Declarative Matrix Language: Linear Algebra Project

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Introduction

I implemented my own language for the creation, manipulation, and output of two-dimensional matrices. It is called 'dml', which is short for 'Declarative Matrix Language'. For more information on dml, check the pdf titled 'Language Documentation of dml'.

Application 3.2

Application 3.2 introduces automated row operations with Matlab, Mathematica, and Mable. Using my program, I was able to represent the same row operations and get the same results. The output has been re-formatted for this document.

Normally, there are some restrictions placed on row operations, including that a row shouldn't be multiplied by 0. In my implementation there are no such constraints and I tried to make it so that the rows could be visualized as a part of a flexible expression instead of having to call a function followed by a list of parameters.

A ::=

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 3 & 8 & 7 & 20 \\ 2 & 7 & 9 & 23 \end{bmatrix}$$

Performing row operations:

$A[1] - 3 * A[0] \rightarrow A[1];$
$A[2] - 2 * A[0] \rightarrow A[2];$
$A[1]/2 \rightarrow A[1];$
$A[2] - 3 * A[1] \rightarrow A[2];$

Result of row operations:

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Swapping is also a row operation, despite it not being used above. Using the previous form, we can now swap A[0] and A[2] to allow forward-substitution instead of back-substitution.

Result of swapping rows:

$$\begin{bmatrix} 0 & 0 & 1 & 3 \\ 0 & 1 & 2 & 4 \\ 1 & 2 & 1 & 4 \end{bmatrix}$$

Application 3.3

After getting the basic row operations complete, row reduction becomes possible to implement. In my language, row reduction works best with decimal values instead of integers (the integer values become truncated). When I set up the problem in a script, I created an integer matrix first and had to cast it to decimal. After the computation was complete I casted it back to an integer matrix to get prettier output.

The first example is the same matrix from Application 3.2, except we are interested in the fully-reduced row echelon form.

A ::=

$$\begin{bmatrix} 1 & 2 & 1 & 4 \\ 3 & 8 & 7 & 20 \\ 2 & 7 & 9 & 23 \end{bmatrix}$$

rref:

$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

There are six systems of equations given in the textbook. I checked to make sure they are all correct.

Matrix 1:

$$\begin{bmatrix} 17 & 42 & -36 & 213 \\ 13 & 45 & -34 & 226 \\ 12 & 47 & -35 & 197 \end{bmatrix}$$

rref:

$$\begin{bmatrix} 1 & 0 & 0 & 39 \\ 0 & 1 & 0 & 27 \\ 0 & 0 & 1 & 44 \end{bmatrix}$$

Matrix 2:

$$\begin{bmatrix} 32 & 57 & -41 & 713 \\ 23 & 43 & -37 & 130 \\ 42 & -61 & 39 & 221 \end{bmatrix}$$

rref:

$$\begin{bmatrix} 1 & 0 & 0 & 17 \\ 0 & 1 & 0 & 49 \\ 0 & 0 & 1 & 64 \end{bmatrix}$$

Matrix 3:

$$\begin{bmatrix} 231 & 157 & -241 & 420 \\ 323 & 181 & -376 & 412 \\ 542 & 161 & -759 & 419 \end{bmatrix}$$

rref:

$$\begin{bmatrix} 1.000000 & -0.000000 & -0.000000 & 38.358214 \\ 0.000000 & 1.000000 & 0.000000 & -18.629199 \\ -0.000000 & -0.000000 & 1.000000 & 22.887814 \end{bmatrix}$$

Matrix 4:

$$\begin{bmatrix} 837 & 667 & -729 & 1659 \\ 152 & -179 & -975 & 1630 \\ 542 & 328 & -759 & 1645 \end{bmatrix}$$

rref:

$$\begin{bmatrix} 1 & 0 & 0 & 77 \\ 0 & 1 & 0 & -69 \\ 0 & 0 & 1 & 23 \end{bmatrix}$$

Matrix 5:

$$\begin{bmatrix} 49 & -57 & 37 & -59 & 97 \\ 73 & -15 & -19 & -22 & 99 \\ 52 & -51 & 14 & -29 & 89 \\ 13 & -27 & 27 & -25 & 73 \end{bmatrix}$$

rref:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 5 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 1 & 4 \end{bmatrix}$$

Matrix 6:

$$\begin{bmatrix} 64 & -57 & 97 & -67 & 485 \\ 92 & 77 & -34 & -37 & 486 \\ 44 & -34 & 53 & -34 & 465 \\ 27 & 57 & -69 & 29 & 464 \end{bmatrix}$$

rref:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 17 \\ 0 & 1 & 0 & 0 & 23 \\ 0 & 0 & 1 & 0 & 37 \\ 0 & 0 & 0 & 1 & 43 \end{bmatrix}$$

Application 3.5

I was not able to get to Application 3.5.