

Not All Votes Count! Programs as Verifiers Improve Self-Consistency of Language Models for Math Reasoning

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Abstract

Large language models (LLMs) have shown increasing proficiency in solving mathematical reasoning problems. However, many current open-source LLMs often still make calculation and semantic understanding errors in their intermediate reasoning steps. In this work, we propose PROVE, a simple yet effective framework that uses program-based verification as a heuristic to filter out potentially incorrect reasoning paths before aggregating the final answers. Instead of relying on vanilla majority voting, our approach rejects solutions whose corresponding program outputs are inconsistent with the generated solution, aggregating only those validated by Python programs. We conducted extensive experiments on 13 open-source LLMs from various model families and sizes, ranging from 0.5B to 13B parameters, across seven math benchmarks. We demonstrate that PROVE consistently outperforms vanilla majority voting as a heuristic for solving mathematical reasoning tasks across all datasets and model sizes. Notably, PROVE increases accuracy on the GSM8K benchmark from 48.85% to 53.83% for Qwen2-0.5B-Instruct, from 65.66% to 73.01% for Llama-3.2-1B-Instruct, from 73.39% to 79.61% for Gemma-2-2b-it, and from 41.32% to 59.51% for Llama-2-7B-chat. Our codes are available at <https://github.com/declare-lab/prove>.

1 Introduction

Large Language Models (LLMs) have demonstrated remarkable success in mathematical reasoning tasks, especially with advanced models like GPT-4 (OpenAI et al., 2024). However, smaller open-source LLMs, such as Mistral, Gemma 2, and Llama 2 (Jiang et al., 2023; Team et al., 2024; Touvron et al., 2023) fall short in mathematical reasoning, often producing inaccurate content and

*Now at Deepmind.

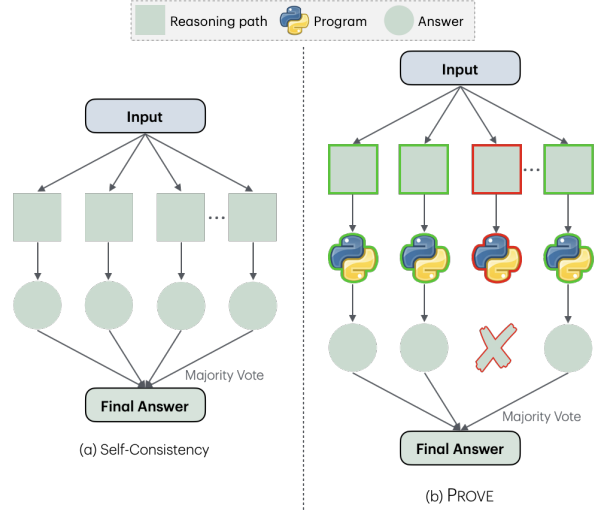


Figure 1: Comparison of self-consistency and PROVE.

failing to handle complex calculations. Many work since then have aimed to tackle those challenges by using different prompting methods such as Plan-and-Solve (PS) Prompting (Wang et al., 2023a) which aims to mitigate calculation errors and improve the overall quality of the generated reasoning by adopting a two-step process: first, devise a plan to divide the entire task into smaller subtasks and then carrying out the subtasks according to the plan. Non-prompting approaches, such as fine-tuning LLMs for reasoning tasks, have also shown performance improvements. However this typically requires large amounts of distilled data from models like GPT-4 (Wang et al., 2024; Gou et al., 2024), which comes with high computational and generation costs.

Despite these advancements, fundamental challenges remain (Banerjee et al., 2024). Autoregressive models lack mechanisms to correct their own errors, meaning that once a mistake is made, it can propagate through the entire response. As a result, LLMs are still vulnerable to hallucinations which can lead to incorrect reasoning or calcula-

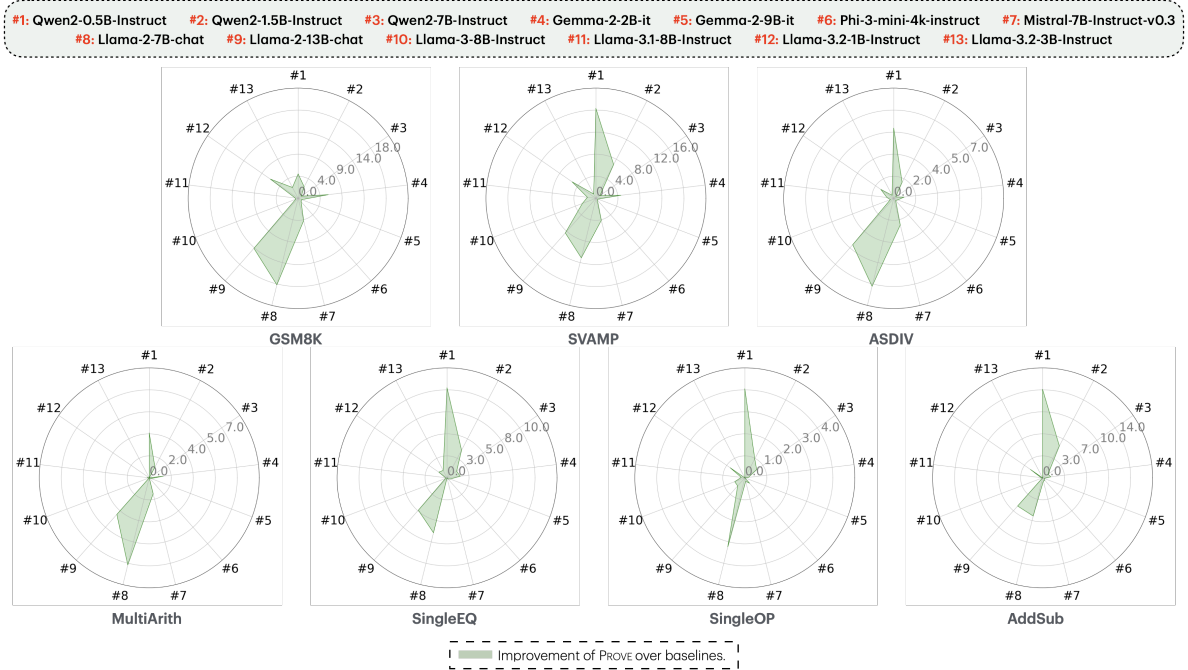


Figure 2: The improvement with PROVE when used with different LLMs on seven benchmarks.

tion mistakes. Given these limitations, a promising direction is to enhance reasoning by leveraging the knowledge already embedded within LLMs themselves (Wang et al., 2023b; Hao et al., 2023; Madaan et al., 2023; Weng et al., 2023; Wang et al., 2023a). Following Wang et al. (2023b), one can sample multiple reasoning paths to solve a question and select the most common answer. While aggregating answers like this improves performance, sometimes the most common answer could still be wrong. Recent efforts have tried to develop better heuristics for selecting answers from multiple reasoning paths, such as verification via formalization (Lin et al., 2024). However, this approach requires few-shot demonstration examples and more capable models, like GPT-3.5, to effectively verify solutions through autoformalization.

In this paper, we introduce PROVE, a *frustratingly simple* approach that utilizes **PRO**grams as **VER**ifiers. Our framework is model-agnostic and eliminates the need for LLM fine-tuning or few-shot prompting. Instead, it leverages the capabilities of current open-source LLMs to translate natural language plans and solutions into Python programs for verification. Unlike methods that rely on simple majority voting across all generated natural language solutions, we aggregate only those solutions that are successfully verified by their corresponding Python programs. While a natural language plan and solution provides an abstract outline

of the key steps in solving a math word problem, verifying its correctness can be challenging. Python program implementations offer a more precise representation, making it easier to accurately identify correct natural language solutions.

We evaluate PROVE using 13 LLMs from various model families and sizes (Abdin et al., 2024; Touvron et al., 2023; Dubey et al., 2024; Jiang et al., 2023; Team et al., 2024; Yang et al., 2024), ranging from 0.5B to 13B parameters across 7 mathematical reasoning datasets (Cobbe et al., 2021; Patel et al., 2021; Miao et al., 2020; Roy and Roth, 2015; Koncel-Kedziorski et al., 2015; Roy et al., 2015; Hosseini et al., 2014). The results demonstrate that PROVE consistently outperforms vanilla majority voting (Wang et al., 2023b) across all model sizes, evaluated on various math reasoning.

Contributions. We summarize the contributions of our paper as follows:

- We propose a novel framework PROVE, that utilizes programs as verifiers, serving as a heuristic to filter out potentially incorrect reasoning paths before aggregating the answers.
- Our experimental results demonstrate that PROVE significantly outperforms baseline methods across seven mathematical reasoning datasets.
- We present comprehensive ablation studies and qualitative analyses that clearly demonstrate the effectiveness of our framework.

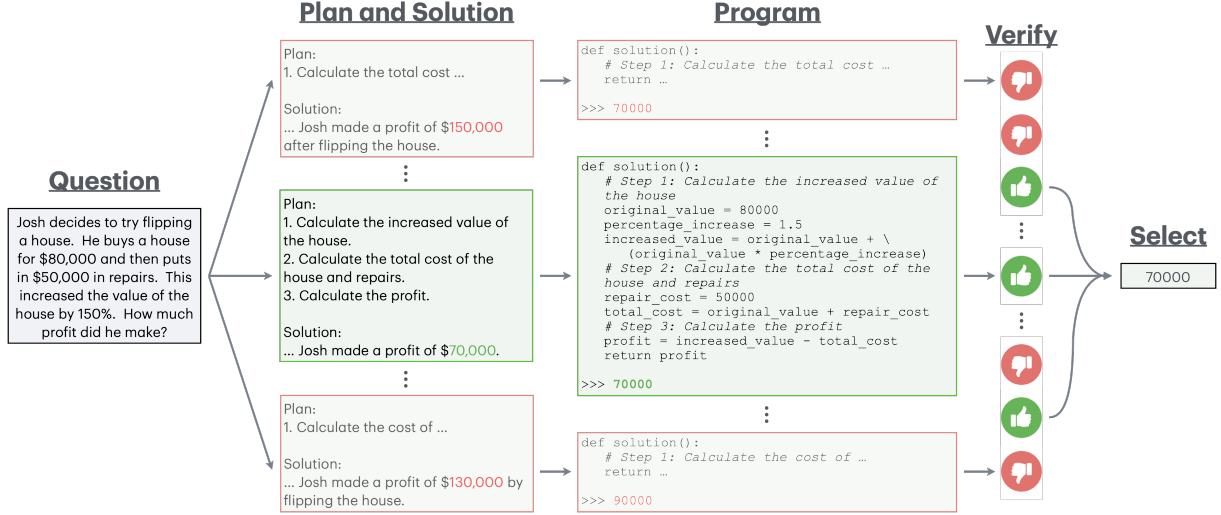


Figure 3: The PROVE Framework.

2 Method

Overview. As illustrated in Figure 3, PROVE framework starts by prompting an LLM to generate a plan and solution to solve a given math word problem. Next, we prompt an LLM to translate the generated plan and solution to a Python program. We then execute the Python program and compare its output to the generated solution. If the program’s output matches the solution, it is considered valid, otherwise, the solution is filtered out. Finally, the remaining valid solutions are undergo a majority voting process to determine the final answer. It’s important to note that all prompts in the PROVE framework are zero-shot.

Generating plan and solution. The first step in our framework involves prompting an LLM to generate a natural language plan and solution to solve a math word problems. For each math word problem x , we prompt an LLM to generate a plan and solution s using the zero-shot prompt proposed by Wang et al. (2023a): “Let’s first understand the problem and devise a plan to solve the problem. Then, let’s carry out the plan and solve the problem step by step”. We sample multiple plans and solutions from the LLM, using a temperature of 0.7, to produce a set of candidate plans and solutions $\{s_1, \dots, s_n\}$.

Translating plan and solution to Python programs. After obtaining the set of candidate plans and solutions $\{s_1, \dots, s_n\}$, we use each plan and solution s_i individually as input to an LLM, prompting it to generate a Python program p_i that implements the plan to solve the given math word prob-

lem. This process results in a corresponding Python program for each candidate plan and solution, yielding the pairs $\{(s_1, p_1), \dots, (s_n, p_n)\}$, which are then used for verification.

Verification and selection. Lastly, we execute the Python program p_i to obtain the output \hat{p}_i , which we use to verify the answer \hat{s}_i from the generated solution s_i . The answer \hat{s}_i is extracted using the answer extraction prompting method proposed by Kojima et al. (2022): “Therefore, the answer (arabic numerals) is”. We consider the answer \hat{s}_i valid if it matches the output \hat{p}_i from the Python program. Once all candidate plans and solutions are verified, we perform majority voting on the remaining valid answers to determine the final answer. If no valid answer remains, we perform majority voting over all invalid answers.

3 Experiments

3.1 Setup

Models and Datasets. PROVE is a model-agnostic framework designed to integrate with any LLM for mathematical reasoning tasks. We evaluated 13 LLMs from various model families and sizes, ranging from 0.5B to 13B parameters. Specifically, evaluated on Phi-3-mini-4k-instruct (3.8B) (Abdin et al., 2024), Llama-2-7B-chat (Touvron et al., 2023), Llama-2-13B-chat, Llama-3-8B-Instruct (Dubey et al., 2024), Llama-3.1-8B-Instruct, Llama-3.2-1B-Instruct, Llama-3.2-3B-Instruct, Mistral-7B-Instruct-v0.3 (Jiang et al., 2023), Gemma-2-2B-it (Team et al., 2024), Gemma-2-9B-it, Qwen2-0.5B-Instruct (Yang et al.,



Figure 4: An actual example of the prompts used in our framework, along with the outputs generated by Phi3-mini-4k-instruct, for the GSM8K task.

2024), Qwen2-1.5B-Instruct, and Qwen2-7B-Instruct. We conducted evaluations across seven mathematical reasoning datasets, including GSM8K (Cobbe et al., 2021), SVAMP (Patel et al., 2021), ASDIV (Miao et al., 2020), MultiArith (Roy and Roth, 2015), SingleEQ (Koncel-Kedziorski et al., 2015), SingleOP (Roy et al., 2015), and AddSub (Hosseini et al., 2014).

Implementation Details. In PROVE, we sample a total of 16 plans and solutions using a temperature of 0.7. To extract the answer from the generated solutions, we rely on the Phi-3-mini-4k-instruct model. We also used Phi-3-mini-4k-instruct to translate the plan and solution into Python programs for verification. With only 3.8 billion parameters, Phi-3-mini-4k-instruct offers more efficient inference. Notably, when Phi-3-mini-4k-instruct

is being evaluated, it performs self-verification by generating not only the plan and solution but also translating the plan and solution to a Python program to verify the results. During the selection phase, we perform majority voting on the remaining valid answer. If no valid answer remains, we perform majority voting over all invalid answers.

Baselines. We compare PROVE with two decoding strategies: single sampling (greedy decoding) and multiple sampling (self-consistency decoding) (Wang et al., 2023b). For the comparison, we use three different prompting techniques: zero-shot CoT (Kojima et al., 2022), zero-shot PoT (Chen et al., 2023), and zero-shot PS (Wang et al., 2023a). Performance is evaluated based on accuracy, where a problem is considered correct only if the final answer matches the ground truth.

Method	Qwen-2			Gemma-2		Phi-3	Mistral	Llama-2		Llama-3	Llama-3.1	Llama-3.2	
	0.5B	1.5B	7B	2B	9B	3.8B	7B	7B	13B	8B	8B	1B	3B
GSM8K													
Direct	37.00	61.56	88.02	67.32	88.63	85.67	59.06	30.55	42.84	79.38	85.67	48.75	79.23
Zero-shot CoT	32.22	57.71	88.25	67.63	89.69	86.35	58.83	29.80	41.24	80.74	87.49	50.72	81.88
Zero-shot PoT	5.23	2.81	63.61	50.80	80.89	84.38	48.52	8.49	26.99	73.46	84.38	42.15	33.06
Zero-shot PS	25.63	51.02	85.90	63.00	89.01	86.50	58.07	28.35	39.80	80.36	86.66	49.05	81.12
Zero-shot CoT @maj16	48.85	71.57	91.66	72.94	91.05	91.96	75.66	38.97	54.44	87.72	92.49	66.11	87.87
Zero-shot PoT @maj16	28.43	37.00	85.67	59.21	86.43	91.36	70.96	27.22	40.86	84.07	90.37	58.45	83.32
Zero-shot PS @maj16	42.00	68.46	90.98	73.39	91.96	92.19	78.54	41.32	54.21	89.61	92.57	65.66	88.86
PROVE	53.83	74.22	92.42	79.61	92.72	93.10	83.24	59.51	68.08	90.14	93.19	73.01	91.36
SVAMP													
Direct	45.70	65.90	92.40	72.90	89.70	91.70	69.30	61.40	66.20	83.00	89.10	70.90	85.70
Zero-shot CoT	41.00	62.10	89.90	74.40	88.70	89.10	71.80	56.40	62.80	85.00	86.80	68.40	86.50
Zero-shot PoT	13.60	5.40	47.00	73.40	91.80	90.70	52.80	25.50	44.10	86.70	88.00	62.60	66.30
Zero-shot PS	35.60	61.00	90.70	73.60	89.80	89.80	73.20	56.00	57.30	83.40	88.30	65.60	87.50
Zero-shot CoT @maj16	57.10	78.30	93.60	80.00	91.30	94.00	84.40	69.70	77.80	90.50	92.30	82.30	91.90
Zero-shot PoT @maj16	47.80	39.20	74.50	79.30	92.50	94.70	80.50	59.70	66.00	90.60	91.50	76.80	92.00
Zero-shot PS @maj16	56.60	79.60	93.70	81.70	91.40	93.60	84.70	71.30	77.80	91.20	92.90	81.50	92.80
PROVE	73.80	86.70	94.60	86.40	92.90	95.10	88.90	82.70	86.40	93.90	94.50	87.70	93.70
ASDIV													
Direct	60.02	75.43	91.94	81.30	91.70	93.32	73.57	61.31	67.51	85.26	89.17	76.91	88.88
Zero-shot CoT	55.10	73.14	91.89	80.68	91.60	91.84	74.52	58.59	65.03	85.35	89.12	75.91	89.22
Zero-shot PoT	18.46	6.35	48.09	71.66	82.20	83.73	54.20	32.40	48.71	78.86	86.74	67.18	59.49
Zero-shot PS	45.99	70.04	91.32	81.82	91.51	91.98	74.62	57.63	62.12	84.40	88.98	72.47	87.74
Zero-shot CoT @maj16	69.18	82.68	93.56	85.35	92.56	94.51	83.92	66.46	74.86	89.46	92.27	84.45	92.70
Zero-shot PoT @maj16	57.49	59.02	77.24	78.24	86.26	92.89	78.77	63.36	66.56	86.74	91.89	79.53	91.27
Zero-shot PS @maj16	68.03	83.35	93.46	87.02	92.99	94.18	84.78	69.27	75.43	89.69	92.22	85.07	92.75
PROVE	74.90	84.83	93.76	87.88	93.35	94.75	87.02	76.67	80.49	89.98	92.83	86.35	93.03
MULTIARITH													
Direct	79.33	95.83	97.67	94.00	98.50	98.33	78.50	74.50	82.67	98.50	97.00	71.83	96.50
Zero-shot CoT	78.17	93.50	98.67	94.00	98.17	98.17	83.67	74.33	76.67	96.67	97.83	85.67	97.83
Zero-shot PoT	18.00	4.33	59.67	88.67	98.00	96.67	61.67	29.83	60.17	97.33	97.83	78.67	62.33
Zero-shot PS	62.33	87.50	97.83	94.33	98.67	98.50	83.50	70.67	70.00	97.83	97.33	83.67	97.00
Zero-shot CoT @maj16	91.83	98.00	98.33	97.17	98.33	98.83	91.50	85.67	92.17	98.83	98.00	97.00	99.00
Zero-shot PoT @maj16	78.33	49.67	90.33	95.00	98.67	98.67	92.83	77.83	79.00	99.00	99.00	95.83	99.00
Zero-shot PS @maj16	91.33	98.33	98.17	97.00	98.67	98.83	94.33	90.33	92.17	98.17	98.50	97.83	98.50
PROVE	95.50	99.33	98.33	98.33	98.83	98.83	95.67	97.67	96.17	99.00	99.17	97.83	99.00
SINGLEEQ													
Direct	72.44	88.19	95.47	91.14	94.88	96.06	85.24	79.92	82.68	93.50	93.31	89.96	94.69
Zero-shot CoT	67.72	85.24	97.44	92.32	96.65	98.23	83.86	78.54	83.07	93.70	94.49	87.40	95.47
Zero-shot PoT	25.00	5.91	49.61	94.49	98.23	98.03	69.49	41.34	69.69	94.49	96.26	84.06	62.40
Zero-shot PS	62.80	81.69	96.65	92.13	97.24	98.03	85.83	75.00	76.57	91.14	95.28	84.45	93.90
Zero-shot CoT @maj16	81.50	90.75	96.85	94.49	97.24	98.82	89.37	83.27	92.13	96.85	97.24	94.09	97.83
Zero-shot PoT @maj16	80.31	71.06	79.33	96.06	98.82	99.02	94.69	86.22	88.78	98.03	98.43	94.49	97.64
Zero-shot PS @maj16	80.71	93.11	97.44	95.08	98.03	98.23	91.14	85.24	91.73	97.24	97.83	94.88	97.24
PROVE	92.13	96.80	98.82	97.64	99.21	99.21	94.69	92.91	97.24	98.23	98.62	96.06	98.82
SINGLEOP													
Direct	77.76	90.75	96.09	91.28	95.37	96.8	90.04	83.45	88.79	93.95	95.73	92.35	96.26
Zero-shot CoT	6.33	89.32	96.26	91.64	95.91	96.26	90.93	85.41	85.94	94.13	96.09	88.43	95.91
Zero-shot PoT	29.54	10.5	44.84	90.57	95.73	95.55	72.78	51.25	68.51	93.24	95.91	84.70	74.02
Zero-shot PS	66.19	88.26	96.26	93.42	96.26	96.80	90.75	81.67	81.32	91.10	95.55	86.83	94.13
Zero-shot CoT @maj16	87.37	92.88	96.26	94.13	96.09	96.80	94.66	88.79	92.70	96.62	96.80	93.77	96.80
Zero-shot PoT @maj16	84.52	77.22	80.60	92.88	95.73	96.80	93.59	89.86	85.77	95.37	96.44	91.99	96.44
Zero-shot PS @maj16	88.08	93.59	96.26	94.66	96.62	97.15	94.84	91.10	91.28	96.44	96.62	93.06	96.62
PROVE	92.53	94.66	96.98	94.84	96.62	97.51	95.02	94.66	93.24	97.16	96.98	94.66	96.80
ADDSUB													
Direct	61.27	80.51	92.91	84.05	90.38	93.16	81.01	73.16	82.53	88.10	89.62	84.56	90.38
Zero-shot CoT	58.73	79.75	94.94	86.33	94.94	95.95	82.28	73.67	81.01	89.37	92.66	84.56	93.42
Zero-shot PoT	49.62	7.34	28.61	53.92	95.95	95.7	70.38	42.53	65.57	93.67	95.44	78.73	88.86
Zero-shot PS	51.39	77.47	92.91	89.11	95.19	94.43	83.29	73.42	75.19	85.32	92.66	77.97	90.89
Zero-shot CoT @maj16	70.89	86.58	95.44	90.89	94.43	96.2	85.82	80.51	86.08	93.42	95.19	89.62	95.95
Zero-shot PoT @maj16	71.39	64.81	63.54	90.89	95.95	96.46	91.14	81.52	85.06	95.19	95.95	82.53	95.95
Zero-shot PS @maj16	71.14	85.82	94.94	91.14	95.7	96.46	88.35	83.29	87.34	93.16	94.43	90.13	94.68
PROVE	85.06	92.15	96.46	92.41	96.20	96.96	91.65	89.37	93.16	95.19	96.20	92.41	95.95

Table 1: Main results showing the comparison of PROVE and baseline methods across 13 LLMs, ranging in size from 0.5B to 13B, on seven mathematical reasoning datasets.

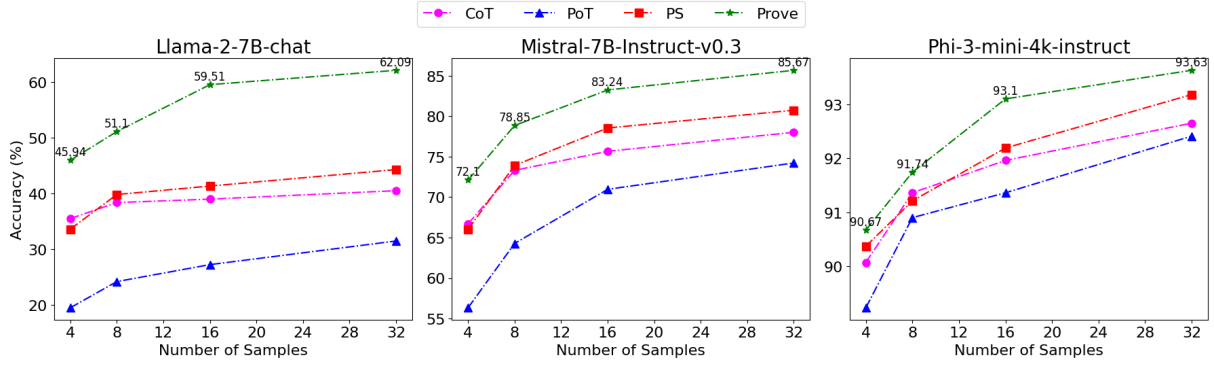


Figure 5: Performance comparison of PROVE and baseline methods across various LLMs evaluated on GSM8K with different number of samples.

3.2 Main Results

We report the main results of our experiments in Table 1. We highlight several key observations:

PROVE improves over baselines consistently on all evaluated LLMs across seven datasets. Our evaluations of PROVE on various model families and sizes reveal consistent improvements over strong baseline models. For instance, on the GSM8K dataset, Mistral 7B achieves an accuracy of 83.24% with PROVE, compared to 78.54% with the best baseline (Zero-shot PS @maj16), reflecting an improvement of 4.7%. In the SVAMP dataset, Mistral 7B reaches 88.90%, surpassing the best baseline by 4.2%. Similarly, in the ASDIV dataset, Mistral 7B achieves an accuracy of 87.02%, exceeding the baseline’s 84.78% by 2.24%. Furthermore, Mistral 7B shows consistent performance gains of up to 3.39% in other datasets, including MultiArith, SingleEq, SingleOp, and AddSub. These improvements are consistent across all twelve LLMs we evaluated.

PROVE provides larger performance gains for smaller models. In our comparison of various model families, including Qwen 2, Gemma 2, Llama 2, and Llama 3.2, we found that smaller models tend to exhibit greater performance gains than their larger counterparts. For example, in the GSM8K dataset, the smaller Llama 2 7B model shows an improvement of 18.19% over the baseline, while the larger Llama 2 13B model only achieves a gain of 13.87%. Similarly, Gemma 2 7B demonstrates a 6.22% increase in accuracy compared to the baseline, whereas the larger Gemma 2 9B model achieves only a 0.76% improvement. The Qwen 2 models also reflect a similar trend, the 0.5B variant outperforms its baseline by 4.98%, while

the 1.5B model improves by 2.65%. In contrast, the larger Qwen 2 7B model shows only a 0.76% improvement. Likewise, Llama 3.2 1B demonstrates a 7.35% improvement compared to its larger variant, the 3B model, which only achieves a 2.5% increase. These results indicate that smaller models benefit more significantly from PROVE, likely because they are more prone to errors, such as miscalculations, making the filtering more beneficial for them.

Newer models closing the gap with larger ones. More recent Llama models, such as Llama 3.2, demonstrate significant progress in closing the performance gap with older, larger models like Llama 2 and Llama 3, despite their smaller size. For example with PROVE on datasets like GSM8K, Llama 3.2 1B outperforms Llama 2 13B achieving 73.01% accuracy versus 68.08%, and Llama 3.2 3B outperforms Llama 3 8B achieving 91.36% over 90.14%. On datasets like ASDIV, the performance of Llama 3.2 1B and Llama 3.2 3B increases the separation even further, reflecting improvements in architecture and training techniques to increase performance of smaller models (Meta, 2024).

3.3 Ablation Study

Impact of the Number of Plans. In Figure 5, we compare the accuracy of CoT, PoT, and PS prompting methods with SC and PROVE across different numbers of sampled solutions on GSM8K. As the number of sampled solutions increases from 4 to 32, PROVE consistently outperforms the baseline methods, demonstrating its effectiveness. Notably, with just 4 samples, the Llama-2-7b-chat and Mistral-7B-Instruct-v0.3 models show significant accuracy gains of 10.46% and 5.39%, respectively. With 32 samples, Llama-2-7b-chat and Mistral-7B-Instruct-v0.3 models have a gain of 17.81% and 4.93%.

<p>Question: A mother goes shopping. She buys cocoa at \$4.20, laundry at \$9.45 and a package of pasta at \$1.35. She pays \$20. How much change does the cashier give back?</p> <p>Generated Solution: To find the change the cashier gives back, we need to subtract the total cost of the items purchased from the amount the mother paid.</p> <p>Step 1: Add the costs of all the items purchased: - Cocoa: \$4.20 - Laundry: \$9.45 - Pasta: \$1.35 Total cost = $\\$4.20 + \\$9.45 + \\$1.35 = \\15.10</p> <p>Step 2: Subtract the total cost from the amount paid: Amount paid = \$20 Change = Amount paid - Total cost Change = \$20 - \$15.10 Change = \$4.90</p> <p>Therefore, the cashier gives back \$4.90 as change.</p> <p>Translated Python Program: def solution(): # Step 1: Add the costs of all the items purchased cocoa_cost = 4.20 laundry_cost = 9.45 pasta_cost = 1.35 total_cost = cocoa_cost + laundry_cost + pasta_cost # Step 2: Subtract the total cost from the amount paid amount_paid = 20 change = amount_paid - total_cost # Return the change return change</p>	<p>Question: Jane counts two zebras with 17 stripes each, a zebra with 36 stripes, and another zebra with half that many stripes. How many stripes do the zebras have on average?</p> <p>Generated Solution: To find the average number of stripes on the zebras, we need to calculate the total number of stripes and then divide it by the number of zebras.</p> <p>Step 1: Calculate the total number of stripes. - Jane's zebras have 17 stripes each, so she has $2 * 17 = 34$ stripes. - The zebra with 36 stripes contributes 36 stripes. - The zebra with half the number of stripes ($36 / 2 = 18$) contributes 18 stripes. Total number of stripes = $34 + 36 + 18 = 88$</p> <p>Step 2: Calculate the average number of stripes. There are 3 zebras in total. To find the average, we divide the total number of stripes by the number of zebras: Average number of stripes = Total number of stripes / Number of zebras = $88 / 3 = 29.33$</p> <p>So, the average number of stripes on the zebras is approximately 29.33.</p> <p>Translated Python Program: def solution(): # Step 1: Calculate the total number of stripes zebra_1_stripes = 17 zebra_2_stripes = 17 zebra_3_stripes = 36 zebra_4_stripes = zebra_3_stripes / 2 total_stripes = (zebra_1_stripes * 2) + zebra_3_stripes + zebra_4_stripes # Step 2: Calculate the average number of stripes per zebra num_zebras = 4 average_stripes = total_stripes / num_zebras return average_stripes</p>
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Figure 6: Case study examples where PROVE gets correct when vanilla majority voting fails. The generated solution is by Llama-3.2-1B-Instruct on a GSM8K type question. In both examples, we show how the translated program can correctly filter out generated solution that have errors in them. Left: highlighted in red shows a calculation error. Right: highlighted in red shows semantic understanding error.

Model	Translation Model	Accuracy (%)
Llama-3.2-1B-Instruct	Phi-3-mini-4k-instruct	73.01
	Llama-3-8B-Instruct	73.54
	Llama-3.1-8B-Instruct	74.53
Llama-2-7B-chat	Phi-3-mini-4k-instruct	59.51
	Llama-3-8B-Instruct	58.15
	Llama-3.1-8B-Instruct	61.94
Mistral-7B-Instruct-v0.3	Phi-3-mini-4k-instruct	83.24
	Llama-3-8B-Instruct	80.29
	Llama-3.1-8B-Instruct	83.32

Table 2: Performance of PROVE on GSM8K, evaluated across various translation models and LLMs.

Model	Prompting Method	Accuracy (%)
Llama-3.2-1B-Instruct	Direct	75.59
	CoT	75.89
	PS	73.01
Llama-2-7B-chat	Direct	59.06
	CoT	59.29
	PS	59.51
Mistral-7B-Instruct-v0.3	Direct	82.34
	CoT	82.79
	PS	83.24

Table 3: Performance of PROVE on GSM8K, with different prompting strategies across various LLMs.

Impact of Translation Model. We study the impact of model selection for translating plans and solutions into Python programs on GSM8K. In our experiments, the default translation model used is Phi-3-mini-4k-instruct. We tested Llama-3-8B-Instruct and Llama-3.1-8B-Instruct for translating the outputs of LLMs, which include Llama-3.2-1B-Instruct, Llama-2-7B-chat, and Mistral-7B-Instruct-v0.3. Table 2 shows the performance variations across different translation models for the different LLMs. We observe that Llama-3.1-8B-Instruct as the translation model, with more parameters than Phi-3-mini-4k-instruct, leads to slight performance improvements for all tested LLMs, having performance gains of 1.52%, 2.43%, and 0.08% for Llama-3.2-1B-Instruct, Llama-2-7B-chat, and Mistral-7B-Instruct-v0.3, respectively.

Impact of Plan and Solve (PS) Prompting. We investigated the impact of PS prompting on our framework by comparing its effectiveness against both direct prompting and CoT prompting in the first stage of PROVE. We evaluated on three different models, namely Llama-3.2-1B-Instruct, Llama-2-7B-Chat, and Mistral-7B-Instruct-v0.3, on GSM8K. As shown in Table 3, the overall performance differences between the prompting methods are relatively small. The largest difference observed is 2.88%, with CoT outperforming PS when evaluating on Llama-3.2-1B-Instruct. Our findings indicate that even with variations in the prompting method during the first stage, PROVE consistently maintains performance, demonstrating that it is prompt-agnostic in this stage of the framework.

3.4 Qualitative Analysis

PROVE gets correct when vanilla majority vote fail. In Figure 6, we show two case study examples of PROVE that correctly filter generated solutions that have errors in them. These two examples are examples that majority voting fails to solve due to the correct answer not being the most common answer, whereas, PROVE is able to successfully filter out incorrect reasoning paths, making the correct answer as the majority answer. In Figure 6 (left), the model did a wrong calculation “ $\$4.20 + \$9.45 + \$1.35 = \15.10 ”, whereas the correct calculation should be “ $\$15.00$ ”. However in our PROVE framework, the translated program is able to detect the miscalculation from the generated solution and filter it out. Similarly in Figure 6 (right), the model had a semantic understanding error where it fails to understand that the total number of zebra should be 4 instead of 3. Again, our translated program is able to point out the correct value and filter it out this generated solution. These examples demonstrate that by leveraging programs as verifiers, we can effectively filter out reasoning paths that contain errors, such as miscalculations and semantic misunderstandings.

Method	Calculation	Missing	Semantic
Zero-shot PS @maj16	20%	12%	68%
PROVE	0%	18%	82%

Table 4: Distribution of error types (calculation error, missing step error, semantic understanding error) for 50 randomly sampled examples from GSM8K, where both PS and PROVE got incorrect, using Llama-3.2-1B-Instruct.

Error Analysis. To better understand the effectiveness of PROVE, we conducted a manual analysis of 50 randomly sampled examples from GSM8K, where both PS and PROVE got incorrect, using Llama-3.2-1B-Instruct. Following (Wang et al., 2023a), we categorized the errors into three types: calculation errors, missing step errors, and semantic understanding errors. The results of this analysis (Table 4) show that PROVE achieves a 0% calculation error rate, significantly lower than PS, which had a 20% calculation error rate. This demonstrates that PROVE effectively minimizes calculation errors, as illustrated in the case study example shown in Figure 6 (left). Additionally, in some instances, it also reduces semantic understanding errors, as depicted in Figure 6 (right).

4 Related Works

Reasoning with Large Language Models. Recent advances in LLMs have demonstrated remarkable capabilities in tackling complex reasoning tasks. Prior research highlights that step-by-step reasoning prompts, such as CoT (Kojima et al., 2022) and PS (Wang et al., 2023a), improve performance compared to directly generating answers. Other techniques, such as multi-step decoding (Yao et al., 2023), explore diverse reasoning paths, while methods like multi-sample reasoning (e.g., majority voting (Wang et al., 2023b)) aggregate solutions to enhance robustness. However, these approaches still heavily depend on the LLM’s generative outputs, which are prone to hallucination, particularly in smaller models. Our method, PROVE, complements these by introducing a verification step that uses Python programs to filter out potentially flawed reasoning paths, resulting in more accurate solution aggregation.

Large Language Models as Verifiers. Using language models to evaluate model generations has been a long standing idea (Kushman et al., 2014; Roy and Roth, 2015; Shen et al., 2021). A common approach involves training a separate verification model to assess the correctness of generated outputs (Cobbe et al., 2021). Other techniques, such as step-by-step verification (Lightman et al., 2024) and ranking multiple reasoning paths to choose the most accurate one (Weng et al., 2023), show promise in reducing errors but often rely heavily on the inherent reasoning abilities of the model, limiting their effectiveness, especially in smaller models. While program-based verification approaches have been proposed (Zhou et al., 2024a; Han et al., 2024; Zhou et al., 2024b), they tend to be more complex, often requiring few-shot prompting and more capable models. In contrast, PROVE is simpler, does not require few-shot exemplars, and is easily adaptable to smaller, open-source LLMs.

5 Conclusion

In this paper, we demonstrate how using programs as verifiers can effectively filter out reasoning paths that contain calculation or semantic understanding errors. Our approach, PROVE, is model-agnostic and does not require fine-tuning or few-shot exemplars for prompting. PROVE consistently outperform baseline methods across 13 LLMs and seven mathematical reasoning datasets.

6 Limitations

One limitation of this approach is that Python programs cannot directly parse LaTeX syntax, making it challenging to evaluate datasets that heavily rely on LaTeX, such as MATH (Hendrycks et al., 2021). As a result, due to the simplicity of our method, PROVE is currently limited to working only on math datasets where the final answer is numeric. Future work will focus on adapting our framework to accommodate LaTeX-intensive math datasets.

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