Multiple Importance Reweighting for Path Guiding Supplemental Document

ZHIMIN FAN, State Key Lab for Novel Software Technology, Nanjing University, China
YIMING WANG, State Key Lab for Novel Software Technology, Nanjing University, China
CHENXI ZHOU, State Key Lab for Novel Software Technology, Nanjing University, China
LING-QI YAN, University of California, Santa Barbara, United States of America
YANWEN GUO, State Key Lab for Novel Software Technology, Nanjing University, China
JIE GUO*, State Key Lab for Novel Software Technology, Nanjing University, China

CCS Concepts: • **Computing methodologies** → **Ray tracing**.

Additional Key Words and Phrases: Path guiding

ACM Reference Format:
Zhimin Fan, Yiming Wang, Chenxi Zhou, Ling-Qi Yan, Yanwen Guo, and Jie Guo. 2025. Multiple Importance Reweighting for Path Guiding Supplemental Document. *ACM Trans. Graph.* 44, 4, Article 113 (August 2025), 2 pages. https://doi.org/10.1145/3618360

1 CONSISTENCY

According to Marin et al. [2019], the reweighting could produce asymptotically unbiased estimations as the number of iterations approaches infinity, under the following conditions:

- **Sample allocation**: The number of samples per iteration increases rapidly enough so that $\sum_{i=1}^{\infty} \frac{1}{n_i} < \infty$. This condition is satisfied when using an exponentially growing number of samples, i.e., $n_i = 2^{i-1}$.
- **Distribution optimality**: The parameter θ^* of the optimal proposal distribution $p^*(\overline{\boldsymbol{x}}) = p(\overline{\boldsymbol{x}}; \theta^*)$ can be expressed as an integral of the product of the unnormalized target distribution $q(\overline{\boldsymbol{x}}) = ||f(\overline{\boldsymbol{x}})||$ and a known function $h(\overline{\boldsymbol{x}})$, i.e.,

$$\theta^* = \int_{\mathcal{P}} q(\overline{\boldsymbol{x}}) h(\overline{\boldsymbol{x}}) \mathrm{d}\overline{\boldsymbol{x}}. \tag{1}$$

- **Distribution continuity**: For all $\overline{\boldsymbol{x}}$, $p(\overline{\boldsymbol{x}}; \theta)$ is continuous to θ and lower semicontinuous[1] to $(\overline{\boldsymbol{x}}, \theta)$. Furthermore, the above h should satisfy

$$\int_{\mathcal{P}} \frac{f^2(\overline{\boldsymbol{x}})}{q^2(\overline{\boldsymbol{x}})} \frac{h^2(\overline{\boldsymbol{x}})}{p(\overline{\boldsymbol{x}}; \theta)} \mathrm{d}\overline{\boldsymbol{x}} < \infty \tag{2}$$

for all θ and depends continuously on θ.

For example, the quadtree model in PPG [Müller et al. 2017] satisfies the above conditions[2].

2 ADDITIONAL RESULTS

We provide full-size images of certain experiments in Figs. 1 and 2. Note that the computational overhead is more severe for high sample rates, so the improvement is minimal in equal-time setting.

REFERENCES

Jean-Michel Marin, Pierre Pudlo, and Mohammed Sedki. 2019. Consistency of Adaptive Importance Sampling and Recycling Schemes. *Bernoulli* 25, 3 (Aug. 2019).
Thomas Müller, Markus Gross, and Jan Novák. 2017. Practical Path Guiding for Efficient Light-Transport Simulation. *Computer Graphics Forum* 36, 4 (July 2017), 91–100.

*Corresponding author.

[1] Mathematically, a function $f : X \to \mathbb{R}$ is *lower semicontinuous* if for every $x_0 \in X$ and $y < f(x_0)$, there exists a neighborhood U of x_0, $f(x) > y$ for all $x \in U$.

Authors' addresses: Zhimin Fan, zhiminfan2002@gmail.com, State Key Lab for Novel Software Technology, Nanjing University, Nanjing, China; Yiming Wang, 02yimingwang@gmail.com, State Key Lab for Novel Software Technology, Nanjing University, Nanjing, China; Chenxi Zhou, 502023320017@smail.nju.edu.cn, State Key Lab for Novel Software Technology, Nanjing University, Nanjing, China; Ling-Qi Yan, lingqi@cs.ucsb.edu, University of California, Santa Barbara, Santa Barbara, United States of America; Yanwen Guo, ywguo@nju.edu.cn, State Key Lab for Novel Software Technology, Nanjing University, Nanjing, China; Jie Guo, guojie@nju.edu.cn, State Key Lab for Novel Software Technology, Nanjing University, Nanjing, China.

[2] As the number of samples tends to infinity, so does the number of samples gathered within each directional quad-tree, yielding a converged structure of each quad-tree. Hence, θ consists of the irradiance values in each node of quadtrees. Since the value of a quadtree node is also an integral in path space with bounded direction on a specific vertex, h is a known function in that region. For distribution continuity, since defensive sampling is always used in path guiding, $1/p(\overline{\boldsymbol{x}}; \theta)$ is finite for all θ. $f(\overline{\boldsymbol{x}})$ and $h^2(\overline{\boldsymbol{x}})$ is also finite. Consequently, the above integral is finite. The continuity is also evident since $p(\overline{\boldsymbol{x}}; \theta)$ is continuous to θ and $p(\overline{\boldsymbol{x}}; \theta) > 0$.

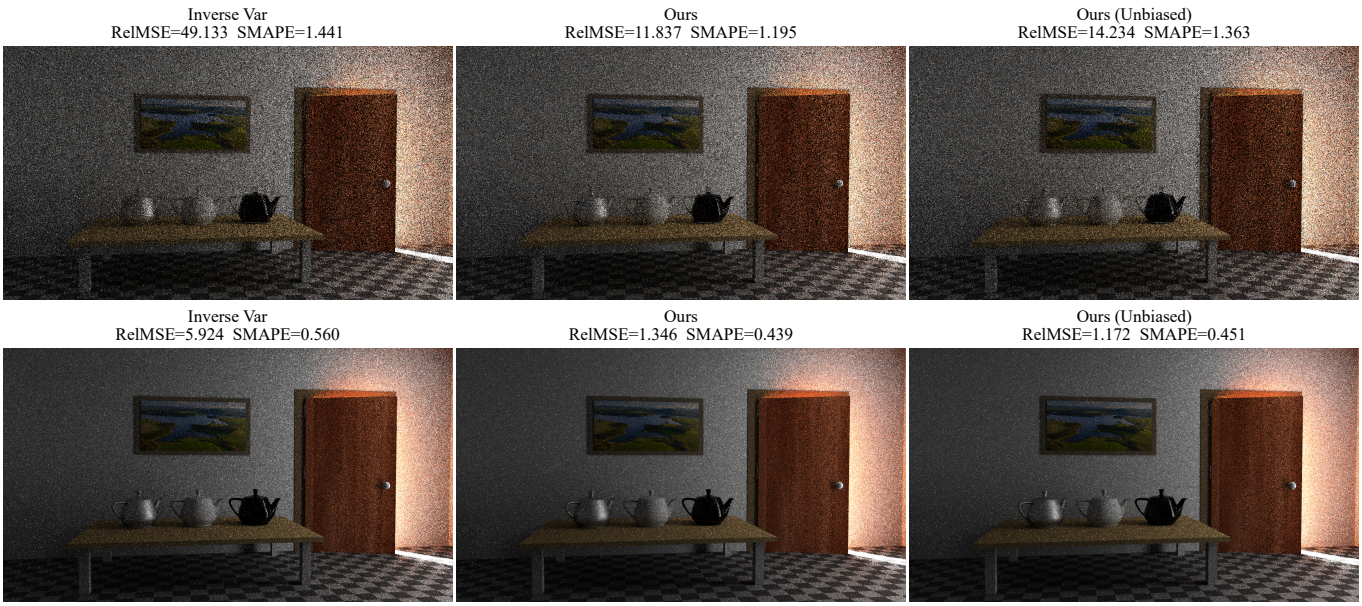

Fig. 1. Validation of the benefit of reweighting through a decorrelated (unbiased) variant at equal sample rates (20 spp and 64 spp, respectively).

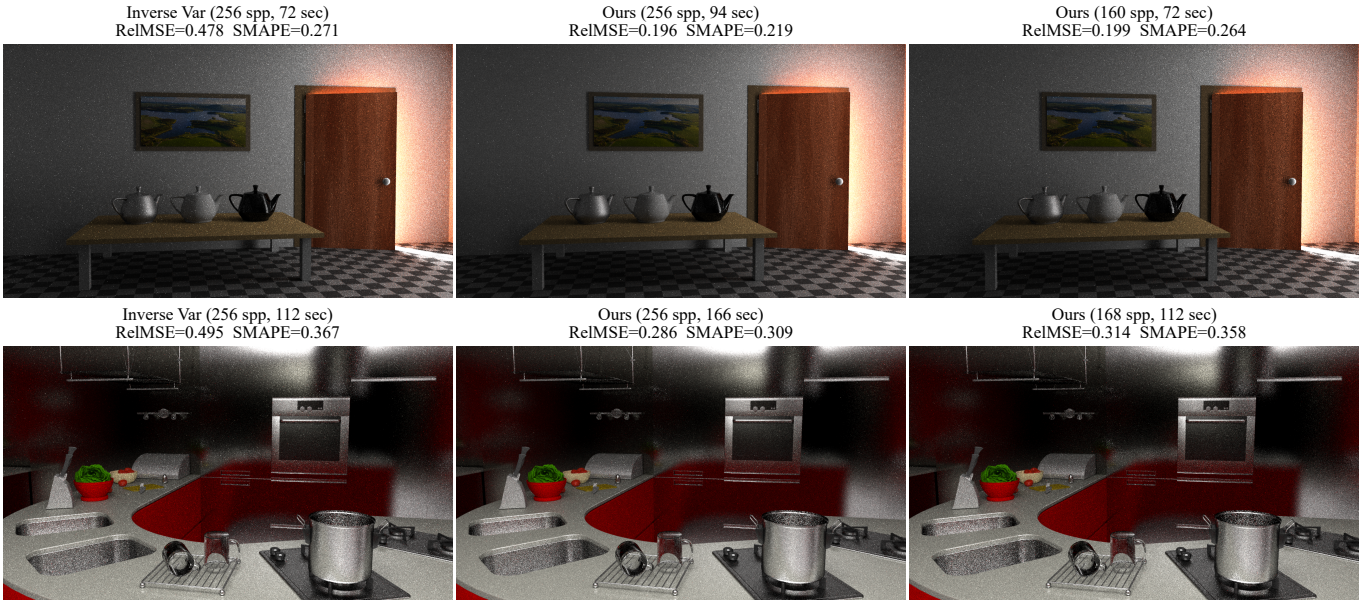

Fig. 2. At high sample rates, the computational overhead becomes more severe, so the variance reduction is not visible in equal time comparisons.