$$q_0 = 0$$

$$\frac{\mu}{h}\Big\{ -1 - \frac{\beta h}{2\mu} + \frac{\sigma h^2}{6\mu}\Big\}_{i-\frac{1}{2}} q_{i-1} +$$

$$+ \Big\{\frac{\mu}{h}\Big[1 + \frac{\beta h}{2\mu} + \frac{\sigma h^2}{3\mu}\Big]_{i-\frac{1}{2}} + \frac{\mu}{h}\Big[1 - \frac{\beta h}{2\mu} + \frac{\sigma h^2}{3\mu}\Big]_{i+\frac{1}{2}}\Big\} q_i +$$

$$+ \frac{\mu}{h}\Big\{ -1 + \frac{\beta h}{2\mu} + \frac{\sigma h^2}{6\mu}\Big\}_{i+\frac{1}{2}} q_{i+1} = \frac{1}{2}\Big\{[hf]_{i-\frac{1}{2}} + [hf]_{i+\frac{1}{2}}\Big\}$$

$$\frac{\mu}{h}\Big\{ -1 - \frac{\beta h}{2\mu} + \frac{\sigma h^2}{6\mu}\Big\}_{N-\frac{1}{2}} q_{N-1} + \frac{\mu}{h}\Big\{1 + \frac{\beta h}{2\mu} + \frac{\sigma h^2}{3\mu}\Big\}_{N-\frac{1}{2}} q_N + \alpha q_N = \frac{1}{2}[hf]_{N-\frac{1}{2}} + \alpha\overline{u}$$

(1)

$$\eta_{i+\frac{1}{2}} = \frac{||\varepsilon_h||_{i+\frac{1}{2}}\sqrt{N}\cdot 100\%}{\sqrt{||u_h||_V^2 + ||\varepsilon_h||_V^2}} \tag{2}$$

$$||\varepsilon_h||_{i+\frac{1}{2}} = \sqrt{\frac{5}{6}\Big\{\frac{h^3}{\mu}\frac{(f - \beta\dot{q} - \sigma q)^2}{(10 + PeSh)}\Big\}_{i+\frac{1}{2}}}$$

$$||\varepsilon_h||_V^2 = \frac{5}{6}\sum_{i=0}^{N-1}\Big\{\frac{h^3}{\mu}\frac{(f - \beta\dot{q} - \sigma q)^2}{(10 + PeSh)}\Big\}_{i+\frac{1}{2}} \tag{3}$$

$$PeSh = \frac{\sigma h^2}{\mu}$$

$$||u_h||_{i+\frac{1}{2}}^2 = \int_{x_i}^{x_{i+1}} [u_h^{'}(x)]^2 dx = \int_{x_i}^{x_{i+1}} \dot{q}_{i+\frac{1}{2}}^2 dx = [h\dot{q}^2]_{i+\frac{1}{2}} \tag{4}$$

$$q_{i+\frac{1}{2}} := \frac{q_i + q_{i+1}}{2}, \quad \dot{q}_{i+\frac{1}{2}} := \frac{q_{i+1} - q_i}{h_{i+\frac{1}{2}}} \tag{5}$$

$$u'(x) \approx u'_{i+\frac{1}{2}}(x) = \dot{q}_{i+\frac{1}{2}} \tag{6}$$