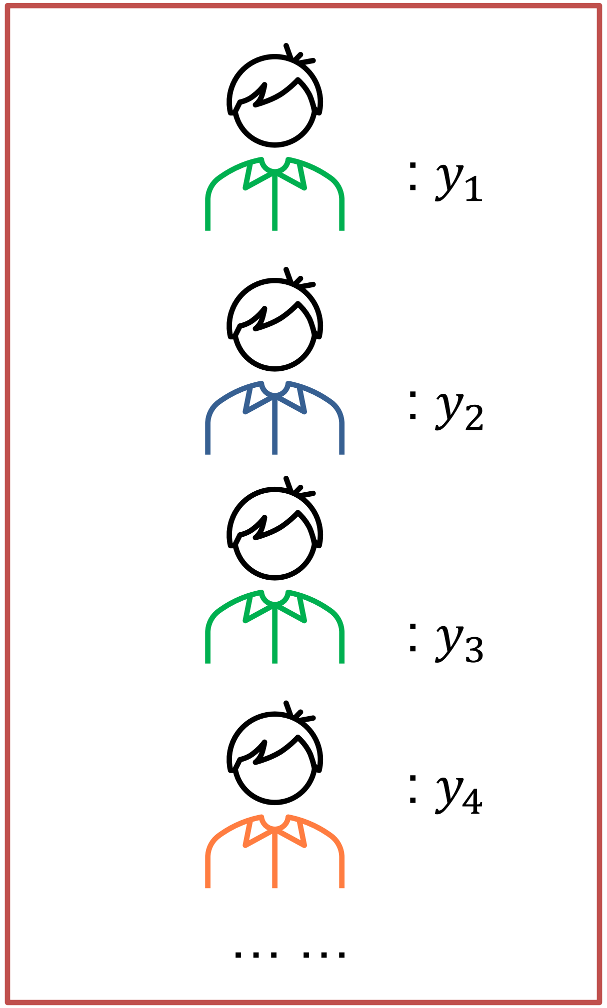
Objective: Minimize the expectation of the comprehensive cost

Tradeoff of incentive c_t :

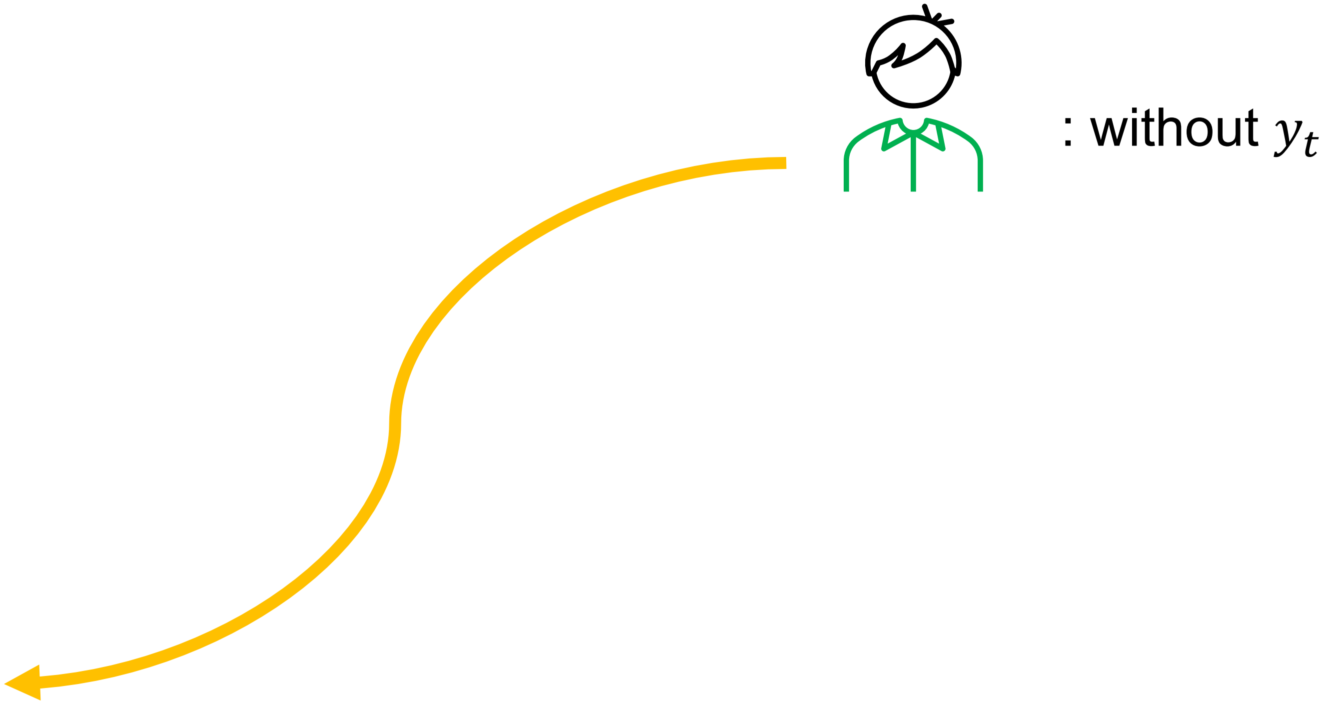
- Too small: Little probability of taking the survey → Lack of data → Regret(h_T) will be large
- Too large: Waste of label cost（incentive）

Value of information

- Training set:



Value of information: The amount of risk reduction of adding a new customer before knowing the true preference



Value of information

Value of information $V(\xi_t;\, \mathcal{S}_{t-1})$

New predictor h_t when including (ξ_t, y_t)

$$V(\xi_t; \mathcal{S}_{t-1}) := \boxed{\beta \cdot \text{Regret}(h_{t-1})} - \boxed{\beta \cdot \mathbb{E}_{y_t}\left[\text{Regret}\left(\boxed{f\left(\mathcal{S}_{t-1} \bigcup \{(\xi_t, y_t)\}\right)}\right) \middle| \xi_t\right]}$$

Risk of predictor h_{t-1}

Expected risk of h_t when knowing (ξ_t, y_t)

It quantifies the expected risk reduction of including the customer t in the training set before knowing y_t

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➢ **Upper bound for the value of information**

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Assortment Problem

- Customers have the no-purchase option 0.
- Suppose y_i follows Gumbel distribution with variance σ
- By MNL choice model, the purchase probability for product i is:

$$\frac{e^{\overline{y_i}/\sigma}}{1+\sum_j e^{\overline{y_j}/\sigma}}$$

- Suppose the price of product i is p_i
- Maximize the revenue of the assortment:

$$\max_{w\in\mathbb{B}^d,\boldsymbol{u}\in\mathbb{R}^d} \frac{\sum_{i\in[d]} u_i p_i w_i}{1+\boldsymbol{u}^T w}$$

$$s.t. \qquad w^T\mathbf{1} = z,$$

$$u_i = e^{\overline{y_i}^i/\sigma}, \qquad \forall i \in [d]$$

Incentives: upper bound for the value of information

Distance to degeneracy:

$$\nu_S(\hat{y}) := \inf_{w^*(y) \neq w^*(\hat{y})} \{\|\hat{y} - y\|\}$$

- It is defined as the distance between the prediction $\hat{y}$ and the closest vector y that leads to a different decision

Suppose the prediction error for $\hat{y}$ is $\rho(\xi)$, then an upper bound for the value of information is:

$$\varkappa\beta\sqrt{2\min\{z, d-z\}}\,\boxed{\mu(\xi)} \cdot \boxed{\rho(\xi)} \cdot \boxed{\mathbb{I}\{\nu_S(\hat{y}) \leq \rho(\xi)\}}$$

Test set distribution

Prediction error
(Training set distribution)

Whether the true optimal
decision is determined

Insights from the upper bound of value of information

Upper bound

$$\varkappa\beta\sqrt{2\min\{z,d-z\}}\,\boxed{\mu(\xi)}\cdot\boxed{\rho(\xi)}\cdot\boxed{\mathbb{I}\{\nu_S(\hat{y})\leq\rho(\xi)\}}$$

1. If one feature has a higher probability in the test set
   - Its value of information gets larger

2. If one feature has a larger proportion in the training set
   - The prediction error for this feature gets smaller
   - The value of information gets smaller

3. If the prediction error for one sample is smaller than $\nu_S(\hat{y})$:
   - The optimal decision for this sample has been determined
   - Regret for this type of customer is zero
   - We will stop surveying this type of customers

Personalized incentives based on the upper bound of value of information

Range of offered incentive: $c_t \in \{0\} \cup [c_{min}, c_{max}]$

Given a type of customer ξ:

If the upper bound of value of information $\leq c_{min}$:

- Provide zero incentive
- Ignore the feedback of this customer

Otherwise, we offer some incentives between $[c_{min}, c_{max}]$

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- ➢ **Guarantees for Assortment Optimization**
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Near-degeneracy function

Near-degeneracy function Ψ :

$$\Psi(\rho) := \mathbb{P}(\nu_S(\mathbb{E}[y|\xi]) \leq \rho)$$

Ψ describes the difficulty in distinguishing the optimal decision from the sub-optimal decision at a certain prediction error level ρ

Guarantees for the problem

Theorem:

After T iterations, the cumulative label cost is at most $\min\left\{\tilde{O}\left(\sum_{t=1}^{T}\Psi(t^{-\frac{1}{2}})\right), \tilde{O}\left(T^{\frac{1}{2}}\right)\right\}$

- Low-noise condition: For some large $\rho_0 > 0$ and $\kappa > 0$, the near-degeneracy function satisfies:

$$\Psi(\rho) \leq \left(\frac{\rho}{\rho_0}\right)^{\kappa}$$

Low-noise condition is closely related to Hu et al. 2022 and Tsybakov's noise condition

Under low-noise conditions:

✓ The cumulative label cost is at most $\tilde{O}\left(T^{1-\kappa/2}\right)$

Comparison with supervised learning

Under the low-noise condition with $\kappa > 2$:

- ➢ Personalized incentives policy requires use finite samples to achieve zero risk
- ➢ Fixed incentive policy requires infinite samples to achieve zero risk

Theorem:

Under various conditions of c_{min}, regarding the comprehensive cost:

✓ Our personalized incentive policy $\leq$ fixed incentive policy

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- ➢ **Numerical Experiments**

Numerical Experiments: Assortment optimization with contextual information

Use synthetic data:

- Number of product: 10
- Number of types of customers: 5

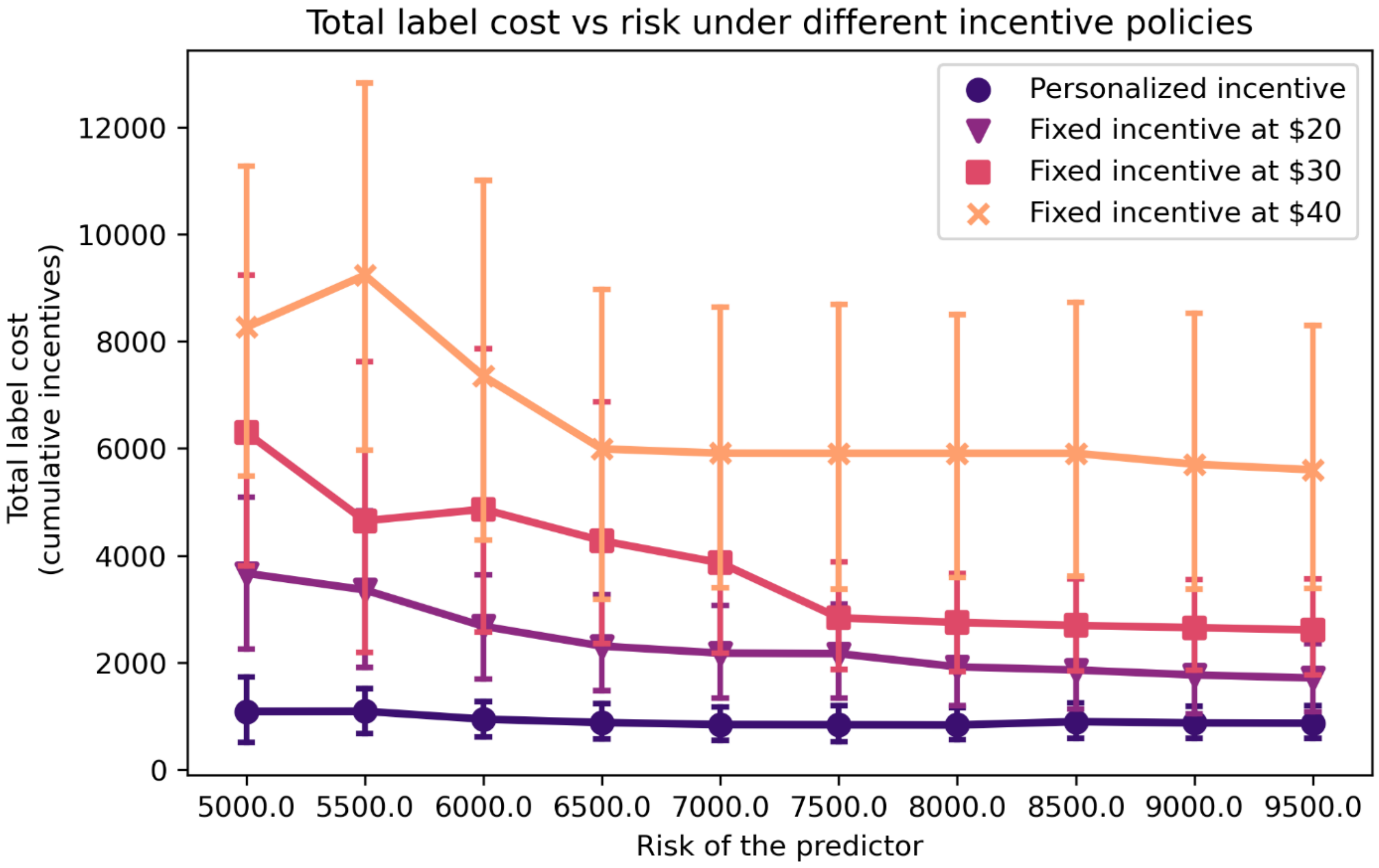
- Dimension of features within each type: 8
- Each type has its own feature and prediction model
- Within each type, we assume the true prediction model is linear

- The offered incentive is either 0 or some value between $[\$20, \$40]$

- $p(c)$ is a linear function between $(\$20, 0.3)$ and $(\$40, 0.9)$

Results: Assortment optimization

- Observation: To achieve the same level of risk, the personalized incentive policy requires much less label cost



Results: Assortment optimization

- When ensuring the excess risk is less than 5000:

	Personalized incentive	Fixed incentive at $20	Fixed incentive at $30	Fixed incentive at $40
Required label cost	1088	3668 (-70%)	6295 (-79%)	8262 (-87%)
Required number of surveyed customers (Size of training set)	30	184 (-84%)	210 (-86%)	206 (-85%)

Thank you

Mo Liu

Personalized incentive given the value of information

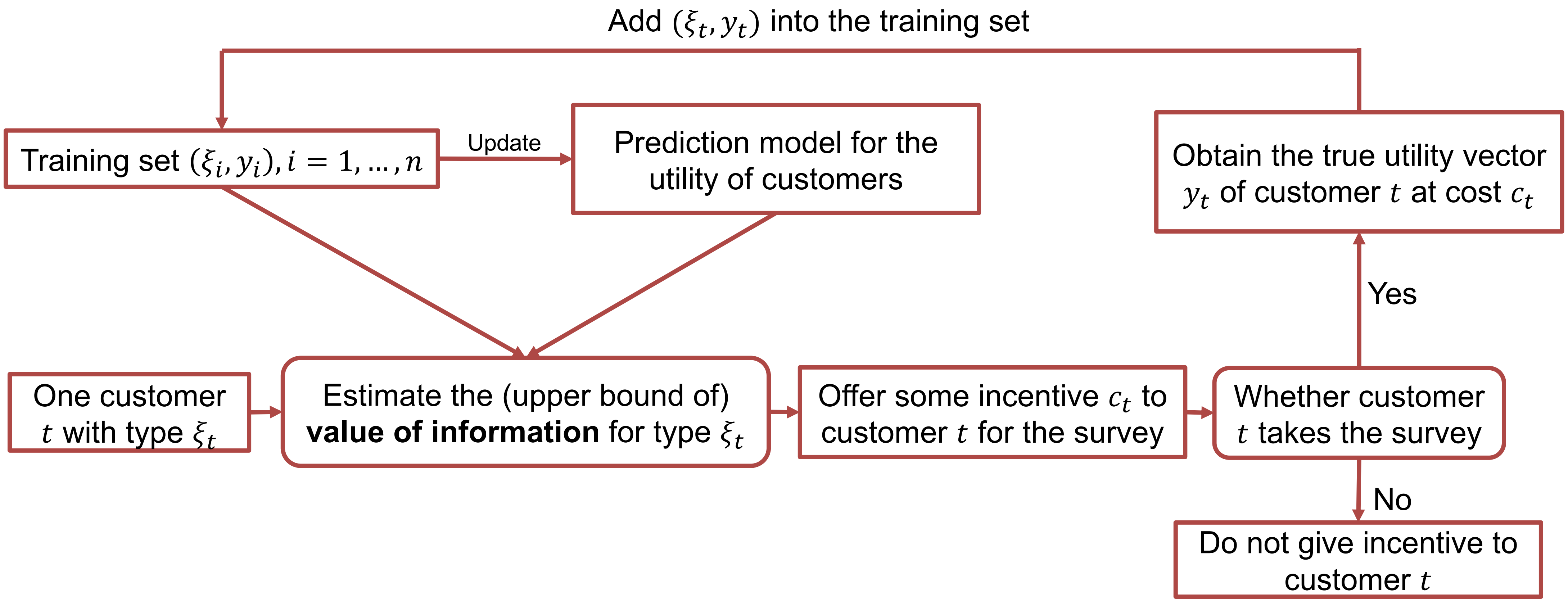
Range of offered incentive: $c_t \in \{0\} \cup [c_{min}, c_{max}]$

- If the offered incentive is zero, we ignore the feedback of this customer
- If the offered incentive is nonzero, the optimal incentive is c^*

$$c^*(V(\xi_t;\, \mathcal{S}_{t-1}), p) \coloneqq \arg \min_{c \in [c_{min}, c_{max}]} \{p(c_t)[c_t - V(\xi_t;\, \mathcal{S}_{t-1})]\}$$

Assumption of $p(c)$: Increasing function with $p(c_{min}) > 0$

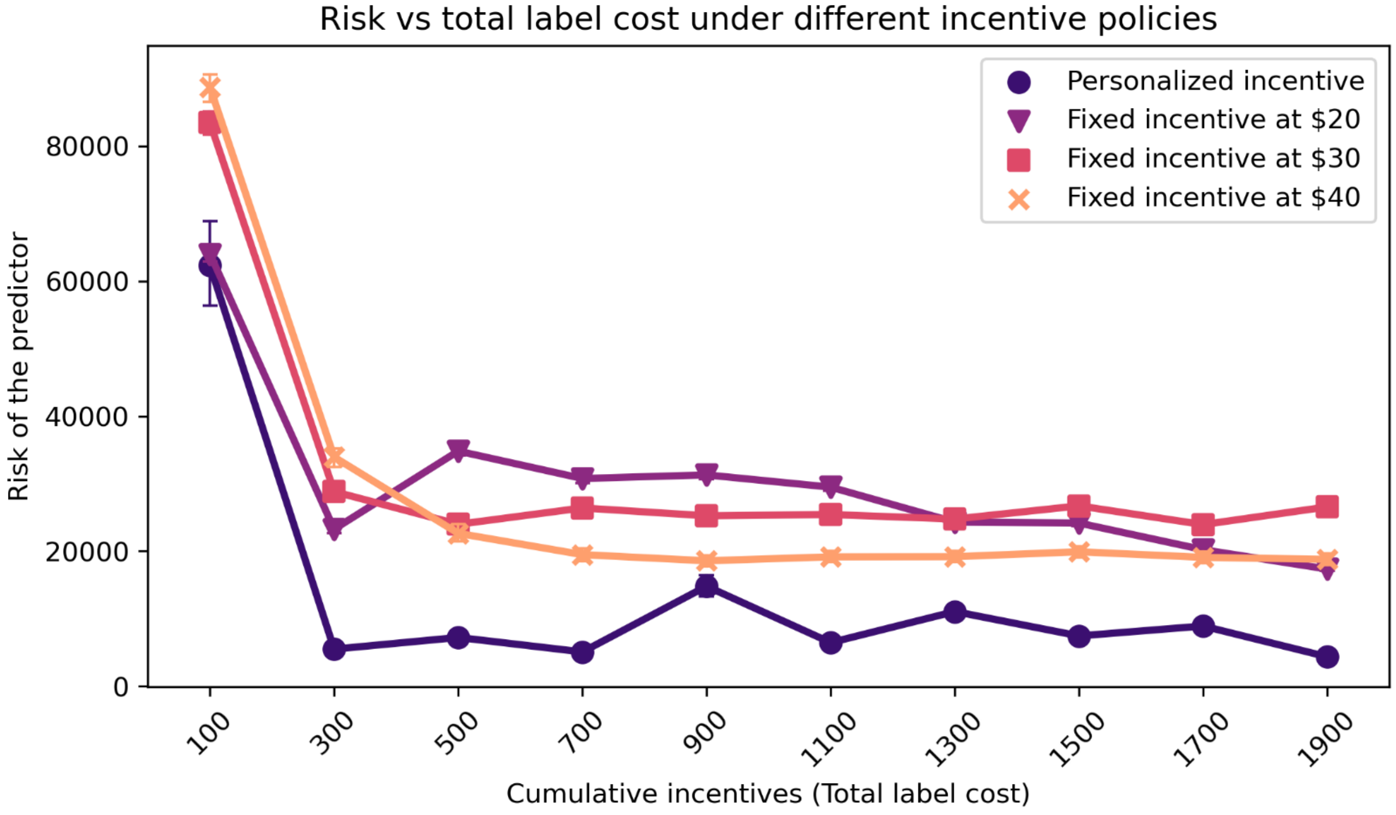
Active label acquisition using value of information



If the upper bound is less than c_{min}, we do not offer any incentive

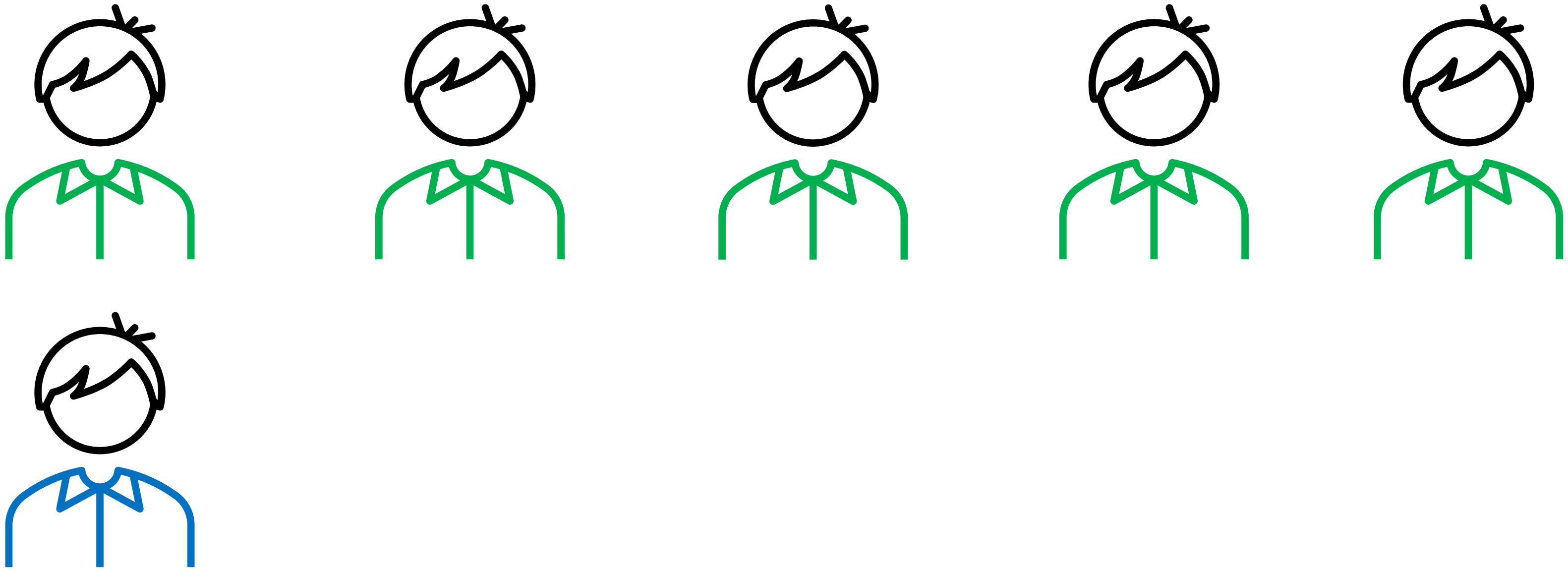
Results: Assortment optimization

- Observation: Using the same amount of label cost, the personalized incentive policy achieves much smaller risk.



Factors that influences the value of information

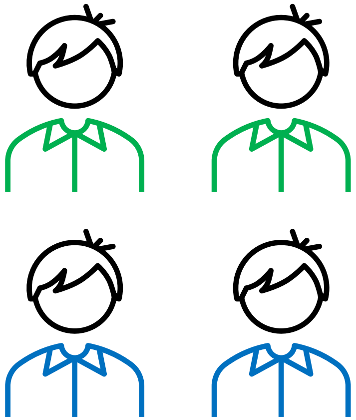
- Training set: (Customers who have taken the survey)



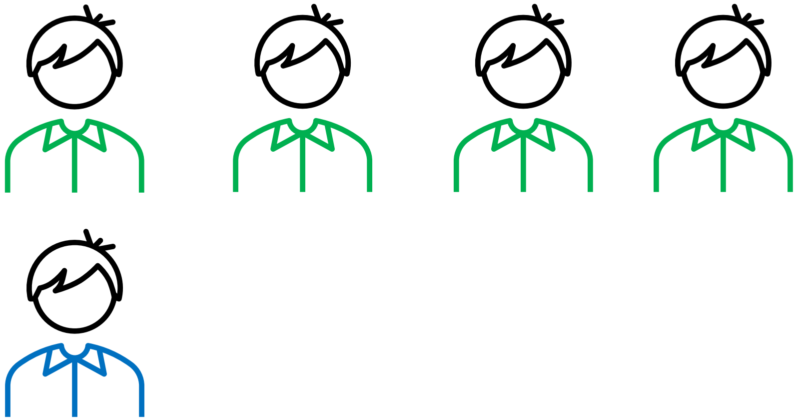
- In the future, which type of customer should receive higher incentive? green shirts or blue shirts?

Factors that influences the value of information

- Training set:



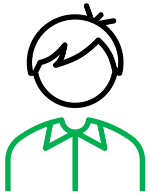
In the test set (future market):



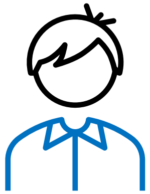
- In the future, which type of customer should receive higher incentive? green shirts or blue shirts?

Factors that influences the value of information

- Training set:



Flavor	Average utility
	2
	9
	4



Flavor	Average utility
	6
	7
	4

- In the future, which type of customer should receive higher incentive? green shirts or blue shirts?

Assortment optimization with MNL model

- Assortment optimization problem:
  - Different ice creams have different prices
  - Customer knows their own utility
  - Customer can choose no-purchase option

Flavor	Utility
	8
	5
	4
No-purchase	6

  - The regret is measured in the risk of revenue:
    - The revenue of the optimal assortment – the revenue of our assortment

Personalized incentive

- In summary, the following factors influence the value of information:
  - Feature distribution in the training set
  - Feature distribution in the real world
  - Difficulty in distinguishing the optimal decisions from sub-optimal decisions
  - Market size
- Besides the value of information, the following factors should also be considered:
  - Probability of taking the survey of customers $p(c)$
  - Range of the offered incentives

Personalized incentive by the upper bound of value of information

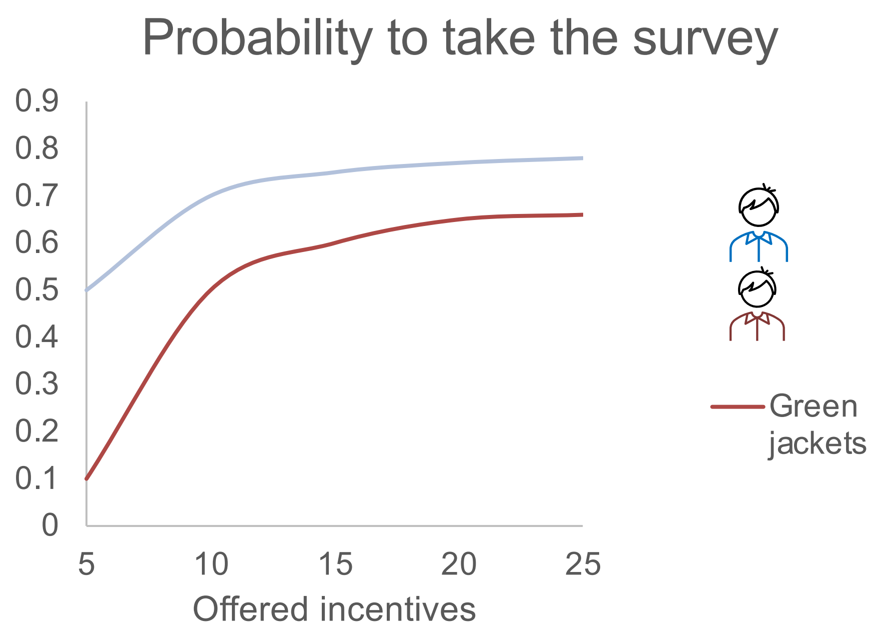
- For any upper bound $U(\xi_T, \mathcal{S}_{T-1})$ for $V(\xi_T, \mathcal{S}_{T-1})$, when $p(c)$ is a concave function, we have:

$$c^*\big(V(\xi_T, \mathcal{S}_{T-1})\big) \le c^*\big(U(\xi_T, \mathcal{S}_{T-1})\big) \le U(\xi_T, \mathcal{S}_{T-1})$$

- Use $U(\xi_T, \mathcal{S}_{T-1})$ as incentive:
  - ✓ Achieve the same order of bound as $c^*\big(U(\xi_T, \mathcal{S}_{T-1})\big)$
  - ✓ Avoid assuming the exact form of $p(c)$
  - ✓ Robust to the form of $p(c)$.

- Assumption of $p(c)$: Increasing function with $p(c_{min}) > 0$.

Personalized incentive



- In the future, which type of customer should receive higher incentive? green shirts or blue shirts?

$$c^*(V(\xi_t;\mathcal{S}_{t-1}),p) := \underset{c\in\{0\}\cup[c_{\min},c_{\max}]}{\arg\min} \{p(c)\,(c - V(\xi_t;\mathcal{S}_{t-1}))\}$$

Example of product selection: ice cream recommendation

- Suppose I own an ice cream shop, and design the following new flavors:

Honey

\+
Special Creams

Chocolate

\+
Special Creams

Strawberry

\+
Special Creams

Recommend a new flavor to a customer

- The customer does not know the utility for each new flavor, so he asks for my recommendation

Honey

+
Special Creams

Chocolate

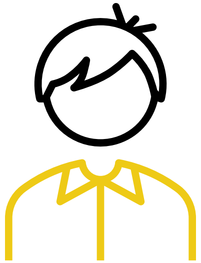
+
Special Creams

Strawberry

+
Special Creams

Ice cream recommendation

- Recommend ice cream based on the type of customers

➢ Customer: 

- Suppose the utilities of three flavors are some number between 1 and 10
- I have a prediction model for customers with yellow shirts:

Flavor	Prediction for utility for the customers with yellow shirts
	8
	5
	4

Ice cream recommendation

- If I only recommend one flavor, which should I pick to maximize his satisfaction level?

Flavor	Prediction for utility
	8
	5
	4

Ice cream recommendation

- If I only recommend one flavor, which should I pick to maximize his satisfaction level?

Flavor	Prediction for utility
	8
	5
	4

- What if the customer's utilities are below?

Flavor	His true utility
	6
	9
	4

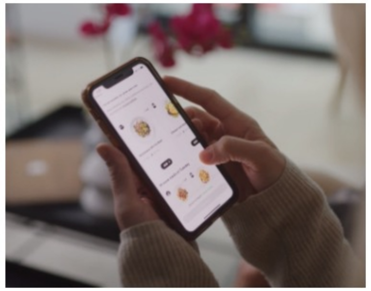
- What is the regret of my recommendation? 9 – 6 = 3

Examples of product selection problem

1. CookUnity: www.cookunity.com

How it works

Small batch meals crafted by top culinary talent. Delivered to your door each week.



01
Set your preferences

Let us know what you love to eat, then choose your meal plan -- from 4 to 16 meals per week.

02
Choose your meals

Our chefs are in constant creation mode. Every weekly menu boasts new craveable meals for you to order.

03
Heat and plate

Every meal comes with Chef heating instructions. Set the table, plate your meal, and savor the experience.

04
Repeat

Choose something new every week or stick with your staples. We'll be in the kitchen cooking up your next mouth-watering meal.

2. Credit card promotions category

Numerical Experiments

- Product-selection:
- Real-world survey data on the students in one university
- Goal: Recommend two student interest groups for each freshman

$$\mathbb{P}(\text{Take the survey when given incentive } c) = \frac{c - C_{\min}}{C_{\max} - C_{\min}} \times 0.6 + 0.2$$

- Interest group: *art and culture, science and technology, social welfare and diversity, entrepreneurship, sports, and others*
- Students type: department

Results of the product selection

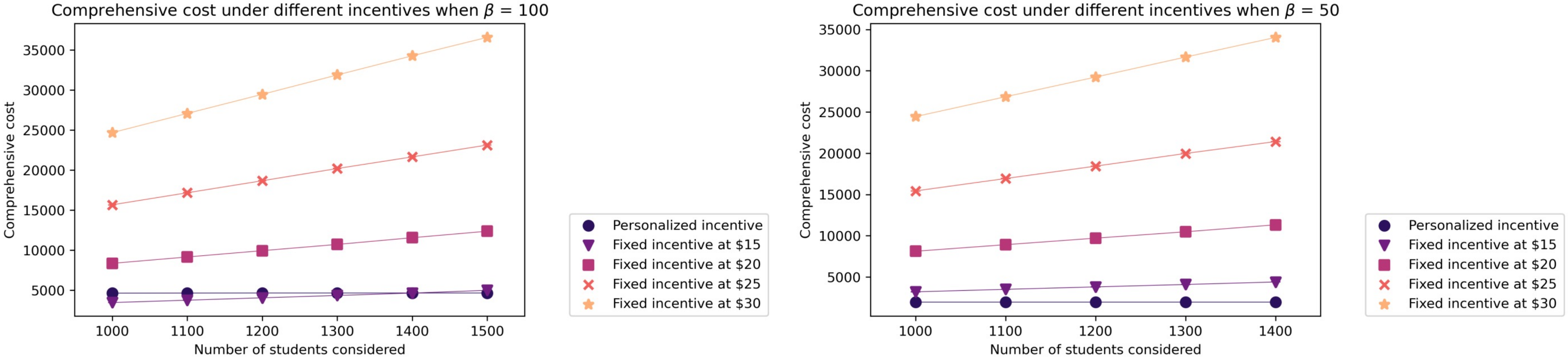


Figure 3 **Comprehensive cost of the active label acquisition algorithms with different market sizes β.**

Results of the product selection

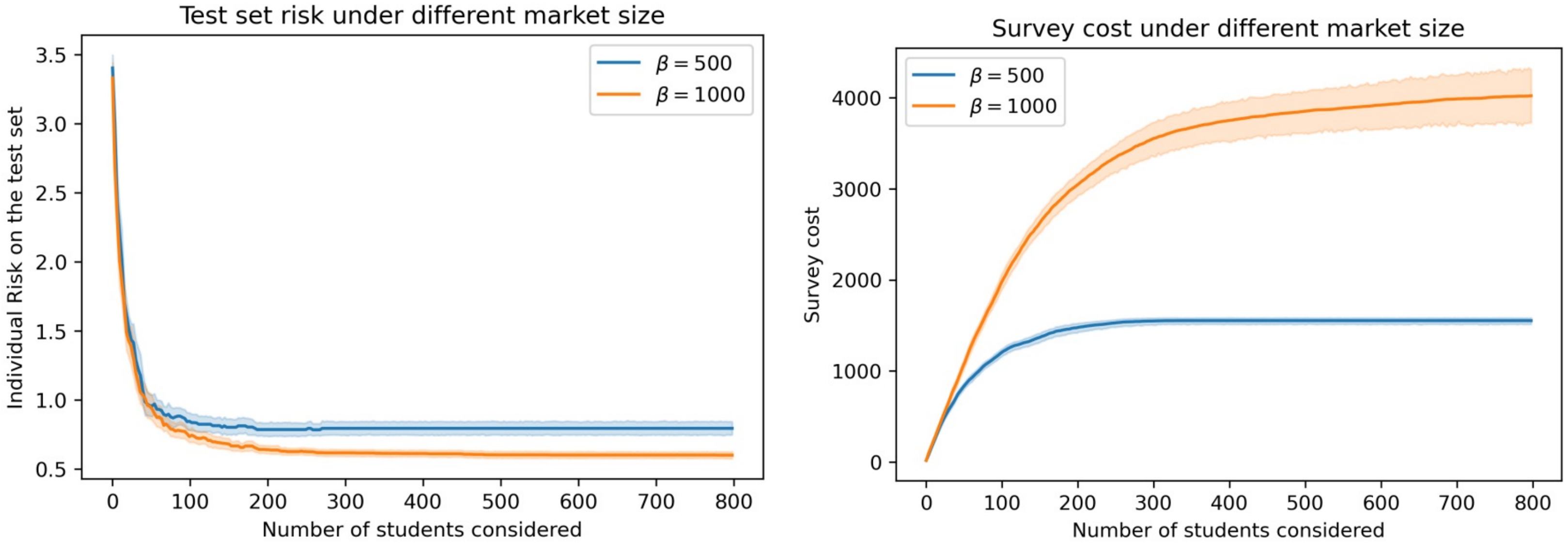


Figure 4 **Risk and the total survey cost with different market sizes β.**