Exercises

Table of Materials

1. MULTICRITERIA DECISION

1.1. Theoretical problem with constraints

1- Let *(P)* the optimization problem:

$$(P) = \begin{cases} \text{minimize } f_1(x,y) = x \\ \text{maximize } f_2(x,y) = y/2 \\ \text{such that} \\ -10 \le x \le 10 \\ -5 \le y \le 5 \end{cases}$$

a. In the decision space define by the axis *x* (horizontal) and *y* (vertical), represent graphically the area of all valid solutions *(x,y)*.

b. In the objective space define by the axis *f1* (horizontal) and *f2* (vertical), represent graphically the surface, line or point, Pareto optimal.

2- The problem is transformed into a single objective problem by the ε-constraint method. *f*1 is the objective and *f*2 is converted as constraint in respect to the value ε.

a. Write the definition of the transformed problem *(Pε)*.

b. Give the optimal solution to the *(Pε)* problem for ε= +3.

3- The exercise returns to the initial problem *(P)*. The constraint: *x* -2*y* ≥ -10 is added to definition of the problem (P).

a. In the decision space define by the axis *x* (horizontal) and *y* (vertical), represent graphically the area of all valid solutions *(x,y)*.

b. In the objective space define by the axis *f1* (horizontal) and *f2* (vertical), compute and represent graphically the surface, line or point, Pareto optimal.

1.2. Pareto – Selection in a Pareto Front

A two-criteria algorithm, where *f1* and *f2* are the fitness to minimize, gets the set *X* of solutions with the following values:

	x1	*x2*	*x3*	*x4*	*x5*	*x6*	*x7*	*x8*	*x9*	*x10*
f1	2.08	5.92	4.18	6.69	7.19	9.89	5.23	9.61	1.90	4.68
f2	5.34	2.08	3.69	6.67	1.80	8.49	7.41	7.28	6.48	1.47

We use the WAR function to manage the selection of solutions for the next step of the algorithm convergence. Calculate the selection probability (3 digits after the decimal point) of each solution of the current set with respect to its rank given by WAR according to the following formula:

$$P_i = \frac{(TP - r_i)}{\sum_{j \in X} (TP - r_j)} \quad \textit{with } TP = \sum_{j \in X} r_j \quad \textit{and } r_j \text{ the rank of the solution } j$$

WAR (Weighted Average Ranking) function: each solution is first ranked according to its quality in each objective, then its overall rank is equal to the average of its ranks over all the objectives. The solutions are therefore ranked on the basis of the average rank by separating the objectives from each other.

Verify that the sum of $P_i = 1$ to check the computation.

Considering that only 4 solutions will be kept for convergence, which ones will be used by the algorithm?