$$F(x_1, x_2, x_3) = 6x_1^2 + 4x_2^2 + 9x_3^2 + \underbrace{4x_1x_2 + 2x_1x_3 + 6x_2x_3}_{\text{DIVISO PER 2 IN MODO TALE DA RENDERLA SIMMETRICA}}$$

$a_{11} = 6 \qquad a_{12} = 2 \Rightarrow a_{21} = 2$

$a_{22} = 4 \qquad a_{13} = 1 \Rightarrow a_{31} = 1$

$a_{33} = 9 \qquad a_{23} = 3 \Rightarrow a_{32} = 3$

$$A = \begin{pmatrix} 6 & 2 & 1 \\ 2 & 4 & 3 \\ 1 & 3 & 9 \end{pmatrix}$$

$$A - \lambda I = \begin{pmatrix} 6-\lambda & 2 & 1 \\ 2 & 4-\lambda & 3 \\ 1 & 3 & 9-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = \begin{vmatrix} 6-\lambda & 2 & 1 & 6-\lambda & 2 \\ 2 & 4-\lambda & 3 & 2 & 4-\lambda \\ 1 & 3 & 9-\lambda & 1 & 3 \end{vmatrix} =$$

$$= (6-\lambda)(4-\lambda)(9-\lambda) + 12 - 4(9-\lambda) - 9(6-\lambda) - 4 + \lambda =$$

$$= (24 - 10\lambda + \lambda^2)(9-\lambda) + 12 - 36 + 4\lambda - 54 + 9\lambda - 4 + \lambda =$$

$$= 216 - 24\lambda - 90\lambda + 10\lambda^2 + 9\lambda^2 - \lambda^3 - 82 + 14\lambda =$$

$$= -\lambda^3 + 19\lambda^2 - 100\lambda + 134$$

$\lambda_1 = 2{,}06$

$\lambda_2 = 5{,}90 \quad \Rightarrow \lambda_1, \lambda_2, \lambda_3 > 0 \quad \Rightarrow$ DEFINITA POSITIVA

$\lambda_3 = 11{,}04$

$$F(x_1, x_2, x_3, x_4) = x_1^2 + 3x_2^2 - 5x_4^2 + 4x_1x_2 - 6x_2x_3 + 2x_1x_4 - 8x_3x_4$$

$a_{11} = 1$ $\quad a_{12} = 2$ $\quad a_{13} = 0$ $\quad a_{14} = 1$

$a_{22} = 3$ $\quad a_{21} = 2$ $\quad a_{23} = -3$ $\quad a_{24} = 0$

$a_{33} = 0$ $\quad a_{31} = 0$ $\quad a_{32} = -3$ $\quad a_{34} = -4$

$a_{44} = -5$ $\quad a_{41} = 1$ $\quad a_{42} = 0$ $\quad a_{43} = -4$

$$A = \begin{pmatrix} 1 & 2 & 0 & 1 \\ 2 & 3 & -3 & 0 \\ 0 & -3 & 0 & -4 \\ 1 & 0 & -4 & -5 \end{pmatrix}$$

$n = 4$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \qquad A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix}$$

$$X^T = [\, x_1 \quad x_2 \quad x_3 \quad x_4 \,]$$

$$F = X^T A \quad X$$

$m\,X^T = 4$

$\Longrightarrow$ CONFORMABILI

$m\,A = 4$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{pmatrix} \quad X^T = [\, x_1 \quad x_2 \quad x_3 \quad x_4 \,]$$

$$\underset{1 \cdot 4}{X^T A} = \quad x_1 a_{11} + x_2 a_{21} + x_3 a_{31} + x_4 a_{41} \qquad x_2 a_{12} + x_3$$

$$A = \begin{pmatrix} 3 & 0 & -2 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix} \qquad F = ?$$

$$F = x^T A \ x$$

$$F = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix} \begin{pmatrix} 3 & 0 & -2 \\ 0 & 1 & 1 \\ -2 & 1 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} =$$

$$= \begin{pmatrix} 3x_1 - 2x_3 & x_2 + x_3 & -2x_1 + x_2 + 4x_3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} =$$

$$= 3x_1^2 - 2x_1x_3 + x_2^2 + x_2x_3 - 2x_1x_3 + x_2x_3 + 4x_3^2 =$$

$$= 3x_1^2 + x_2^2 + 4x_3^2 - 4x_1x_3 + 2x_2x_3$$

$$F(x_1, x_2) = -x_1^2 + 4x_1x_2 - 2x_2^2$$

$a_{11} = -1 \qquad a_{12} = 2$

$a_{21} = 2 \qquad a_{22} = -2$

$$A = \begin{pmatrix} -1 & 2 \\ 2 & -2 \end{pmatrix}$$

$$A - \lambda I = \begin{pmatrix} -1-\lambda & 2 \\ 2 & -2-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = (-1-\lambda)(-2-\lambda) - 4 =$$

$$= \lambda^2 + 3\lambda + 2 - 4 = \lambda^2 + 3\lambda - 2$$

$$\lambda^2 + 3\lambda - 2 = 0$$

$$\Delta = 9 + 8 = 17$$

$$\lambda = \frac{-3 \pm \sqrt{17}}{2} \qquad \lambda_1 = \frac{-3 - \sqrt{17}}{2} \qquad \lambda_2 = \frac{-3 + \sqrt{17}}{2}$$

$\lambda_1 < 0$ e $\lambda_2 > 0 \implies F$ INDEFINITA

$$F(x_1, x_2, x_3) = 3x_2^2 + 8x_3^2 + 4x_1x_2 + 6x_1x_3 + 12x_2x_3$$

$a_{11} = 0 \qquad a_{12} = 2 \qquad a_{13} = 3$

$a_{21} = 2 \qquad a_{22} = 3 \qquad a_{23} = 6$

$a_{31} = 3 \qquad a_{32} = 6 \qquad a_{33} = 8$

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 3 & 6 \\ 3 & 6 & 8 \end{pmatrix}$$

$$A - \lambda I = \begin{pmatrix} -\lambda & 2 & 3 \\ 2 & 3-\lambda & 6 \\ 3 & 6 & 8-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = \left| \begin{array}{ccc|cc} -\lambda & 2 & 3 & -\lambda & 2 \\ 2 & 3-\lambda & 6 & 2 & 3-\lambda \\ 3 & 6 & 8-\lambda & 3 & 6 \end{array} \right| =$$

$$= -\lambda(3-\lambda)(8-\lambda) + 36 + 36 - 4(8-\lambda) + 36\lambda - 9(3-\lambda) =$$

$$= (-3\lambda + \lambda^2)(8 - \lambda) + 72 - 32 + 4\lambda + 36\lambda - 27 + 9\lambda =$$

$$= -24\lambda + 3\lambda^2 + 8\lambda^2 - \lambda^3 + 13 + 48\lambda =$$

$$= -\lambda^3 + 11\lambda^2 + 25\lambda + 13$$

	-1	11	25	13
-1		1	-12	-13
	-1	12	13	0

$$-(\lambda + 1)(\lambda^2 - 12\lambda - 13) = 0$$

$$\lambda^2 - 12\lambda - 13 = 0$$

$$\Delta = 144 + 52 = 196$$

$$\lambda = \frac{12 \pm 14}{2} \begin{cases} -1 \\ 13 \end{cases}$$

$$-(\lambda + 1)^2 (\lambda - 13) = 0$$

$$-(\lambda + 1)^2 = 0 \qquad \lambda_1 = -1 < 0$$

$$-\lambda + 13 = 0 \qquad \lambda_2 = 13 > 0$$

$\Longrightarrow$ <u>INDEFINITA</u>

$$A = \begin{pmatrix} 0 & 2 & 3 \\ 2 & 3 & 6 \\ 3 & 6 & 8 \end{pmatrix}$$

FORMA QUADRATICA = ?

$$F = x^T A x \qquad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \qquad x^T = [\, x_1 \quad x_2 \quad x_3 \,]$$

$$x^T A = [\, 2x_2 + 3x_3 \quad 2x_1 + 3x_2 + 6x_3 \quad 3x_1 + 6x_2 + 8x_3 \,]$$

$1 \cdot 3$

$$x^T A x = [\, 2x_1 x_2 + 3x_3 x_1 + 2x_1 x_2 + 3x_2^2 + 6x_3 x_2 + 3x_1 x_3 + 6x_2 x_3 + 8x_3^3 \,]$$

$1 \cdot 1$

$$A = \begin{pmatrix} 3 & -4 \\ -4 & 6 \end{pmatrix} \qquad F = ?$$

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad x^T = [\, x_1 \quad x_2 \,]$$

$$\underset{1 \cdot 2}{x^T A} = [\, 3x_1 - 4x_2 \quad -4x_1 + 6x_2 \,]$$

$$\underset{1 \cdot 1}{x^T A x} = [\, 3x_1^2 - 4x_2 x_1 - 4x_1 x_2 + 6x_2^2 \,]$$

$$F(x_1, x_2, x_3) = 2x_1^2 - x_2^2 + 5x_3^2 + 6x_1x_2 - 3x_1x_3$$

$a_{11} = 2 \qquad a_{12} = 3 \qquad a_{13} = -\frac{3}{2}$

$a_{21} = 3 \qquad a_{22} = -1 \qquad a_{23} = 0$

$a_{31} = -\frac{3}{2} \qquad a_{32} = 0 \qquad a_{33} = 5$

$$A = \begin{pmatrix} 2 & 3 & -\frac{3}{2} \\ 3 & -1 & 0 \\ -\frac{3}{2} & 0 & 5 \end{pmatrix}$$

$$F(x_1, x_2, x_3) = 3x_1^2 + 3x_2^2 + 3x_3^2 - 2x_1x_2 - 2x_1x_3 - 2x_2x_3$$

$a_{11} = 3 \qquad a_{12} = -1 \qquad a_{13} = -1$

$a_{21} = -1 \qquad a_{22} = 3 \qquad a_{23} = -1$

$a_{31} = -1 \qquad a_{32} = -1 \qquad a_{33} = 3$

$$A = \begin{pmatrix} 3 & -1 & -1 \\ -1 & 3 & -1 \\ -1 & -1 & 3 \end{pmatrix}$$

$$A - \lambda I = \begin{pmatrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 3-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = \left| \begin{matrix} 3-\lambda & -1 & -1 \\ -1 & 3-\lambda & -1 \\ -1 & -1 & 3-\lambda \end{matrix} \right| \begin{matrix} 3-\lambda & -1 \\ -1 & 3-\lambda \\ -1 & -1 \end{matrix} \Bigg| =$$

$$= (3-\lambda)^3 - 1 - 1 - 3 + \lambda - 3 + \lambda - 3 + \lambda =$$

$$= 27 - 27\lambda + 9\lambda^2 - \lambda^3 - 11 + 3\lambda = -\lambda^3 + 9\lambda^2 - 24\lambda + 16$$

	-1	9	-24	16
1		-1	8	-16
	-1	8	-16	$//$

$$-(\lambda - 1)(\lambda^2 - 8\lambda + 16) = 0$$

$$-(\lambda - 1)(\lambda - 4)^2 = 0$$

$-\lambda + 1 = 0 \qquad \lambda_1 = 1 > 0$

$\Longrightarrow$ def. POSITIVA

$-(\lambda - 4)^2 = 0 \qquad \lambda - 4 = 0 \qquad \lambda_2 = 4 > 0$

$$A = \begin{pmatrix} 4 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 4 \end{pmatrix}$$

SEGNO DI A = ?

$$A - \lambda I = \begin{pmatrix} 4-\lambda & 0 & 3 \\ 0 & -2-\lambda & 0 \\ 3 & 0 & 4-\lambda \end{pmatrix}$$

$$\det(A - \lambda I) = \left|\begin{matrix} 4-\lambda & 0 & 3 \\ 0 & -2-\lambda & 0 \\ 3 & 0 & 4-\lambda \end{matrix}\right| \begin{matrix} 4-\lambda & 0 \\ 0 & -2-\lambda \\ 3 & 0 \end{matrix}\Bigg| =$$

$$= (4-\lambda)^2(-2-\lambda) - 9(-2-\lambda) =$$

$$= (-2-\lambda)(4 - 2\lambda + \lambda^2 - 9) = (-2-\lambda)(\lambda^2 - 2\lambda - 8)$$

$$\lambda^2 - 2\lambda - 8 = 0$$

$$\Delta = 4 + 32 = 36$$

$$\lambda = \frac{2 \pm 6}{2} \quad \begin{matrix} \lambda_1 = \frac{2-6}{2} = -2 \\ \lambda_2 = 4 \end{matrix}$$

$$(\lambda+2)(\lambda-4)(-2-\lambda)=0$$

$$\lambda_1 = -2 \qquad \lambda_1 < 0 \quad e \quad \lambda_2 > 0$$

$$\lambda_2 = 4$$

$$\Downarrow$$

INDEFINITA

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix}$$

$$F(x) = x^T A x$$

$$\underset{1 \cdot 3}{x^T A} = (x_1 \quad x_2 \quad x_3) \begin{pmatrix} 1 & 0 & 3 \\ 0 & -2 & 0 \\ 3 & 0 & 1 \end{pmatrix} =$$

$$= [x_1 + 3x_3 \qquad -2x_2 \qquad 3x_1 + x_3]$$

$$[x_1 + 3x_3 \qquad -2x_2 \qquad 3x_1 + x_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} =$$

$$= x_1^2 + 3x_3x_1 - 2x_2^2 + 3x_1x_3 + x_3^2$$