$= \frac{1}{n-1} E\left(\sum X_i^2 - n\overline{X}^2\right)$

例 6、4 $\hat{\theta}_1 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / n$, $\hat{\theta}_2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / (n-1)$

$E(x_i) = u$, $V(x_i) = \sigma^2 = E(x_i^2) - u^2$

$= E(x_i^2) - E^2(x_i)$

则 $E(\bar{x}) = u$, $V(\bar{x}) = \frac{\sigma^2}{n} = E(\bar{x}^2) - u^2$

$$E(\hat{\theta}_1) = E\left(\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}\right) = \frac{1}{n} E\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$= \frac{1}{n}(n\sigma^2 + nu^2 - \sigma^2 - nu^2) = \frac{n-1}{n}\sigma^2$$

$$E(\hat{\theta}_2) = E\left(\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}\right) = \frac{1}{n-1} E\left(\sum_{i=1}^{n} x_i^2 - n\bar{x}^2\right)$$

$$\uparrow \quad = \frac{1}{n-1}(n\sigma^2 + nu^2 - \sigma^2 - nu^2) = \sigma^2$$

满足不偏性.