Homework 4

Problem 1.

bond price $p(t,T)$ PDE:

$$\frac{1}{2} P_{rr} (\gamma r(t) + \sigma) + P_r (\mu - k r(t)) + P_t - r(t) P = 0$$

inf $P = \exp\{ A(T,t) - B(T,t) r(t) \}$

$$P_{rr} = B^2 (T,t) \cdot P$$

$$P_r = -B(T,t) \cdot P$$

$$P_t = P \cdot A_t - P \cdot r(t) B_t$$

$$A_t = \mu B - \frac{1}{2} \sigma^2 B$$

$$B_t = kB + \frac{1}{2} \gamma B^2 - 1$$

Therefore. $\frac{1}{2} P_{rr} (\gamma r(t) + \sigma) + P_r (\mu - k r(t)) + P_t - r(t) \cdot P$

$$= \frac{1}{2} P \cdot B^2 (\gamma r(t) + \sigma) - P \cdot B (\mu - k r(t)) + P (\mu B - \frac{1}{2} \sigma^2 B) - P \cdot r(t) (kB + \frac{1}{2} \gamma B^2 - 1) - r(t) P,$$

$$= P r(t) - P \cdot r(t) = 0$$

$P(t,T) = \exp\{ A(t,T) - B(t,T) r(t) \}$, where. $\frac{dA}{dt} = \mu B - \frac{1}{2} \sigma B^2$. $\frac{dB}{dt} = kB + \frac{1}{2} \gamma B^2 - 1$