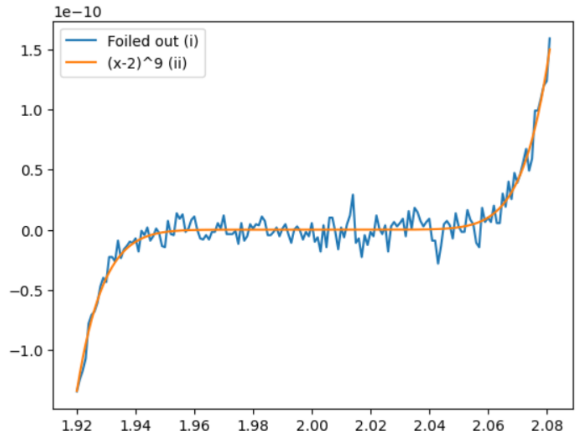
```python
import numpy as np
import matplotlib.pyplot as plt

x = np.arange(1.920, 2.081, 0.001)
poly1 = x**9 - 18*(x**8) + 144*(x**7) - 672*(x**6) + 2016*(x**5) - 4032*(x**4) + 5376*(x**3) - 4608*(x**2) + 2304*x - 512
poly2 = (x-2)**9
plt.plot(x, poly1)
plt.plot(x, poly2)
plt.legend(["Foiled out (i)", "(x-2)^9 (ii)"])
plt.show()
```



3. $P_2(x)$ for $f(x) = (1+x+x^3)\cos(x)$ about $x_0 = 0$

$\cos(x)(1+3x^2) - \sin(x)(1+x+x^3) \to \cos(x)(6x) - \cos(x)(1+x+x^3)$

$P_2 : 1 + x - \frac{x^2}{2} + \left[(\xi(x)^3 - 17\xi(x) + 1)\sin(\xi(x)) + (3 - 9\xi(x)^2)\cos(\xi(x))\right]\frac{x^3}{6}$

a) $P_2(0.5) = 1 + 0.5 - \frac{(0.5)^2}{2} + \frac{1}{48}\left[(\xi(x)^3 - 17\xi(x) + 1)\sin(\xi(x)) + (3 - 9\xi(x)^2)\cos(\xi(x))\right]$ $\quad$ $\boxed{P_2(0.5) = 1.375}$

$0 < \xi(x) < 0.5$, when $\cos(\xi(x)) = 1$, $\sin(\xi(x)) = 0$ vice versa So,

$(\xi(0.5)^3 - 17\xi(0.5) + 1)\sin(\xi(0.5)) + (3 - 9\xi(0.5)^2)\cos(\xi(0.5)) \le 3$ $\quad (\xi(0.5) = 0)$

so $|f(0.5) - P_2(0.5)| \le \frac{3}{48} = 1/16 = 0.0625$

Actual error: $|f(0.5) - P_2(0.5)| = 0.0511 < 0.0625$

b) $|f(x) - P_2(x)| \le \frac{\left(\frac{1}{16}\right)}{(1/2)^3}|x|^3 = \frac{1}{2}|x|^3$

c) $\int_0^1 f(x)\,dx \approx \int_0^1 (1 + x + 1/2 x^2)\,dx = x + 1/2 x^2 + 1/6 x^3 \Big|_0^1 = 1.\overline{666}$

d) $\frac{1}{6}\left|\int_0^1 x^3\sin(\xi(x))(\xi(x)^3 - 17\xi(x) + 1)\,dx + \int_0^1 x^3\cos(\xi(x))(3 - 9\xi(x)^2)\,dx\right|$

$\le \frac{1}{6}\left[\int_0^1 x^3(\xi(x)^3 - 17\xi(x) + 1\right.$

2. : $\sqrt{x+1} - 1$, $x \simeq 0$. $(-1+\sqrt{x+1}) \cdot \left(\frac{-1-\sqrt{x+1}}{-1-\sqrt{x+1}}\right) = \frac{1+-x-1}{-1-\sqrt{x+1}} = \frac{-x}{-1-\sqrt{x+1}} = \boxed{\frac{x}{1+\sqrt{x+1}}}$

ii.) $\cancel{\frac{\sin(x)-\sin(y)}{(\sin(x)-\sin(y))}}$ $x \simeq y$ $\cancel{\left(\frac{\sin(x)+\sin(y)}{\sin(x)+\sin(y)}\right) = \frac{\sin^2(x)-\sin^2(y)}{\sin(x)+\sin(y)} = \frac{1-\cos^2(x)-\sin^2(y)}{\sin(x)+\sin(y)}} = \frac{1-\left(\frac{1+\cos 2x}{2}\right)-\sin^2(y)}{\sin(x)+\sin(y)}$

$\cancel{(\sin(x)-\sin(y))\left(\frac{\tan(x)\cos(x)}{\sin(x)}\right) = \tan(x)\cos(x) - \frac{\sin(y)\cos(x)\tan(x)}{\sin(x)}}$ $= \frac{\frac{1}{2} - \frac{1}{2}\cos(2x) - \sin^2(y)}{\sin(x)+\sin(y)}$

↳ hmm cross

ii)

$\sin(x)-\sin(y) = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

iii) $\frac{1-\cos(x)}{\sin(x)}\left(\frac{1+\cos(x)}{1+\cos(x)}\right) = \frac{1-\cos^2(x)}{\sin(x)+\sin(x)\cos(x)}$ $-\frac{\sin^2 x}{\sin(x)+\sin(x)\cos(x)}$ $+\frac{\sin^2(x)}{\sin(x)+\sin(x)\cos(x)}$

$= \frac{1-1+\sin^2(x)}{\sin(x)+\sin(x)\cos(x)} = \frac{\sin(x)}{1+\cos(x)}$ //

1. c) $(x-2)^9$ is an exact, smooth plot vs jagged, variable plot of same-ish shape.

$(x-2)^9$ minimizes the # of arithmetic calculations, minimizing round-off errors!!

$(x-2)^9$ is more accurate!