

Math Problems

1. Elementary algebra

1.1. $\frac{2^{17}}{2^3 \cdot 2^5} = \frac{2^{17}}{2^8} = 2^{17-8} = 2^9$

1.2

$G^2 \cdot G^x = G^6$ $a^x \cdot a^y = a^{x+y} \rightarrow G^2 \cdot G^4 = G^6 \Rightarrow \underline{x=4}$

1.3.

$x \cdot y = 5$ $x^3 y^3 = ?$ $x^3 y^3 = (xy)^3 \Rightarrow x^3 y^3 = (xy)^3 \Rightarrow 5^3 = 125$ $x^3 y^3 = \underline{125}$

1.4.

$\sqrt[3]{\frac{2^{10}}{4^3}} = \frac{2^{\frac{10}{3}}}{\sqrt[3]{4^3}} = \frac{2^{\frac{10}{3}}}{2^{\frac{6}{3}}} = \frac{2^{\frac{10}{3}}}{2^2} = 2^{\frac{10}{3}-2} = 2^{\frac{4}{3}} = 2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} = \sqrt[3]{2^4} = \sqrt[3]{16}$

1.5

a, TRUE b, TRUE c, FALSE d, TRUE

1.6

$\frac{2x-5}{2} \geq 4$ $2x-5 \geq 8$ $2x \geq 13$ $\underline{x \geq \frac{13}{2}}$

Function

2.1

$f(x) = \frac{9}{5}x + 32$

When $x = f(x) = x$ $\frac{9}{5}x + 32 = x$ $\frac{4}{5}x + 32 = 0$

$\frac{4}{5}x = -32$ $x = \underline{-40}$

2.2.

$f(x) = 5x + 4$ $5y + 4 = 14$ $5y = 10$ $\underline{y=2}$

2.4

$1,03^x = 2$ $\ln 1,03^x = \ln 2$ $x \ln 1,03 = \ln 2$

2.3.

$10^{x^2-2x+2} = 100$

$10^u = 100$ when $u = 2$

$x^2 - 2x + 2 = 2$ $x^2 - 2x = 0$

$x^2 - 2x = 0$

$x(x-2) = 0$

$\underline{x_1 = 0}$

$\underline{x_2 = 2}$

$x = \frac{\ln 2}{\ln 1,03}$

$x \approx 23,45$

2.5.

$\ln(1/e) = \ln e^{-1} = \underline{-1}$

Calculus

3.1

$$\sum_{i=0}^{\infty} \left(\frac{1}{8^i} + 0,5^i \right) = \sum_{i=0}^{\infty} \frac{1}{8^i} + \sum_{i=0}^{\infty} 0,5^i$$

$$\sum \frac{1}{8^i} \rightarrow r = \frac{1}{8} \quad \frac{1}{1-\frac{1}{8}} = \frac{1}{\frac{7}{8}} = \frac{8}{7} \quad \sum 0,5^i \rightarrow r = \frac{1}{2} \quad \frac{1}{1-\frac{1}{2}} = \frac{1}{\frac{1}{2}} = 2$$

$$\frac{8}{7} + 2 = \frac{8}{7} + \frac{14}{7} = \frac{22}{7}$$

3.2

$$\lim_{x \rightarrow 3} \frac{x-3}{2} \Rightarrow \text{When } x \rightarrow 3 \quad x-3 \rightarrow 0 \quad \text{Thus. } \lim_{x \rightarrow 3} \frac{x-3}{2} = \frac{0}{2} = \underline{0}$$

3.3

$$f(x) = x^2 - 4 \quad f(-1, -3)$$

$$f'(x) = 2x \rightarrow 2x = -2 \quad \underline{m = -2}$$

3.4

$$\frac{d}{dx} \frac{x^2+3}{x+2} = \frac{2x(x+2) - (x^2+3) \cdot 1}{(x+2)^2} = \frac{2x^2+4x-x^2-3}{(x+2)^2} = \frac{x^2+4x-3}{(x+2)^2}$$

3.5

$$\frac{d}{dx} 4x^3 + 4 = \frac{d}{dx} 12x^2 = \underline{24x}$$

3.6

No, because $\frac{1}{x}$ has no value at $x=0 \rightarrow$

It's not part of its domain \rightarrow No

3.8

$$f(x, y) = x^2 y^3 \quad f(2, 3) = 2^2 \cdot 3^3 = 4 \cdot 27 = \underline{108}$$

3.9

$$f(x, y) = \ln(x-y) \quad \ln(x-y) \text{ is defined where } x-y > 0$$

\Rightarrow where $\underline{x > y}$

3.10

$$\frac{\partial}{\partial x} x^5 + xy^3 = \frac{\partial}{\partial x} 5x^4 + y^3 = 20x^3$$

Linear Algebra

$$(3 \times 2)(2 \times 3) = (3 \times 3)$$

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 4 & 1 \\ 2 & 1 & 2 \end{bmatrix}$$

4.1

$$A \cdot B = \begin{bmatrix} 8 & 11 & 8 \\ 6 & 17 & 6 \\ 5 & 6 & 5 \end{bmatrix}$$

4.2

$$B \cdot A = ?$$

$$B \cdot A = \begin{bmatrix} 19 & 9 \\ 10 & 11 \end{bmatrix}$$

4.3

$$A = \begin{bmatrix} 3,3 & 5,1 & 4,7 \\ 2 & 6,1 & 1,23 \\ 4 & 5,76 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 3,3 & 2 & 4 \\ 5,1 & 6,1 & 5,76 \\ 4,7 & 1,23 & 0 \end{bmatrix}$$

4.4

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$$

$$\det(A) = 2 \cdot 5 - 3 \cdot 4 = 10 - 12 = \underline{\underline{-2}}$$

Probability

5.1

$$\Omega = \{(H,H,H,H), (H,H,H,T), (H,H,T,H), \dots, (T,T,T,T)\}$$

$$2 \cdot 2 \cdot 2 \cdot 2 = 2^4 = 16 \text{ Possible outcomes}$$

5.3

21
possible
outcomes

2	3	4	5	6	7	8	9	10	11	12
1,1	1,2	1,3	1,4	1,5	1,6	2,6	3,6	4,6	5,6	6,6
		2,2	2,3	2,4	2,5	3,5	4,5	5,5		
				3,3	3,4	4,4				

$$E_X = \frac{2 \cdot 1}{21} + \frac{3 \cdot 1}{21} + \frac{4 \cdot 2}{21} + \frac{5 \cdot 2}{21} + \frac{6 \cdot 3}{21} + \frac{7 \cdot 3}{21} + \frac{8 \cdot 3}{21} + \frac{9 \cdot 2}{21} + \frac{10 \cdot 2}{21} + \frac{11 \cdot 1}{21} + \frac{12 \cdot 1}{21} =$$

$$= \frac{147}{21} = 7 \quad \underline{\underline{E_X = 7}}$$