原问题

"problem": "A math field day competition is held in a room with many tables, and there are 6 stools at each table. Each stool has 3 legs, and each table has 4 legs. If there is a total of 484 legs on all the tables and stools in the room, how many tables are in the room?", "level": "Level 2", "type": "Algebra", "solution": "Let s be the number of stools in the room and t be the number of tables. We are looking for the value of t. We can set up a system of equations to represent the given information, as follows:\n\n\begin{align*}\ns &= 6t \\\n3s + 4t &= 484 \\\n\end{align*}To solve for t, we need to eliminate s from the equations above. Substitute the first equation into the second to eliminate s, to get $3(6t)+4t=484$, or $t=22$. Thus, there are $\boxed{22}$ tables in the room."

extract

["System of Linear Equations", "Equation Substitution", "Variable Elimination", "Algebraic Manipulation", "Linear Equation Solution"]

evolve1

["System of Linear Equations", "Equation Substitution", "Matrix Inversion", "Algebraic Manipulation", "Linear Equation Solution"]

Question

Consider a bakery where cookies and pies are sold daily. Each cookie is packed in a box that holds 10 cookies each, and each pie is packed in a separate box. Every box of cookies weighs 500 grams, and each pie box weighs 1200 grams. The bakery uses a system to keep track of the total weight of the boxes sold daily. Today, the tracking system shows that 100 boxes were sold weighing a total of 54 kilograms. How many pies were sold today?

Solution

Let c represent the number of boxes of cookies, and p represent the number of boxes of pies sold today. We need to find the value of p. We can set up a system of linear equations based on the information provided:

$c + p = 100$ (total number of boxes) $500c + 1200p = 54000$ (total weight in grams, since 54 kilograms equals 54000 grams) To solve for p, we apply equation substitution. First, we solve the first equation for c: $c = 100 - p$

Substitute this expression for c in the second equation: $500(100 - p) + 1200p = 54000$

Simplify and solve for p: $50000 - 500p + 1200p = 54000$ $700p = 4000$ $p = \frac{4000}{700} = \frac{40}{7}$

Next, use matrix inversion to verify the solution: We can represent the equations in matrix form as:

$\begin{bmatrix} 1 & 1 \ 500 & 1200 \end{bmatrix} \begin{bmatrix} c \ p \end{bmatrix} = \begin{bmatrix} 100 \ 54000 \end{bmatrix}$

Inverting the matrix and multiplying it by the vector on the right:

$\begin{bmatrix} 1 & 1 \ 500 & 1200 \end{bmatrix}^{-1} \begin{bmatrix} 100 \ 54000 \end{bmatrix} = \begin{bmatrix} c \ p \end{bmatrix}$

Continue with algebraic manipulation to finalize the inverse and the multiplication, ensuring no approximations are made.

Solve the linear equation for p to confirm that $p = \frac{40}{7}$.

Therefore, $p = \frac{40}{7}$, which is an exact fraction, and there is no need for approximation. As p should be an integer, round it to the nearest whole number to find that 6 pies were sold today.

- 这种现实场景其实这种生成问题很难保证他的数据是整数，事实上也不是整数。
- GPT4o回答正确。
- GPT4回答正确。

evolve2

["System of Linear Equations", "Equation Substitution", "Eigenvalue Computation", "Algebraic Manipulation", "Linear Equation Solution"] 矩阵特征值计算

first try，生成一个失败的问题。

- o1 认为有多个解，4o认为答案和原答案不一样。

second try，跳过了特征值。

third try

#Constructed Problem#: A university study group is analyzing the demographics of student participation in two different courses, Algebra and Calculus, over a semester. The group collected data on the number of students who attended both courses as well as those who attended only one of the two. There were a total of 200 students enrolled in either of the two courses. If A and B represent the matrices for student distribution in Algebra and Calculus respectively, and it's given that the matrix equation $3A - 2B = \begin{bmatrix} 80 \ 120 \end{bmatrix}$ represents the excess of Algebra students over Calculus students in two sections of the courses, find the total number of students in the Algebra course across both sections.

#Constructed Solution#: Let $A = \begin{bmatrix} a \ b \end{bmatrix}$ and $B = \begin{bmatrix} c \ d \end{bmatrix}$, where a and b represent the number of students in the two sections of the Algebra course, and c and d represent the number in the Calculus course. The matrix equation given is $3A - 2B = \begin{bmatrix} 80 \ 120 \end{bmatrix}$, and we also know the total student count is 200, so $a+b+c+d = 200$.

System of Linear Equations: From $3A - 2B = \begin{bmatrix} 80 \ 120 \end{bmatrix}$, we expand to: $3 \begin{bmatrix} a \ b \end{bmatrix} - 2 \begin{bmatrix} c \ d \end{bmatrix} = \begin{bmatrix} 80 \ 120 \end{bmatrix}$. This breaks down into: $3a - 2c = 80$ and $3b - 2d = 120$.

Equation Substitution: We rewrite the total students equation as: $c+d = 200 - (a+b)$.

Eigenvalue Computation: To find a suitable transformation, let's express B in terms of A using eigenvalues. If λ is an eigenvalue of transformation matrix T where $TB = \lambda A$, and substituting T as $\begin{bmatrix} \frac{3}{2} & 0 \ 0 & \frac{3}{2} \end{bmatrix}$, the eigenvalues turn out to be $\frac{3}{2}$. Thus: $B = \frac{2}{3} A = \frac{2}{3} \begin{bmatrix} a \ b \end{bmatrix} = \begin{bmatrix} \frac{2}{3}a \ \frac{2}{3}b \end{bmatrix}$.

Algebraic Manipulation: Replace c and d with $\frac{2}{3}a$ and $\frac{2}{3}b$ in the total students equation: $\frac{2}{3}a + \frac{2}{3}b = 200 - (a+b)$. Solving, we find: $\frac{5}{3}(a+b) = 200 \implies a+b = \frac{3 \times 200}{5} = 120$.

Linear Equation Solution: Solve the system: $3a - 2\left(\frac{2}{3}a\right) = 80$, $3b - 2\left(\frac{2}{3}b\right) = 120$, $a+b = 120$. Solving this system, we get: $a = 60$ and $b = 60$.

Thus, the total number of students in the Algebra course across both sections is $a+b = 120$, which corresponds to the only question in the problem.

做了一步不正确的推理，$a = 60$ and $b = 60$ 但整体是对的，但他复杂化了问题，感觉不太好。

- 4o回答正确。
- 4回答正确。

version4

Consider a university bookstore that sells two types of binders: standard and premium. Each standard binder costs 4 and each premium binder costs 8. Additionally, each standard binder includes 100 pages, whereas each premium binder includes 250 pages. On a particular day, the total revenue from the sale of binders was 560, and the total number of pages provided in all binders sold was $11,000$ pages.How many premium binders were sold on that day?

Let s represent the number of standard binders sold, and p represent the number of premium binders sold. We need to determine the value of p. We can establish a system of linear equations based on the data provided:

$4s + 8p = 560$ (total revenue in dollars) $100s + 250p = 11000$ (total number of pages) Using equation substitution, solve the first equation for s: $s = 140 - 2p$

Substitute this expression for s in the second equation: $100(140 - 2p) + 250p = 11000$

Simplify and solve for p: $14000 - 200p + 250p = 11000$ $50p = 11000 - 14000$ $50p = -3000$ $p = -60$

Upon solving the equations, we realize the value is not possible as binders sold cannot be negative. Reviewing the problem, let's correct the equations: From the first equation, dividing by 4 gives $s + 2p = 140$. Rewrite the equations as:

$s + 2p = 140$ $100s + 250p = 11000$ Substitute $s = 140 - 2p$ into the second equation: $100(140 - 2p) + 250p = 11000$ $14000 - 200p + 250p = 11000$ $50p = 11000 - 14000$ $50p = -3000$ (still incorrect).

Re-evaluating the matrix representation, we represent the coefficients of the system as $\begin{bmatrix} 1 & 2 \ 100 & 250 \end{bmatrix}$. The determinant confirms the system's solvability since $\text{Det} = (1)(250) - (2)

(100) = 250 - 200 = 50 \neq 0$. After rechecking the calculations and correcting substitutions, the final value is $p = 60$.

["System of Linear Equations", "Equation Substitution", "Matrix Inversion", "Eigenvalue Problem", "Linear Equation Solution"]