Variational Autoencoders

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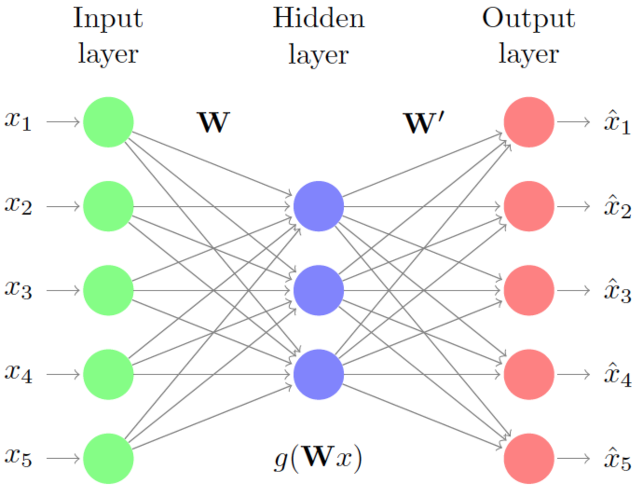
Autoencoders: Overview I



Autoencoders: Overview II

$$\underset{\boldsymbol{A} \in \mathbb{R}^{p \times q},\ \boldsymbol{A}'\boldsymbol{A} = \boldsymbol{I}_q}{\text{minimize}} \sum_{i=1}^{n} \| x_i - \boldsymbol{A}\boldsymbol{A}' x_i \|_2^2.$$

$$\underset{\boldsymbol{W} \in \mathbb{R}^{q \times p}}{\text{minimize}} \sum_{i=1}^{n} \| x_i - \boldsymbol{W}' g(\boldsymbol{W} x_i) \|_2^2,$$

Autoencoders: Uses

- Pre-training deep neural networks
- Dimensionality reduction and feature extraction
- Reconstruction and deblurring
- Anomaly detection
- Generative modeling: the rest of what follows

Variational Autoencoders: Problem Overview

Generative model:

$$
\begin{aligned}
z &\sim \mathcal{N}(0, I) \\
x|z &\sim p_\theta(x|z)
\end{aligned}
$$

Preview:

Encoder: $q_\phi(z|x) = \mathcal{N}(\mu_\phi(x), \sigma_\phi(x)I)$
Decoder: $p_\theta(x|z) = \mathcal{N}(\mu_\theta(x), \sigma_\theta(x)I)$

(Can use whatever probability model is appropriate for the decoder based on the data, i.e. Bernoulli. Mean and variance parameters for each MVN are the output from neural networks.)

Variational Autoencoders: Algorithm Overview

Problem: maximize marginal likelihood (over θ)

$$p_\theta(x) \equiv \int_z p(z)p_\theta(x|z)dz$$

Form lower bound on log-likelihood

$$\log p_\theta(x) \geq \mathcal{L}(\theta, q) = \int_z q(z|x) \log \frac{p_\theta(x,z)}{q(z|x)}$$

Recall, the EM algorithm is EXACTLY (coordinate ascent):

$$\begin{aligned} q_{t+1} &= p_{\theta_t}(z|x) \\ \theta_{t+1} &= \text{argmax}_\theta \mathcal{L}(\theta, q_{t+1}) \end{aligned}$$

Variational Autoencoders: Algorithm Details

What if $p_{\theta_t}(z|x)$ is intractable?

Instead, parametrize q by a neural network q_ϕ and perform gradient ascent after calculating $\nabla_{\theta,\phi}\mathcal{L}(\theta,\phi)$

An approximation to the gradient is:

$$\nabla_{\theta,\phi}\mathcal{L}(\theta,\phi) \approx \nabla_{\theta,\phi}\frac{1}{L}\sum_{k=1}^{L}\log p_\theta(x, z_l) - \log q_\phi(z_l|x)$$

$$z_l \sim q_\phi(z|x)$$

Variational Autoencoders: Reparametrization Trick

This is almost everything we need: the gradient with respect to ϕ is very noisy since the sample is generated from $q_\phi(z|x)$ (i.e. it is weird to sample from a distribution and then take a derivative w.r.t the parameter governing that distribution).

Reparametrization trick: find $\epsilon \sim \pi(\epsilon)$ such that $z \sim g_\phi(\epsilon, x)$

$$\mathcal{L}(\theta, \phi) \approx \frac{1}{L} \sum_{k=1}^{L} \log p_\theta(x, g_\phi(\epsilon^l, x)) - \log q_\phi(g_\phi(\epsilon^l, x)|x)$$

$$\epsilon^l \sim \pi(\epsilon)$$

Variational Autoencoders: Use Case

Variational Autoencoders: Encoder / Decoder

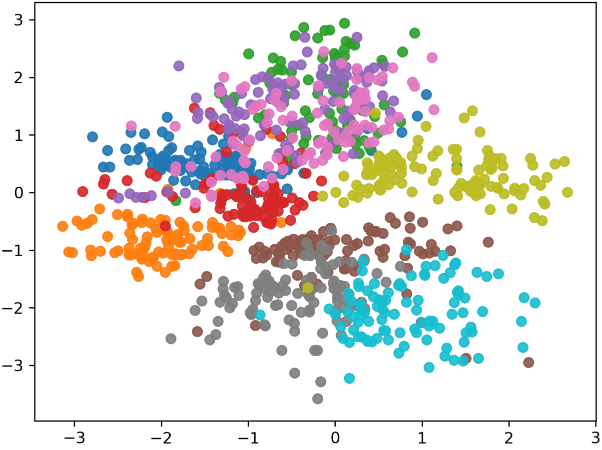
Bernoulli decoder:

$$\log p(\mathbf{x}|\mathbf{z}) = \sum_{i=1}^{D} x_i \log y_i + (1 - x_i) \cdot \log(1 - y_i)$$
$$\text{where } \mathbf{y} = f_\sigma(\mathbf{W}_2 \tanh(\mathbf{W}_1 \mathbf{z} + \mathbf{b}_1) + \mathbf{b}_2)$$

Gaussian encoder:

$$\begin{aligned} \log p(\mathbf{x}|\mathbf{z}) &= \log \mathcal{N}(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\sigma}^2 \mathbf{I}) \\ \text{where } \boldsymbol{\mu} &= \mathbf{W}_4 \mathbf{h} + \mathbf{b}_4 \\ \log \boldsymbol{\sigma}^2 &= \mathbf{W}_5 \mathbf{h} + \mathbf{b}_5 \\ \mathbf{h} &= \tanh(\mathbf{W}_3 \mathbf{z} + \mathbf{b}_3) \end{aligned}$$

Variational Autoencoders: Latent Variable Visualization



Variational Autoencoders: Generating Random Images