Proof the First for CS250

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Consider the relation f over the real numbers ,

$$f : \mathbf{R} \longrightarrow \mathbf{R}\,, where\ f(x)\ =\ x^3\ \ldots \tag{1}$$

1 Prove or disprove: f is a function.

Proof. A relation F from A to B is a function, if and only if:

1. Every element of A is the first element of an ordered pair of F.
2. No distinct ordered pairs in F have the same first element.

Any real number, if raised to an odd power, retains its positivity or negativity. Therefore ...

$$\forall (x, y) \in F \mid \{isUnique(x)\} \tag{2}$$

...satisfying property (1).

Since the the set of the first element in each ordered pair of F is equal to the real numbers, **R**, which is continuuous from - ∞ to ∞, there can be no ordered pairs in F that have the same first element.

...satisfying property (2).

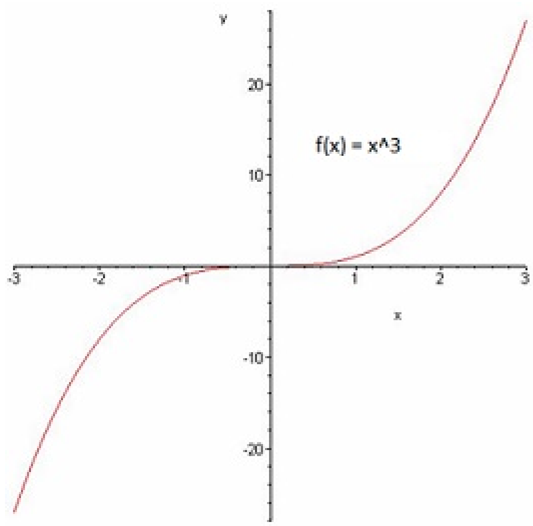
Conclusion: f is a function. □

2 Prove or disprove: f is injective (one-to-one).

Proof. A function is injective (one-to-one) if every element of the codomain is mapped to by at most one element of the domain.

One method of testing whether or not a function is injective is the performance of a horizontal line test on a graph of the function in question. If you can draw a horizontal line at any place on the graph which touches the function in two places or more, the function is not injective.

Below is a graph of the function, $f(x) = x^3$



As indicated by the graph above, there is no horizontal line that can possibly be drawn on the graph of $f(x) = x^3$ which would touch the function in multiple places.

Given the erroneous nature of images, it would be possible to argue from the graph alone that a horizontal line drawn at $y = 0$ would pass through the function more than once.

Consider the function $f(x) = x^3$.

$$When\ x = 0\ ,\ f(0) = 0^3 = 0 \tag{3}$$

There is no other possible value for x such that $x^3 = 0$.

Conclusion: f is injective.

$\square$

3 Prove or disprove: f is surjective (onto).

Proof. A function f (from set A to B) is surjective if and only if for every y in B, there is at least one x in $A \parallel f(x) = y$. In other words, f is surjective if and only if $f(A) = B$.

$$f(x) = x^3 = y, \qquad x, y \in \mathbf{R} \ldots \tag{4}$$

$$\therefore \sqrt[3]{x^3} = \sqrt[3]{y} \tag{5}$$

$$\therefore x = y^{1/3} \tag{6}$$

$$\therefore \forall x \in \mathbf{R}\ \exists y \parallel f(x) = y \tag{7}$$

Conclusion: f is surjective.

□

4 Prove or disprove: f is bijective (both one-to-one and onto).

Proof. A function is considered bijective if it has been proven to be both injective and surjective. We have previously concluded in this document that $f(x) = x^3$ is a function, and that it is both injective and surjective.

Conclusion: f is bijective. □