

应该把现有的条件组成一个有向图结构（mind map）

允许多做一些假设。在之前的设计上证明有效。

先提取问题中的premise

You are a Mathematics Expert in the field of extracting key information. Your target is to extract key mathematical premises and the question in one given problem. The result in #Mathematical Premises# MUST be the json array in the json format,each item in the array is a string describe the mathematical premise. String in the result MUST keep the same mathematical format and language specification as #Original Problem#. You CANNOT omit the non-text parts such as the table and code in #Original Problem#. Also, please do not omit the input in #Original Problem#. You SHOULD complicate the #Mathematical Premises# and #Mathematical Question# using the following method: Check each token in the problem and identify the definition part and question part of the problem in #Original Problem#. Identify each premise in the definition part and append it into #Mathematical Premises#. Try you best to find the implied premise in the #Original Problem#. Each item in the list in #Mathematical Premises# MUST contain ONLY ONE mathematical premise. Fill in the #Mathematical Question# with the question part. Try your best to make sure we can reconstruct the problem using the extracted #Mathematical Premises# and #Mathematical Question#. #Original Problem#: #Mathematical Premises#:

#Mathematical Question#:

#Mathematical Premises#:

```
[
  "The fifth term of a geometric sequence of positive numbers is $11$.",
  "The eleventh term of the geometric sequence is $5$."
]
```

#Mathematical Question#: What is the eighth term of the sequence? Express your answer in simplest radical form.

["The series is a geometric series with first term 1 and common ratio 7", "The series has 2005 terms, from 7^0 to 7^{2004} ", "We are interested in the remainder when the sum of this series is divided by 1000"

Determine the value of N , the remainder when $1 + 7 + 7^2 + \cdots + 7^{2004}$ is divided by 1000 .

细粒度提取每一步条件，方法，结论

You are a Mathematics Expert in the field of extracting key information in the process. Your target is to extract the triple of {premises, method and conclusion} in each step of a given mathematical solution process in #Original Solution#. The result in #Extract Triples# MUST be a json array in the json format, and each triple in the array describes a single step in the process, including three attributes: premises, method and conclusion. String in #Extract Triples# MUST keep the same mathematical format and language specification as #Original Solution#. You SHOULD make sure the triples are in the order of their

appearances in the original solution. Strings in #Initial Premises# indicate the initial premise. You SHOULD complicate the #Extract Triples# using the following method: Check each step in the process of solution and extract detailed premises, method and the conclusion. Each triple in #Extract Triples# SHOULD be fine-grained. Premises in LATER steps MUST STRICTLY equal to conclusions in FORMER steps or #Initial Premises#. FORMER conclusion are NOT ALLOWED to appear in LATER conclusion. #Initial Premise# are NOT ALLOWED to appear in conclusion. The conclusion in each triple SHOULD contain ONLY ONE mathematical result. Try you best to find the implied premise used in the method and conclusion in each triple. You SHOULD check the connection between each step in #Original Solution# and reflect the connection in the exacted triples. ANY other information is STRICTLY forbidden to use in the method and conclusion of each triple in #Extract Triples# except its corresponding premises in the same triple. The method in each triple SHOULD perform All the mathematical calculation process and get calculation results. The conclusion of each triple SHOULD be an intermediate conclusion on this question and describe clearly. The conclusion of the LAST triple in the #Extract Triples# MUST answer the #Original Question# in detail. Split the conclusion into two conclusions if it contains more than ONE conclusions. Try your best to make sure we can reconstruct the solution only using #Extract Triples#. #Initial Premises#: #Original Question#: #Original Solution#: #Extract Triples#:

The fifth term of a geometric sequence of positive numbers is \$11\$. The eleventh term of the geometric sequence is \$5\$. The eighth term is $11r^3$ and the eleventh term can be expressed as $11r^6 = 5$. $r^6 = \frac{5}{11}$. $r^3 = \sqrt[3]{\frac{5}{11}}$. $11r^3 = 11 \cdot \sqrt[3]{\frac{5}{11}} = \sqrt[3]{55}$.

```
[
  {
    "premises": [
      "The fifth term of a geometric sequence of positive numbers is $11$.",
      "The eleventh term of the geometric sequence is $5$."
    ],
    "method": "Using the properties of geometric sequences, express the eighth term and the eleventh term in terms of the common ratio $r$ and the fifth term. Set up an equation for the eleventh term based on these expressions.",
    "conclusion": "The eighth term is $11r^3$ and the eleventh term can be expressed as $11r^6 = 5$."
  },
  {
    "premises": [
      "The eleventh term can be expressed as $11r^6 = 5$."
    ],
    "method": "Solve for $r^6$ from the equation $11r^6 = 5$.",
    "conclusion": "$r^6 = \frac{5}{11}$"
  },
  {
    "premises": [
      "$r^6 = \frac{5}{11}$"
    ],
    "method": "Take the cube root of both sides of the equation $r^6 = \frac{5}{11}$ to find $r^3$.",
    "conclusion": "$r^3 = \sqrt[3]{\frac{5}{11}}$"
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    },
    {
      "premises": [
        "The eighth term is  $11r^3$ ",
        " $r^3 = \sqrt{\frac{5}{11}}$ "
      ],
      "method": "Substitute  $r^3 = \sqrt{\frac{5}{11}}$  into the expression for the eighth term.",
      "conclusion": " $11r^3 = 11 \cdot \sqrt{\frac{5}{11}} = \sqrt{55}$ "
    }
  ]

```

我觉得应该更加细粒度的提取。但这个问题和solution并不好。

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[
  {
    "premises": [
      "The series is a geometric series with first term 1 and common ratio 7",
      "The series has 2005 terms, from  $7^0$  to  $7^{2004}$ "
    ],
    "method": "Apply the formula for the sum of a geometric series,  $S_n = a \frac{r^n - 1}{r - 1}$ , where  $a$  is the first term,  $r$  is the common ratio, and  $n$  is the number of terms.",
    "conclusion": " $1 + 7 + 7^2 + \dots + 7^{2004} = \frac{7^{2005} - 1}{6}$ "
  },
  {
    "premises": [
      "We are interested in the remainder when the sum of this series is divided by 1000",
      " $1 + 7 + 7^2 + \dots + 7^{2004} = \frac{7^{2005} - 1}{6}$ "
    ],
    "method": "Use Fermat-Euler's theorem, noting that  $\varphi(1000) = 400$  to simplify  $7^{2005} \bmod 1000$ .  $7^{2005} \equiv 7^{400 \cdot 5 + 5} \pmod{1000}$  simplifies to  $7^5$  due to  $7^{400} \equiv 1 \pmod{1000}$ .",
    "conclusion": " $\frac{7^{2005} - 1}{6} \equiv \frac{7^5 - 1}{6} \pmod{1000}$ "
  },
  {
    "premises": [
      " $\frac{7^{2005} - 1}{6} \equiv \frac{7^5 - 1}{6} \pmod{1000}$ "
    ],
    "method": "Calculate  $7^5 - 1$  and then divide by 6, finally applying modulo 1000 to the result.",
    "conclusion": "The remainder when  $1 + 7 + 7^2 + \dots + 7^{2004}$  is divided by 1000 is 801"
  }
]

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```
}
]
```

evolve triples

You are a Mathematics Expert in writing a continuation of mathematical process. Your target is to construct ONE json object in the #Constructed Triple# including three attributes: premises, method and conclusion to extend the #Process Triples#. The constructed triple MUST be in the same form of the object in #Process Triples#. You SHOULD complicate the #Constructed Triple# using the following method: The premises of the #Constructed Triple# MUST contain the conclusion of the LAST triple in #Process Triples#, while premises in #Initial Premises# and other conclusions of #Process Triples# are optional. You can add at most ONE assumption in the premises to make the whole process more coherent. The method of #Constructed Triple# is ONLY ALLOWED to use information in the premises of the #Constructed Triple#. The method of #Constructed Triple# MUST treat the conclusion of the LAST triple in #Process Triples# as key information and extend the conclusion, while treating other premises as side information. ANY assumption, supposition, new scenario or new case is STRICTLY forbidden in The method of #Constructed Triple#. All the calculation MUST be done in the method of #Constructed Triple#. The conclusion should describe clearly the meaning of the method and avoid being verbose. You MUST generate ONE mathematical malleable conclusion with a simplified number WITHOUT approximation in the conclusion of #Constructed Triple#. Adding #Constructed Triple# to #Process Triples# SHOULD push the whole process into a more complicated one for AI systems. Try your best to make sure each triple in original #Process Triples# is indispensable for the final conclusion in #Constructed Triple#. "#Constructed Triple#,"constructed triple" is NOT ALLOWED to appear in the answer. #Initial Premises#: #Process Triples#: #Constructed Triple#:

原问题直接尝试非常失败，我尝试从中间断开

```
{ "premises": [ "$r^6 = \frac{5}{11}$" ], "method": "Taking the sixth root of both sides of the equation $r^6 = \frac{5}{11}$ to solve for the common ratio $r$.", "conclusion": "The common ratio $r$ is $(\frac{5}{11})^{\frac{1}{6}}$." }
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```
{ "premises": [ "The common ratio $r$ is $(\frac{5}{11})^{\frac{1}{6}}$", "The fifth term of a geometric sequence of positive numbers is $11$." ], "method": "Use the expression for the fifth term in terms of the first term and common ratio, $a \cdot r^4 = 11$, and substitute the given common ratio to find the first term $a$.", "conclusion": "The first term of the sequence, $a$, is $11 \div (\frac{5}{11})^{\frac{2}{3}} = 11 \cdot (\frac{11}{5})^{\frac{2}{3}}$." }
```

这一步开始算错了， $\frac{2}{3} - \frac{19}{6} = -\frac{15}{6} \neq -\frac{13}{6}$ 。尝试了GPT4，4o，o1都能正确回答这个问题 log10 (第12项)

```
{ "premises": [ "The first term of the sequence, $a$, is $11 \cdot (\frac{11}{5})^{\frac{2}{3}}$", "The common ratio $r$ is $(\frac{5}{11})^{\frac{1}{6}}$." ], "method": "Calculate the value of the twentieth term of the geometric sequence using the first term $a$ and the common ratio $r$, expressed as $a \cdot r^{19}$.", "conclusion": "The twentieth term of the geometric sequence is $11 \cdot (\frac{11}{5})^{\frac{2}{3}}$." }
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\cdot (\frac{5}{11})^{\frac{19}{6}} = 11 \cdot (\frac{11}{5})^{\frac{2}{3}} - \frac{19}{6} = 11 \cdot (\frac{11}{5})^{-\frac{13}{6}}$. " }
```

```
{ "premises": [ "The twentieth term of the geometric sequence is $11 \cdot (\frac{11}{5})^{-\frac{13}{6}}$." ], "method": "Simplify the expression for the twentieth term of the geometric sequence by computing $(\frac{11}{5})^{-\frac{13}{6}}$ explicitly.", "conclusion": "The twentieth term of the geometric sequence is $11 \cdot \left(\frac{5}{11}\right)^{\frac{13}{6}} = 11 \cdot \left(\frac{5^{13}}{11^{13}}\right)^{\frac{1}{6}} = 11 \cdot \frac{5^{\frac{13}{6}}}{11^{\frac{13}{6}}} = \frac{5^{\frac{13}{6}}}{11^{\frac{7}{6}}}$." }
```

```
{ "premises": [ "The twentieth term of the geometric sequence is $\frac{5^{\frac{13}{6}}}{11^{\frac{7}{6}}}$." ], "method": "Calculate the logarithm base 10 of the twentieth term of the geometric sequence to convert the expression into a power of 10 for simpler numerical representation.", "conclusion": "The logarithm base 10 of the twentieth term of the geometric sequence is $\log_{10}\left(\frac{5^{\frac{13}{6}}}{11^{\frac{7}{6}}}\right) = \frac{13}{6} \log_{10}(5) - \frac{7}{6} \log_{10}(11)$." }
```

```
{ "premises": [ "The logarithm base 10 of the twentieth term of the geometric sequence is $\frac{13}{6} \log_{10}(5) - \frac{7}{6} \log_{10}(11)$." ], "method": "Using logarithmic identities, convert $\frac{13}{6} \log_{10}(5) - \frac{7}{6} \log_{10}(11)$ to a single logarithmic term by applying the logarithm property $\log_b(a) - \log_b(c) = \log_b\left(\frac{a}{c}\right)$. Simplify the expression further using powers.", "conclusion": "The logarithm base 10 of the twentieth term of the geometric sequence is $\log_{10}\left(\frac{5^{\frac{13}{6}}}{11^{\frac{7}{6}}}\right)$, which simplifies to $\log_{10}\left(\left(\frac{5}{11}\right)^{\frac{13-7}{6}}\right) = \log_{10}\left(\left(\frac{5}{11}\right)^1\right) = \log_{10}\left(\frac{5}{11}\right)$." }
```

```
{ "premises": [ "The logarithm base 10 of the twentieth term of the geometric sequence is $\log_{10}\left(\frac{5}{11}\right)$." ], "method": "Convert the logarithmic value $\log_{10}\left(\frac{5}{11}\right)$ to an approximate decimal value using logarithmic computation.", "conclusion": "The value of $\log_{10}\left(\frac{5}{11}\right)$ is approximately $-0.1427$." }
```

在前序步骤都给出的情况下仍然计算错误，应该是受到了想求目标的不确定性影响。

两个问题：

- 从何处断开？甚至其实只取一个片段都是可以的。（这里我是人为观察出来的，有些问题的求解步骤是类似的或者有共性，类似于数列求解，会求首项、公比等等，这种求解步骤适合往后延申。有些步骤是问题特化的，例如求解第八项的数值，如果是问题特化的结论，感觉很难进化。应该“变异”换成一个更难的特化问题？）
 - 如何判断两种不同的步骤？其实有很大一部分问题可以说是完全特化的，例如鸡兔同笼问题，共性问题往往有一个解题模板，几何问题、数列问题等等。
- 尝试能否让大模型先生成下一步想求的目标，再生成计算过程。（问题导向，这样可以把所有已知结论转化成条件，说不定从特化结论中也能获取一些新的信息）

我们应该把问题步骤分成两种来考虑，或者我们直接对任何问题都采用两种方式，然后消除进化失败。

尝试

- 尝试先生成问题，再生成solution。
- 这样就可以避免之前出现的问题，并且我们可以从很多不同的地方断开进行这样的操作，然后消除进化失败。 You are a Mathematics Expert in writing a continuation of mathematical process. Your target is to construct ONE json object in the #Constructed Quadruple# including four attributes: premises,question,method and conclusion based on #Initial Premises# and #Process Triples#. You SHOULD complicate the #Constructed Quadruple# using the following method: The premises of the #Constructed Quadruple# MUST contain the conclusion of the LAST triple in #Process Triples#, while premises in #Initial Premises# and other conclusions of #Process Triples# are optional. You can add at most ONE assumption in the premises to make the constructed question solvable. The question SHOULD be meaningful and challenge the famous AI systems a bit more than #Process Triples#. The question of #Constructed Quadruple# SHOULD explore the conclusion of the LAST triple in #Process Triples# by extension and avoid being verbose. The question of #Constructed Quadruple# MUST treat the conclusion of the LAST triple in #Process Triples# as key information while treating other premises as side information. The question MUST NOT contain any assumption or condition. Try your best to AVOID generate ANY intermediate conclusion in the solution of the question of #Constructed Quadruple#. The method of #Constructed Quadruple# is STRICTLY ONLY ALLOWED to use information in the premises of the #Constructed Quadruple#. Try your best to AVOID using similar mathematical skills in the method of each triple in #Process Triples#. ANY assumption, supposition, new scenario or new case is STRICTLY forbidden in the method of #Constructed Quadruple#. The method of #Constructed Quadruple# SHOULD perform All the mathematical calculation process. You MUST generate ONE mathematical malleable conclusion with a simplified number WITHOUT approximation in the conclusion of #Constructed Quadruple# answering the question. The constructed conclusion SHOULD avoid being verbose. Adding premises,method and conclusion of #Constructed Quadruple# to #Process Triples# SHOULD push the whole process into a more complicated one for AI systems. "#Constructed Quadruple#","constructed Quadruple" is NOT ALLOWED to appear in the answer. #Initial Premises#: #Process Triples#: #Constructed Quadruple#:

```
{ "premises": [ "$r^6 = \frac{5}{11}$" ], "method": "Using the known value of $r^6 = \frac{5}{11}$, find $r$ by calculating the sixth root of $\frac{5}{11}$. Express the first term $a$ of the geometric sequence using the relationship $a = \frac{11}{r^4}$, derived from the given information that the fifth term $a r^4 = 11$.", "conclusion": "The first term of the geometric sequence is $\frac{11}{(\frac{5}{11})^{2/3}}$ simplified to $\frac{11}{11^{2/3}} \times 11^{2/3}$." }
```

```
{ "premises": [ "The first term of the geometric sequence is $\frac{11}{11^{2/3}} \times 11^{2/3}$." ], "method": "Use the formula for the $n$-th term of a geometric sequence, which is given by $a r^{n-1}$. Here, $a$ is the first term, which is $\frac{11}{11^{2/3}} \times 11^{2/3}$, and $r$ is the common ratio, calculated as $(\frac{5}{11})^{1/6}$. Substitute these values into the formula to find the tenth term.", "conclusion": "The tenth term of the sequence is $\frac{11}{11^{2/3}} \times 11^{2/3} \times (\frac{5}{11})^{9/6}$ simplified to $\frac{11^{1/3}}{11^{3/2}} \times 11^{3/2}$." }
```


这一步生成的毫无关系，没有很好的利用他的前提（第10项的值），甚至利用了一些不在其中的premise,但起码计算是正确的（前15项的和）。

```
{ "premises": [ "The tenth term of the sequence is  $\frac{11^{1/3}}{5^{3/2}} \times 11^{3/2}$ $.", "method": "Use the formula for the sum of the first  $n$  terms of a geometric sequence,  $S_n = a \frac{1 - r^n}{1 - r}$ , where  $a$  is the first term and  $r$  is the common ratio. The first term  $a$  is  $\frac{11 \times 11^{2/3}}{5^{2/3}}$ , and  $r = (\frac{5}{11})^{1/6}$ $. Calculate  $r^{15}$  and substitute all values to find  $S_{15}$ $.", "conclusion": "The sum of the first 15 terms of the sequence is  $\frac{\frac{11 \times 11^{2/3}}{5^{2/3}} \left(1 - \left(\frac{5}{11}\right)^{2.5}\right)}{1 - \left(\frac{5}{11}\right)^{1/6}}$ $. " }
```

把前15项转到了前20项，但是也没有很好的利用前提，利用了一些不在其中的premise，计算正确。

```
{ "premises": [ "The sum of the first 15 terms of the sequence is  $\frac{\frac{11 \times 11^{2/3}}{5^{2/3}} \left(1 - \left(\frac{5}{11}\right)^{2.5}\right)}{1 - \left(\frac{5}{11}\right)^{1/6}}$ $.", "method": "Extend the sum of the first 15 terms to 20 terms using the geometric series sum formula  $S_n = a \frac{1 - r^n}{1 - r}$ , where  $a$  is the first term  $\frac{11 \times 11^{2/3}}{5^{2/3}}$  and  $r$  is the common ratio  $(\frac{5}{11})^{1/6}$ $. Calculate  $r^{20}$  and substitute all values to find  $S_{20}$ $.", "conclusion": "The sum of the first 20 terms of the sequence is  $\frac{\frac{11 \times 11^{2/3}}{5^{2/3}} \left(1 - \left(\frac{5}{11}\right)^{3.333}\right)}{1 - \left(\frac{5}{11}\right)^{1/6}}$ $. " }
```

- 修改意见 应该让method进行（perform）计算，问题尽可能避免产生中间结论，尝试利用一些更加高级的技巧？
- method 避免使用类似的方法
- question需要使得每个步骤都有意义，而不是我们可以跳过某些triple去尝试解决这个问题。
- process triple是解决这个question的一部分。
- 可以指定method使用的知识技巧的水平，小学，初中，高中，本科早期，本科后期，研究生水平，研究水平
- [elementary school level,middle school level,high school level,early undergraduate-level, late undergraduate-level,graduate-level,research-level]
- 我认为可能被原始triple影响了，应该直接分成必选条件和可选条件。
- 生成一个问题和一个solution
- 这种问题“有点难”避免我们可以通过其他步骤更快的到达答案。（把条件组织起来？） You are a Mathematics Expert in constructing mathematical problems and solutions. Your target is to construct the premises for ONE question in #Constructed Premises# , the constructed question in #Constructed Question# and corresponding solution in #Constructed Solution# based on the given #Required Premises# and #Optional Premises#. The #Constructed Premises# SHOULD be in the

same format as #Required Premises# and #Optional Premises#. You SHOULD complicate the #Constructed Premises# ,#Constructed Question# and #Constructed Solution# using the following method: The #Constructed Premises# MUST contain #Required Premises# , while #Optional Premises# are optional. The #Constructed Question# MUST treat the #Required Premises# as key information. The #Constructed Question# SHOULD require *early undergraduate-level* mathematical skills to solve. The #Constructed Question# MUST raise ONLY ONE question. You SHOULD add ONE premise in the #Constructed Premises# to make the #Constructed Question# solveable and meet the above requirements. The #Constructed Solution# SHOULD be LIMITED to Less than or equal to three detailed steps. The #Constructed Solution# SHOULD AVOID generate ANY premises in #Required Premises# , #Optional Premises# or #Constructed Premises#. ANY assumption, supposition, new scenario or new case is STRICTLY forbidden in the method of #Constructed Solution#. You MUST generate ONE mathematical conclusion with a simplified number WITHOUT approximation in #Constructed Solution# answering the question. The #Constructed Solution# SHOULD avoid being verbose. #Required Premises#: ["The eighth term of the sequence, $11r^3$, simplifies to $\sqrt{55}$ \$.] #Optional Premises#: ["The fifth term of a geometric sequence of positive numbers is 11 \$.", "The eleventh term of the geometric sequence is 5 \$.", " $r^6 = \frac{5}{11}$ ", " $r^3 = \sqrt{\frac{5}{11}}$ \$.",] #Constructed Premises#:

#Constructed Question#:

#Constructed Solution#:

用concise中的方法，把我们的结构组织成一个树形（ordered sentence 我用反拓扑序），然后生成答案。

You are a Mathematics Expert in constructing mathematical problems and solutions. Your target is to construct ONE question in #Constructed Question# and corresponding solution in #Constructed Solution# based on several paragraphs with multiple ordered premises in #Ordered Premises#. Each paragraph of Premises in #Ordered Premises# are organized in order, the final premise of each paragraph SHOULD be indispensable for the constructed question and solution. You SHOULD complicate the #Constructed Question# and #Constructed Solution# using the following method: The #Constructed Question# SHOULD require *graduate-level* mathematical skills to solve. The #Constructed Question# MUST raise ONLY ONE question. The answer to the raised question is FORBIDDEN to equal to ANY premises. You CAN make ONE constraint in the #Constructed Question# to make the #Constructed Question# solveable and meet the above requirements. The #Constructed Solution# SHOULD be LIMITED to less than or equal to three detailed steps. ANY assumption, supposition, new scenario or new case is STRICTLY forbidden in the method of #Constructed Solution#. You MUST generate ONE mathematical conclusion with a simplified number WITHOUT approximation in #Constructed Solution# answering the question. The #Constructed Solution# SHOULD avoid being verbose. #Ordered Premises#: We are interested in the remainder when the sum of this series is divided by 1000.The series is a geometric series with first term 1 and common ratio 7.The series has 2005 terms, from 7^0 to 7^{2004} \$. $1 + 7 + 7^2 + \cdots + 7^{2004} = \frac{7^{2005}-1}{6}$ \$. $\frac{7^{2005}-1}{6} \equiv \frac{7^5 - 1}{6} \pmod{1000}$ \$.The remainder when $1 + 7 + 7^2 + \cdots + 7^{2004}$ is divided by 1000 is 801 \$. #Constructed Question#:

#Constructed Solution#:

取消独占一行或多行的任何latex数学公式或符号，将全部的公式改成只用\$包裹起来的markdown行内公式。

Constructed Question: Calculate the exact value of $7^{2005} \bmod 6000$ given that $7^5 \equiv 16801 \bmod 18000$. Use the result from premise 5 that $\frac{7^5 - 1}{6} \equiv \frac{7^{2005} - 1}{6} \pmod{1000}$.

Constructed Solution: Start by using the given equivalence: From the information given, $\frac{7^5 - 1}{6} \equiv \frac{7^{2005} - 1}{6} \pmod{1000}$, we know that $\frac{7^{2005} - 1}{6} \equiv 801 \pmod{1000}$. Thus, $7^{2005} - 1 \equiv 4806 \pmod{6000}$ since $801 \times 6 = 4806$.

Add 1 to both sides to isolate 7^{2005} : We have $7^{2005} - 1 \equiv 4806 \pmod{6000}$, adding 1 to both sides gives $7^{2005} \equiv 4807 \pmod{6000}$.

Result: The exact value of $7^{2005} \bmod 6000$ is 4807 . This result does not violate the premises and adheres to the constraints and requirements set in the constructed question.

这个问题过程利用了我们已知的结论没有计算错误。o1计算正确，o2计算错误，他们都运用了中国剩余定理，而不是欧拉公式。 $7^{2005} - 1 \equiv 4806 \pmod{6000}$ since $801 \times 6 = 4806$. The exact value of $7^{2005} \bmod 6000$ is 4807 .

Constructed Question: Determine the smallest positive integer n such that the remainder of $\frac{7^{2005n} - 1}{6}$ divided by 1000 equals 1 .

Constructed Solution: Formulate the expression and apply the modulus: Given that $\frac{7^{2005} - 1}{6} \equiv 801 \pmod{1000}$, we first express 7^{2005n} in terms of 7^{2005} using the identity $7^{2005n} = (7^{2005})^n$. Now, we need to find n such that $\frac{(7^{2005})^n - 1}{6} \equiv 1 \pmod{1000}$.

Utilize the value of $7^{2005} \bmod 6000$: Since $7^{2005} \equiv 4807 \pmod{6000}$ and $4807 - 1 = 4806$, we apply this to find $(4807^n - 1) \equiv 6 \pmod{6000}$. Using $6000 = 6 \times 1000$, we solve $\frac{4807^n - 1}{6} \equiv 1 \pmod{1000}$.

Solve for n : We need to check the minimum n for which $4807^n \equiv 7 \pmod{6000}$ to ensure $\frac{4807^n - 1}{6} \equiv 1 \pmod{1000}$. Calculating, $n = 1000$ (the smallest such n) ensures $(4807^{1000} - 1)/6 \equiv 1 \pmod{1000}$.

Conclusion: The smallest positive integer n is 1000 .

这个问题，我有点不太会算，o1说没有答案，这么整数我感觉算错了。不太像是答案。

消除进化失败

- 用method和给定的premise列，分析它运用了哪些premise，如果没有用到必选premise 判定失败。
- check method中的计算是否正确
- method中做了新的假设，也判定失败。