

原问题

"problem": "The sphere with radius 1 and center $(0,0,1)$ rests on the xy -plane. A light source is at $P = (0,-1,2)$. Then the boundary of the shadow of the sphere can be expressed in the form $y = f(x)$, for some function $f(x)$. Find the function $f(x)$." "solution": "Let $O = (0,0,1)$ be the center of the sphere, and let $X = (x,y,0)$ be a point on the boundary of the shadow. Since OX is on the boundary, \overline{PX} is tangent to the sphere; let T be the point of tangency. Note that $\angle PTO = 90^\circ$. Also, lengths OP and OT are fixed, so $\angle OPT = \angle OPX$ is constant for all points X on the boundary.

`\nimport three;\nimport solids;\n\nsize(250);\nncurrentprojection = perspective(6,3,2);\n\ntriple O = (0,0,1), P = (0,-1,2), X = (3, 3^2/4 - 1, 0), T = P + dot(O - P, X - P)/dot(X - P,X - P)*(X - P);\n\nreal x;\n\npath3 shadow = (-1,1/4 - 1,0);\n\nfor (x = -1; x <= 3.1; x = x + 0.1) {\n shadow = shadow--(x,x^2/4 - 1,0);\n\n\ndraw(surface(shadow--(3,9/4 - 1,0)--(3,3,0)--(-1,3,0)--(-1,1/4 - 1,0)--cycle),gray(0.8),nolight);\n\ndraw((3,0,0)--(-2,0,0));\n\ndraw((0,3,0)--(0,-1.5,0));\n\ndraw(shadow);\n\ndraw(shift((0,0,1))*surface(sphere(1)),gray(0.8));\n\ndraw(O--P,dashed + red);\n\ndraw(P--X,red);\n\ndraw(O--T,dashed + red);\n\n\ndot("O", O, SE, white);\n\ndot("P", P, NW);\n\ndot("X", X, S);\n\ndot(T, red);\n\nlabel("T", T, W);\n\n}\n\nIf we take $X = (0,-1,0)$ and $T = (0,-1,1)$, we see that $\angle OPX = 45^\circ$. Hence, the angle between \overrightarrow{PX} and \overrightarrow{PO} is 45° . This means $\left[\begin{pmatrix} x \\ y + 1 \\ -2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right] \left[\begin{pmatrix} x \\ y + 1 \\ -2 \end{pmatrix} \right] \left[\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \right] = \cos 45^\circ = \frac{1}{\sqrt{2}}$. Then $\frac{(y + 1)(1) + (-2)(-1)}{\sqrt{x^2 + (y + 1)^2 + (-2)^2}} = \frac{1}{\sqrt{2}}$, or $y + 3 = \sqrt{x^2 + (y + 1)^2 + 4}$. Squaring both sides, we get $y^2 + 6y + 9 = x^2 + y^2 + 2y + 1 + 4$. Solving for y , we find $y = \frac{x^2}{4} - 1$. Thus, $f(x) = \boxed{\frac{x^2}{4} - 1}$.`

extract knowledge version2

["Spherical geometry", "Tangent lines to spheres", "Right angles in 3D geometry", "Fixed lengths and constant angles", "3D vector operations", "Dot product and projection in 3D", "Parametric equations for curves", "3D visualization and graphing", "Linear algebra in vector calculations", "Trigonometric ratios and their geometric interpretations", "Solving systems of equations", "Quadratic functions and their transformations"]

iter1

try 1

evolve knowledge method 1 version 2

["Spherical geometry", "Tangent lines to spheres", "Right angles in 3D geometry", "Fixed lengths and constant angles", "3D vector operations", "Cross product in 3D vector spaces", "Parametric equations for curves", "3D visualization and graphing", "Linear algebra in vector calculations", "Trigonometric ratios and their geometric interpretations", "Solving systems of linear equations using matrix methods", "Quadratic functions and their transformations"]

generate solution and problem

先solution 后 problem的问题生成的感觉不够满意，solution总是会到处乱求（没有确切的目标）

generate problem and solution

- first try失败，生成了一个看起来挺像回事的问题，但是实际数值是错的。
- second try:失败

version3

- first try失败，生成了一个非常困难的问题，o1推理不出来，原本的solution也是没有生成答案的。
- second try失败，可能原问题太难了。