A **metric space** is an ordered pair (M, d) where M is a set and d is a metric on M, i.e., a function

$$d : M \times M \to \mathbb{R}$$

such that for any $x, y, z \in M$, the following holds:[2]

1. $d(x, y) = 0 \iff x = y$ identity of indiscernibles
2. $d(x, y) = d(y, x)$ symmetry
3. $d(x, z) \leq d(x, y) + d(y, z)$ subadditivity or triangle inequality

Given the above three axioms, we also have that $d(x, y) \geq 0$ for any $x, y \in M$. This is deduced as follows:

$d(x, y) + d(y, x) \geq d(x, x)$	by triangle inequality
$d(x, y) + d(x, y) \geq d(x, x)$	by symmetry
$2d(x, y) \geq 0$	by identity of indiscernibles
$d(x, y) \geq 0$	we have non-negativity