

原问题

"problem": "In a certain hyperbola, the center is at $(2,0)$, one focus is at $(2,6)$, and one vertex is at $(2,-3)$. The equation of this hyperbola can be written as $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$. Find $h + k + a + b$.", "level": "Level 3", "type": "Intermediate Algebra", "solution": "The center of the hyperbola is $(h,k) = (2,0)$. The distance between the center and one vertex is $a = 3$, and the distance between the center and one focus is $c = 6$. Then $b^2 = c^2 - a^2 = 6^2 - 3^2 = 27$, so $b = 3\sqrt{3}$. Therefore, $h + k + a + b = 2 + 0 + 3 + 3\sqrt{3} = \boxed{3\sqrt{3} + 5}$."

Extract

["Definition of Hyperbola Center", "Distance from Center to Vertex (a)", "Distance from Center to Focus (c)", "Relationship between a, b, and c in Hyperbola", "Calculation of b using Pythagorean Identity", "Simplifying Complex Expressions", "Summation of Coordinates and Constants"]

Evolve

["Definition of Hyperbola Center", "Distance from Center to Vertex (a)", "Distance from Center to Focus (c)", "Derivation of Hyperbola Equation using a, b, and c", "Calculation of b using Pythagorean Identity", "Simplifying Complex Expressions", "Summation of Coordinates and Constants"]

"problem": "Consider a hyperbola where the center is positioned at $(1, -2)$, with one of its vertices at $(1, 4)$ and a focus located at $(1, 10)$. The equation of this hyperbola can be expressed in the form: $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$. Calculate the sum $h + k + a + b$."

"solution": "First, identify the hyperbola's center as $(h, k) = (1, -2)$, utilizing the knowledge point 'Definition of Hyperbola Center.'"

Next, calculate the distance from the center to one vertex a , using the point 'Distance from Center to Vertex (a)'. Given the coordinates, $a = |4 - (-2)| = 6$.

Proceeding with the knowledge point 'Distance from Center to Focus (c)', compute the distance c from the center to one focus: $c = |10 - (-2)| = 12$.

With these values, apply 'Derivation of Hyperbola Equation using a, b, and c' to establish the hyperbola equation. Before that, calculate b using the 'Calculation of b using Pythagorean Identity': $b^2 = c^2 - a^2 = 12^2 - 6^2 = 144 - 36 = 108$, therefore, $b = \sqrt{108} = 6\sqrt{3}$.

Lastly, compute the sum $h + k + a + b$ by adding these values together, aligning with the knowledge point 'Summation of Coordinates and Constants'. Thus, $h + k + a + b = 1 + (-2) + 6 + 6\sqrt{3}$.

Hence, the sum is $h + k + a + b = 1 - 2 + 6 + 6\sqrt{3} = 5 + 6\sqrt{3}$.

GPT-4在没有提示的情况下回答正确。 GPT-4o在没有提示的情况下回答正确。

Evolve2

["Definition of Hyperbola Center", "Distance from Center to Vertex (a)", "Distance from Center to Focus (c)", "Derivation of Hyperbola Equation using a, b, and c", "Introduction of Rotated Axes for Hyperbola", "Simplifying Complex Expressions", "Summation of Coordinates and Constants"]

Question

Consider a hyperbola with its center located at $(3, -1)$. This hyperbola has one of its vertices at $(3, 6)$ and a focus at $(3, 10)$. Additionally, the axes of this hyperbola are rotated by 45 degrees with respect to the coordinate axes. Find the sum of the coordinates of the center, the value of a , and the simplified form of the expression $a + b$ if b is derived using the distances a and c .

Solution

Definition of Hyperbola Center: The center of the hyperbola is given as $(h, k) = (3, -1)$.

Distance from Center to Vertex (a): The distance from the center to one vertex (a) is calculated using the coordinates. The vertex is at $(3, 6)$, so $a = |6 - (-1)| = 7$.

Distance from Center to Focus (c): Next, calculate the distance from the center to a focus (c). The focus is at $(3, 10)$, hence $c = |10 - (-1)| = 11$.

Derivation of Hyperbola Equation using a, b, and c: Utilize these values to derive the hyperbola equation. For a hyperbola, $c^2 = a^2 + b^2$, thus calculating $b^2 = c^2 - a^2 = 11^2 - 7^2 = 121 - 49 = 72$. Consequently, $b = \sqrt{72} = 6\sqrt{2}$.

Introduction of Rotated Axes for Hyperbola: The hyperbola's axes are rotated, but for the calculation of $a + b$, the rotation does not affect the values of a and b . Therefore, $a + b = 7 + 6\sqrt{2}$.

Simplifying Complex Expressions: Simplify the expression $a + b$ derived in the previous step. The simplified expression remains $7 + 6\sqrt{2}$.

Summation of Coordinates and Constants: Finally, sum up the coordinates of the center and the values of a and the simplified expression $a + b$. The result is $h + k + a + (a + b) = 3 - 1 + 7 + (7 + 6\sqrt{2}) = 16 + 6\sqrt{2}$.

The final answer to the problem is $16 + 6\sqrt{2}$.

- 这里它受Simplifying Complex Expressions的影响，在Question里说的很奇怪。
- GPT-4o理解错了题意，回答错误。
- GPT-4也理解错了题意，回答错误。

Second Try

Question

Consider a hyperbola where the center is positioned at $(3, -5)$, with one of its vertices at $(3, 1)$ and a focus located at $(3, 13)$. This hyperbola has been rotated 45 degrees counterclockwise. Calculate the sum of the coordinates of the center, the vertex, and the focus.

Solution

First, identify the hyperbola's center as $(h, k) = (3, -5)$, utilizing the knowledge point "Definition of Hyperbola Center."

Next, calculate the distance from the center to one vertex a , using the point "Distance from Center to Vertex (a)." Given the coordinates, $a = |1 - (-5)| = 6$.

Proceeding with the knowledge point "Distance from Center to Focus (c)", compute the distance c from the center to one focus: $c = |13 - (-5)| = 18$.

With these values, apply "Derivation of Hyperbola Equation using a, b, and c" to establish the hyperbola equation. Calculate b using the relationship $c^2 = a^2 + b^2$: $b^2 = c^2 - a^2 = 18^2 - 6^2 = 324 - 36 = 288$, therefore, $b = \sqrt{288} = 12\sqrt{2}$.

Considering the hyperbola has been rotated, utilize the knowledge point "Introduction of Rotated Axes for Hyperbola." This information is crucial but affects only the orientation, not the direct computation in this specific problem context.

Lastly, compute the sum $h + k + a + b$ by adding these values together, aligning with the knowledge point "Summation of Coordinates and Constants". Thus, the sum $h + k + a + b = 3 + (-5) + 6 + 12\sqrt{2}$.

回答的问题和question完全不一样，不对

Third Try

Question

Consider a hyperbola with a rotated axes system, where the center is located at the point $(3, 1)$. One vertex of this hyperbola is positioned at $(7, 1)$, and one of its foci is at $(11, 1)$. The hyperbola is oriented such that its transverse axis is parallel to the x-axis. You are required to calculate the sum of the hyperbola's center coordinates, h and k , the distance a from the center to the vertex, and the length b of the semi-minor axis.

Solution

Step 1: Definition of Hyperbola Center: Identify the hyperbola's center as $(h, k) = (3, 1)$.

Step 2: Distance from Center to Vertex (a): Calculate the distance from the center to the vertex a . Given the coordinates, $a = |7 - 3| = 4$.

Step 3: Distance from Center to Focus (c): Determine the distance c from the center to the focus. $c = |11 - 3| = 8$.

Step 4: Derivation of Hyperbola Equation using a, b, and c: With $a = 4$ and $c = 8$, calculate b using the relationship $c^2 = a^2 + b^2$. Thus, $b^2 = c^2 - a^2 = 64 - 16 = 48$ and $b = \sqrt{48} = 4\sqrt{3}$.

Step 5: Introduction of Rotated Axes for Hyperbola: Confirm that the given hyperbola's equation can be represented in standard form relative to the rotated axes, with vertices and foci indicating no rotation is necessary (aligned along x-axis).

Step 6: Simplifying Complex Expressions: Simplify the expression b from the previous step: $b = 4\sqrt{3}$.

Step 7: Summation of Coordinates and Constants: Finally, compute the sum $h + k + a + b$: $h + k + a + b = 3 + 1 + 4 + 4\sqrt{3} = 8 + 4\sqrt{3}$.

This computed sum, $8 + 4\sqrt{3}$, represents the final answer, matching the construction requirements by using all specified knowledge points in order and maintaining mathematical integrity throughout.

- 计算正确，但是最后这个ask有歧义，导致4o又理解的不一样，但4o整体计算正确。
- GPT-4也理解错误，但整体计算正确。

Evolve 3

["Definition of Hyperbola Center", "Distance from Center to Vertex (a)", "Distance from Center to Focus (c)", "Derivation of Hyperbola Equation using a, b, and c", "Application of Rotation Matrices in Hyperbola Axes Transformation", "Simplifying Complex Expressions", "Summation of Coordinates and Constants"]

try1

Question

Consider a hyperbola with its center located at point $(h, k) = (-2, 5)$. One vertex of this hyperbola is positioned at $(-6, 5)$, and one of its foci is at $(-10, 5)$. Additionally, the transverse axis is parallel to the x-axis, and the conjugate axis is rotated by an angle of 30 degrees relative to the y-axis. You are required to calculate the equation of this hyperbola in its rotated axes system.

Solution

Step 1: Definition of Hyperbola Center: Identify the hyperbola's center as $(h, k) = (-2, 5)$.

Step 2: Distance from Center to Vertex (a): Calculate the distance from the center to the vertex a . Given the coordinates, $a = |-6 + 2| = 4$.

Step 3: Distance from Center to Focus (c): Determine the distance c from the center to the focus. $c = |-10 + 2| = 8$.

Step 4: Derivation of Hyperbola Equation using a, b, and c: With $a = 4$ and $c = 8$, calculate b using the relationship $c^2 = a^2 + b^2$. Thus, $b^2 = c^2 - a^2 = 64 - 16 = 48$ and $b = \sqrt{48} = 4\sqrt{3}$.

Step 5: Application of Rotation Matrices in Hyperbola Axes Transformation: To address the rotation of the axes, the standard form equation $\frac{(x')^2}{a^2} - \frac{(y')^2}{b^2} = 1$ needs to be transformed. With a rotation angle of 30 degrees, the rotation matrix R is applied, leading to the new variables: $x' = x \cos(30^\circ) + y \sin(30^\circ)$ and $y' = -x \sin(30^\circ) + y \cos(30^\circ)$. Plugging these into the standard form gives the equation: $\frac{(x \cos(30^\circ) + y \sin(30^\circ))^2}{16} - \frac{(-x \sin(30^\circ) + y \cos(30^\circ))^2}{48} = 1$.

Step 6: Simplifying Complex Expressions: Simplify the equation using trigonometric values $\cos(30^\circ) = \frac{\sqrt{3}}{2}$ and $\sin(30^\circ) = \frac{1}{2}$, resulting in $\frac{(\frac{\sqrt{3}}{2}x + \frac{1}{2}y)^2}{16} - \frac{(-\frac{1}{2}x + \frac{\sqrt{3}}{2}y)^2}{48} = 1$.

Step 7: Summation of Coordinates and Constants: There is no explicit summation of coordinates or constants required in the final step of the solution, but the computation is entirely consistent with the problem's

demands and the given hyperbola's rotated axes.

This derived equation, $\frac{(\frac{\sqrt{3}}{2}x + \frac{1}{2}y)^2}{16} - \frac{(-\frac{1}{2}x + \frac{\sqrt{3}}{2}y)^2}{48} = 1$, represents the final answer, matching the construction requirements by using all specified knowledge points in order and maintaining mathematical integrity throughout.

- step7没有work, 其实是有问题的。
- 算错了, 双曲线它没有考虑它给定的中心, 直接用了原点为中心。

try2

#Constructed Problem#: Consider a hyperbola whose center is located at the point $(4, -3)$ on a coordinate plane. The hyperbola's transverse axis is aligned with the x-axis. A vertex of the hyperbola is at $(9, -3)$, and one of its foci is situated at $(13, -3)$. The axes of this hyperbola have been rotated by an angle with respect to the standard coordinate axes. Calculate the sum of the squares of the coordinates of the center, the length of the semi-major axis (a), and the length of the semi-minor axis (b).

#Constructed Solution#: Step 1: Definition of Hyperbola Center: Identify the hyperbola's center as $(h, k) = (4, -3)$.

Step 2: Distance from Center to Vertex (a): Calculate the distance from the center to the vertex a . Given the coordinates, $a = |9 - 4| = 5$.

Step 3: Distance from Center to Focus (c): Determine the distance c from the center to the focus. $c = |13 - 4| = 9$.

Step 4: Derivation of Hyperbola Equation using a , b , and c : Using the hyperbola equation relationship $c^2 = a^2 + b^2$, calculate b with $a = 5$ and $c = 9$. Therefore, $b^2 = c^2 - a^2 = 81 - 25 = 56$ and $b = \sqrt{56} = 2\sqrt{14}$.

Step 5: Application of Rotation Matrices in Hyperbola Axes Transformation: Verify that the axes of the hyperbola have been rotated, but because the transformation aligns with the standard coordinate system (vertices and foci lie along the x-axis), the rotation does not affect the hyperbola's standard form equation or calculations.

Step 6: Simplifying Complex Expressions: Simplify the expression b from the previous step: $b = 2\sqrt{14}$.

Step 7: Summation of Coordinates and Constants: Calculate the sum of the squares of the coordinates of the center, h and k , and the lengths a and b . Hence, $h^2 + k^2 + a + b = 4^2 + (-3)^2 + 5 + 2\sqrt{14} = 16 + 9 + 5 + 2\sqrt{14} = 30 + 2\sqrt{14}$.

The final computed result, $30 + 2\sqrt{14}$, represents the answer to the problem, utilizing all specified knowledge points in the required order and maintaining mathematical precision throughout.

- 有歧义, 还是问题有歧义。我们还是找个问题形式不会特别容易引起歧义的吧。