Lecture 6
Priority Queues

Priority Queue

A **priority queue** holds *comparable data*

- Given *x* and *y*, is *x* less than, equal to, or greater than *y*
- Meaning of the ordering can depend on your data
- Many data structures require this: dictionaries, sorting
- Typically elements are comparable types, or have two fields: the *priority* and the *data*

Priority Queue vs Queue

Queue: follows First-In-First-Out ordering

Example: serving customers at a pharmacy, based on who got there first.

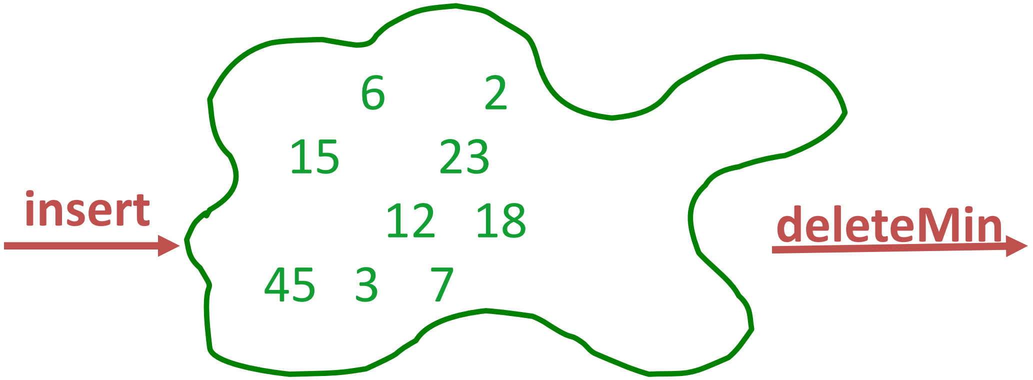
Priority Queue: compares priority of elements to determine ordering

Example: emergency room, serves patients with priority based on severity of wounds

Priorities

- Each item has a "priority"
  - The *lesser* item is the one with the *greater* priority
  - **So "priority 1" is more important than "priority 4"**
  - Can resolve ties arbitrarily

- Operations:
  - **`insert`**
  - **`deleteMin`**
  - **`is_empty`**



- **`deleteMin`** *returns* and *deletes* the item with greatest priority (lowest priority value)
- **`insert`** is like **`enqueue`**, **`deleteMin`** is like **`dequeue`**
  - But the whole point is to use priorities instead of FIFO

Priority Queue Example

Given the following, what values are **a**, **b**, **c** and **d**?

`insert` *element1* with priority *5*

`insert` *element2* with priority *3*

`insert` *element3* with priority *4*

a = **`deleteMin`** `// a = ?`

b = **`deleteMin`** `// b = ?`

`insert` *element4* with priority *2*

`insert` *element5* with priority *6*

c = **`deleteMin`** `// c = ?`

d = **`deleteMin`** `// d = ?`

Priority Queue Example Solutions

`insert` *element1* with priority *5*
`insert` *element2* with priority *3*
`insert` *element3* with priority *4*
a = **`deleteMin`** **`// a = element2`**
b = **`deleteMin`** **`// b = element3`**
`insert` *element4* with priority *2*
`insert` *element5* with priority *6*
c = **`deleteMin`** **`// c = element4`**
d = **`deleteMin`** **`// d = element1`**

Some Applications

- Run multiple programs in the operating system
  - "critical" before "interactive" before "compute-intensive", or let users set priority level
- Select print jobs in order of decreasing length
- "Greedy" algorithms (we'll revisit this idea)
- Forward network packets in order of urgency
- Select most frequent symbols for data compression (Huffman CSE 143)
- Sorting (first **`insert`** all, then repeatedly **`deleteMin`**)

Possible Implementations

- Unsorted Array
  - **`insert`** by inserting at the end
  - **`deleteMin`** by linear search

- Sorted Circular Array
  - **`insert`** by binary search, shift elements over
  - **`deleteMin`** by moving “front”

More Possible Implementations

- Unsorted Linked List
  - `insert` by inserting at the front
  - `deleteMin` by linear search
- Sorted Linked List
  - `insert` by linear search
  - `deleteMin` remove at front
- Binary Search Tree
  - `insert` by search traversal
  - `deleteMin` by find min traversal

One Implementation: Heap

Heaps are implemented with Trees

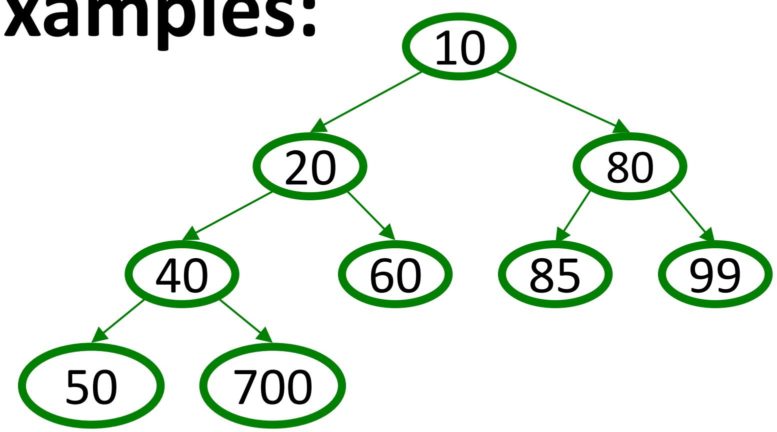
A *binary min-heap* (or just *binary heap* or *heap*) is a data structure with the properties:

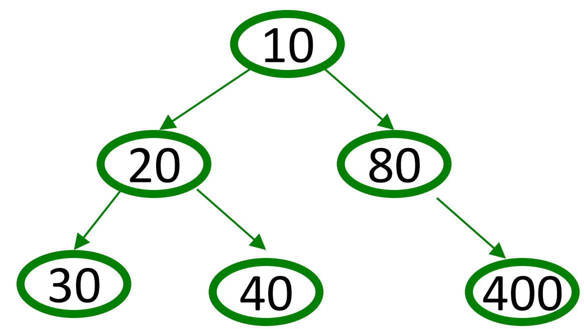
- Structure property: A *complet*e binary tree
- Heap property: The priority of every (non-root) node is greater than the priority of its parent
  - ***Not*** a binary search tree

Structure Property: Completeness

- A Binary Heap is a **complete** binary tree:
  - A binary tree with all levels full, with a possible exception being the bottom level, which is filled left to right

Examples:





are these trees *complete*?

Structure Property: Completeness

- A Binary Heap is a **complete** binary tree:
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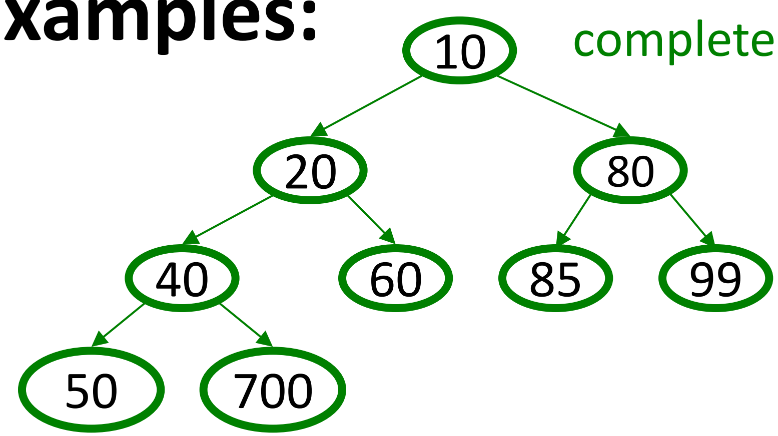
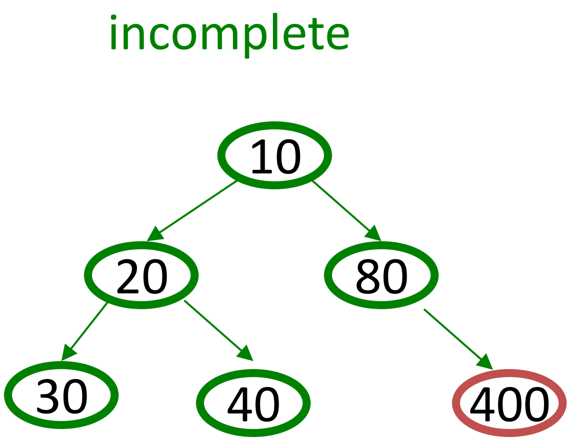
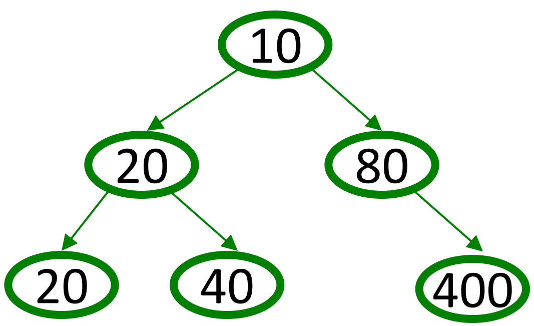
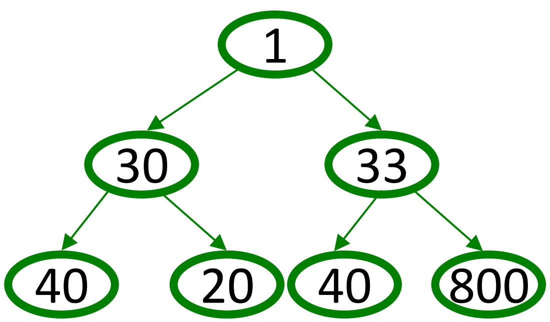




Heap Order Property

- The priority of every (non-root) node is greater than (or equal to) that of it's parent. AKA the children are always greater than the parents.





which of these follow the *heap order property*?

Heap Order Property

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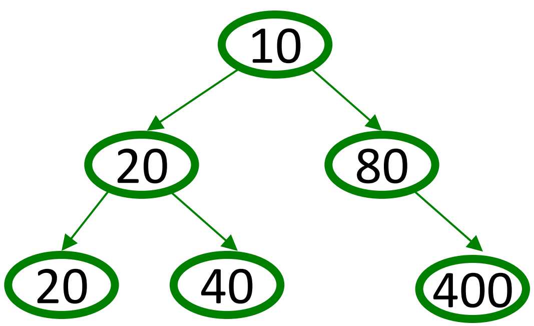
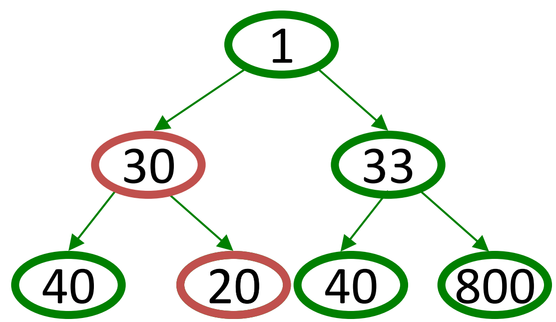
heap property



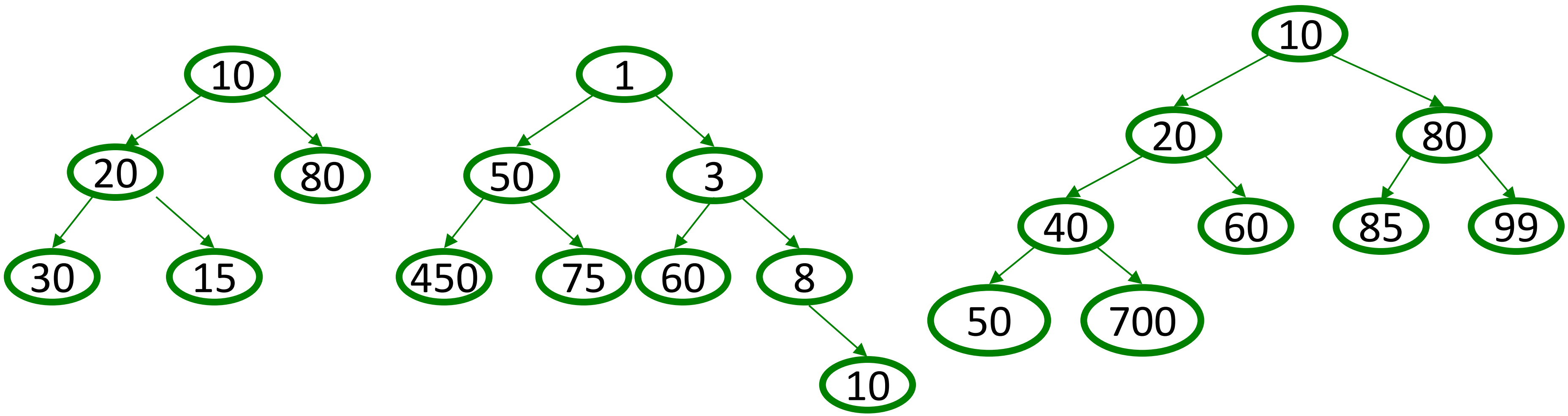
not the heap property



Heaps

A *binary min-heap* (or just *binary heap* or just *heap*) is:

- Structure property: A *complete* binary tree
- Heap property: The priority of every (non-root) node is greater than (or equal to) the priority of its parent. AKA the children are always greater than the parents.
  - ***Not*** a binary search tree



which of these are *heaps*?

Heaps

A *binary min-heap* (or just *binary heap* or just *heap*) is:

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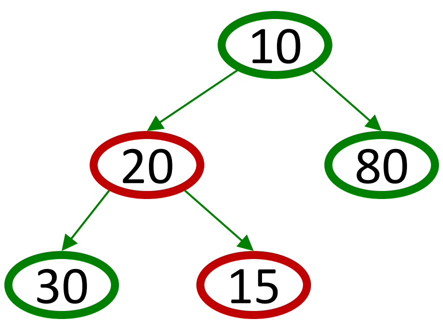
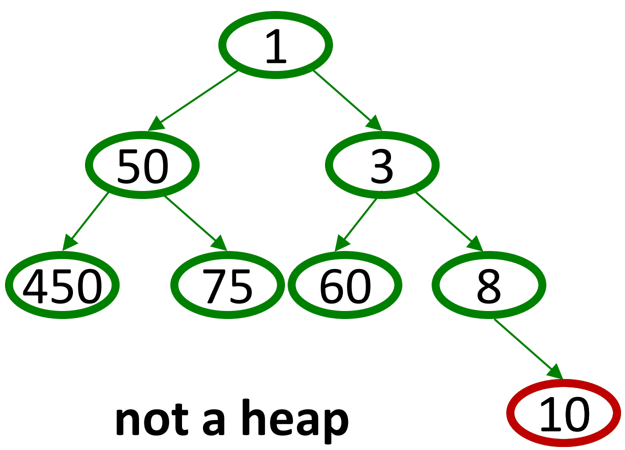
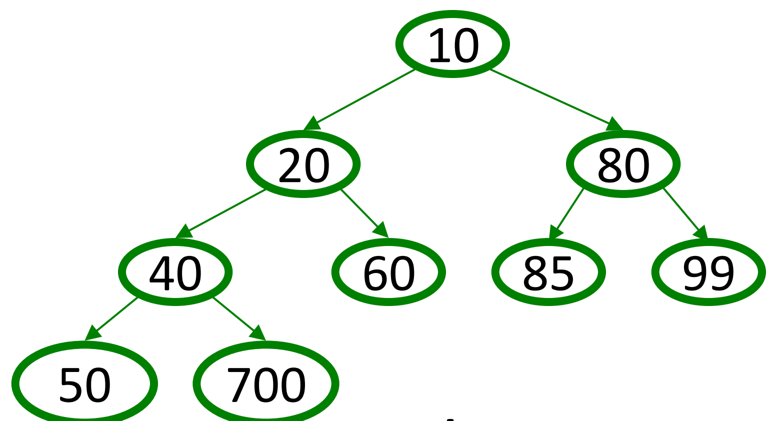


not a heap



not a heap



a heap

Heaps

- Where is the highest-priority item?

- What is the height of a heap with *n* items?

- How do we use heaps to implement the operations in a Priority Queue ADT?

Heaps

- Where is the highest-priority item?

  At the root (at the top)

- What is the height of a heap with *n* items

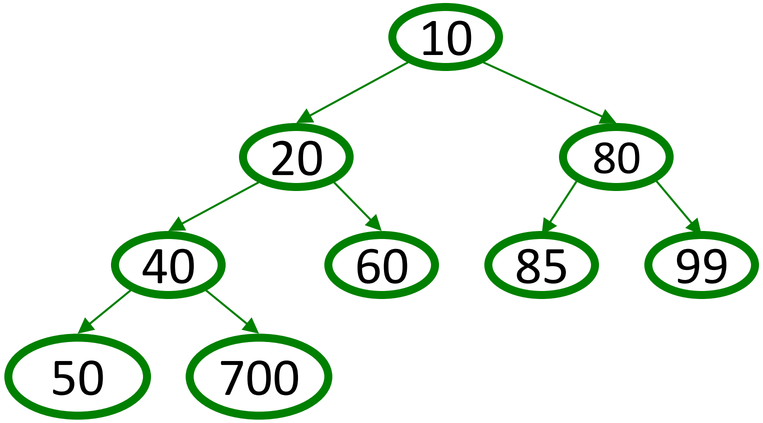
  $\log_2 n$ (We'll look at computing this next week)

- How do we use heaps to implement the operations in a Priority Queue ADT?

  See following slides

Operations: basic idea

- **`deleteMin`**:
  1. **`answer = root.data`**
  2. Move right-most node in last row to root to restore structure property
  3. "Percolate down" to restore heap property
- **`insert:`**
  1. Put new node in next position on bottom row to restore structure property
  2. "Percolate up" to restore heap property

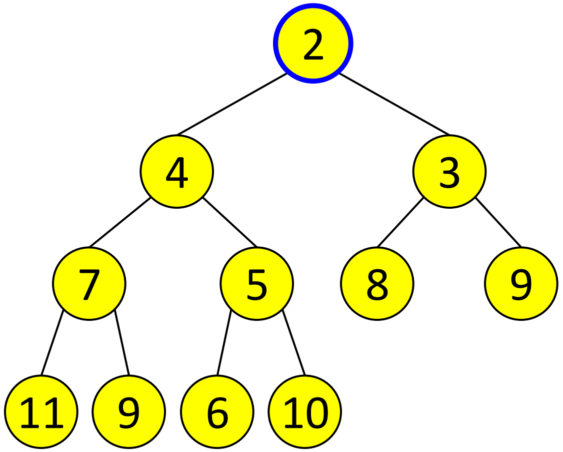


Overall strategy:

- *Preserve structure property*
- *Break and restore heap property*

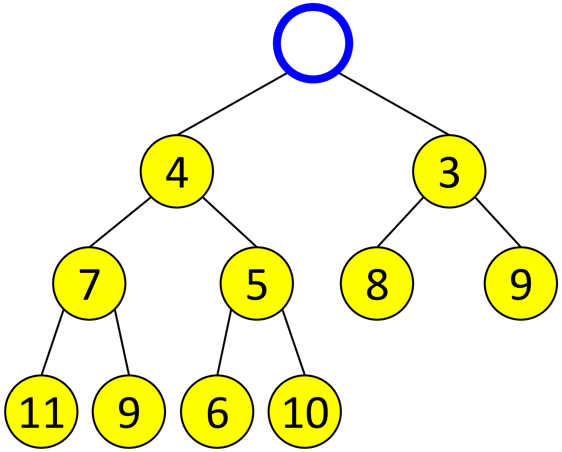
deleteMin

1. Delete (and later return) value at root node

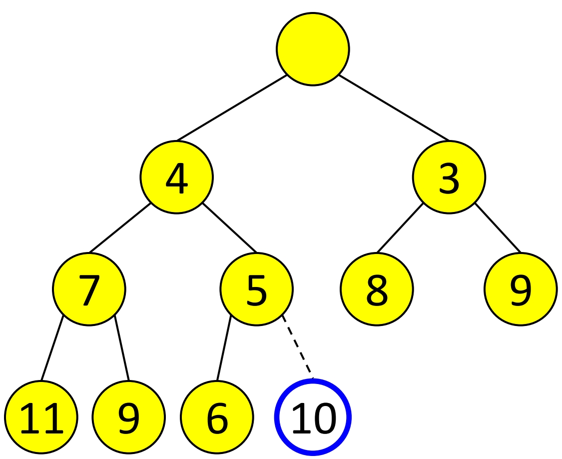


2. Restore the Structure Property

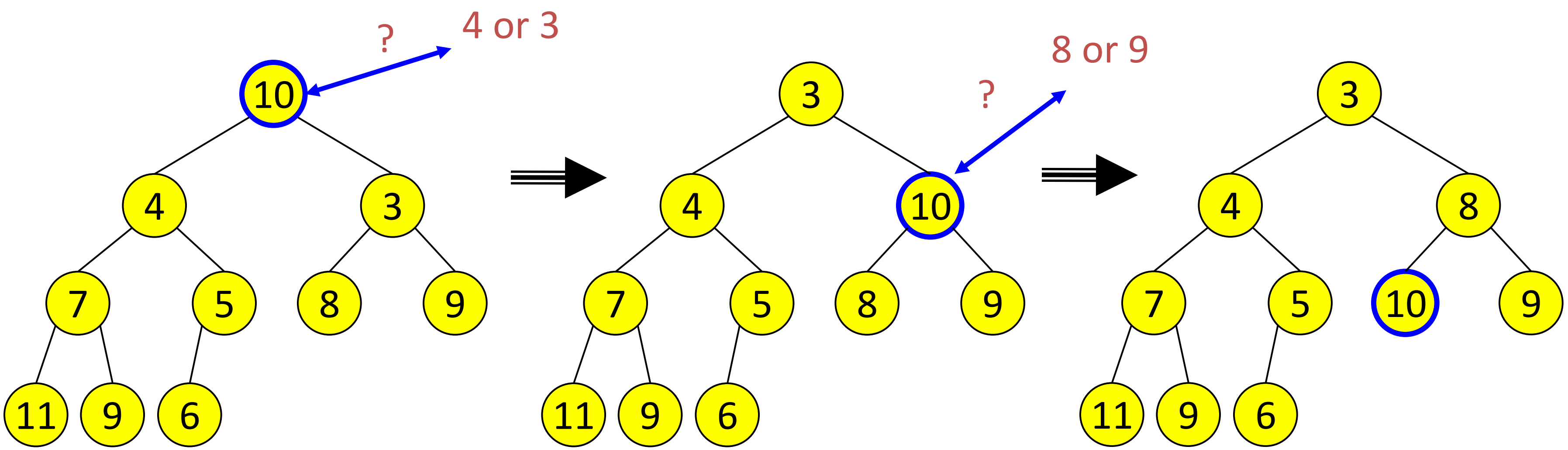
- We now have a "hole" at the root
  - Need to fill the hole with another value



- When we are done, the tree will have one less node and must still be complete



3. Restore the Heap Property

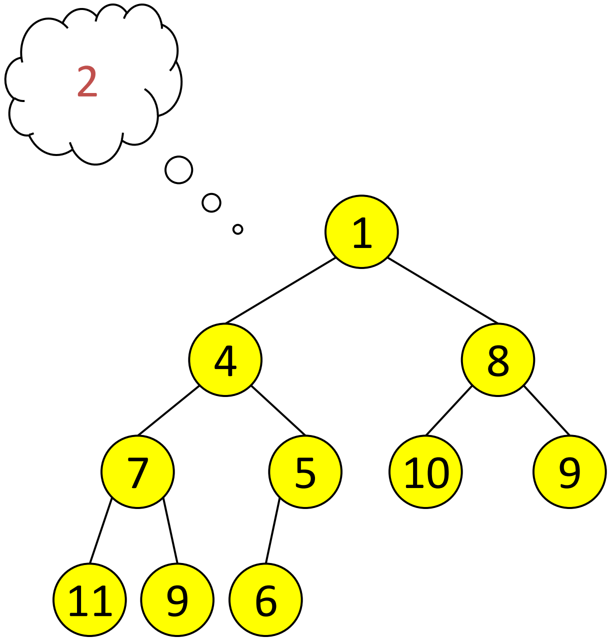


Percolate down:

- Keep comparing with both children
- Swap with lesser child and go down one level
  - What happens if we swap with the larger child?
- Done if both children are ≥ item or reached a leaf node

Insert

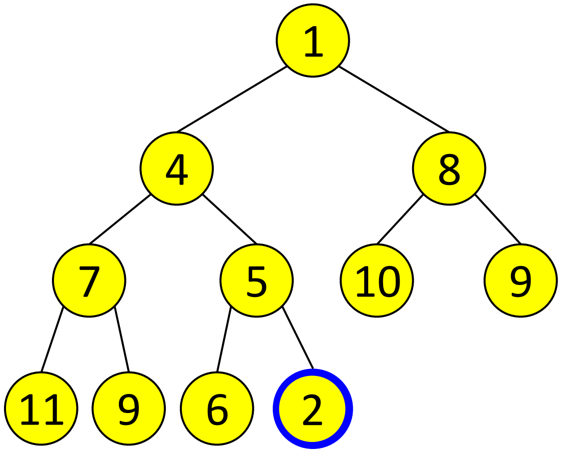
- Add a value to the tree
- Afterwards, structure and heap properties must still be correct
- Where do we insert the new value?



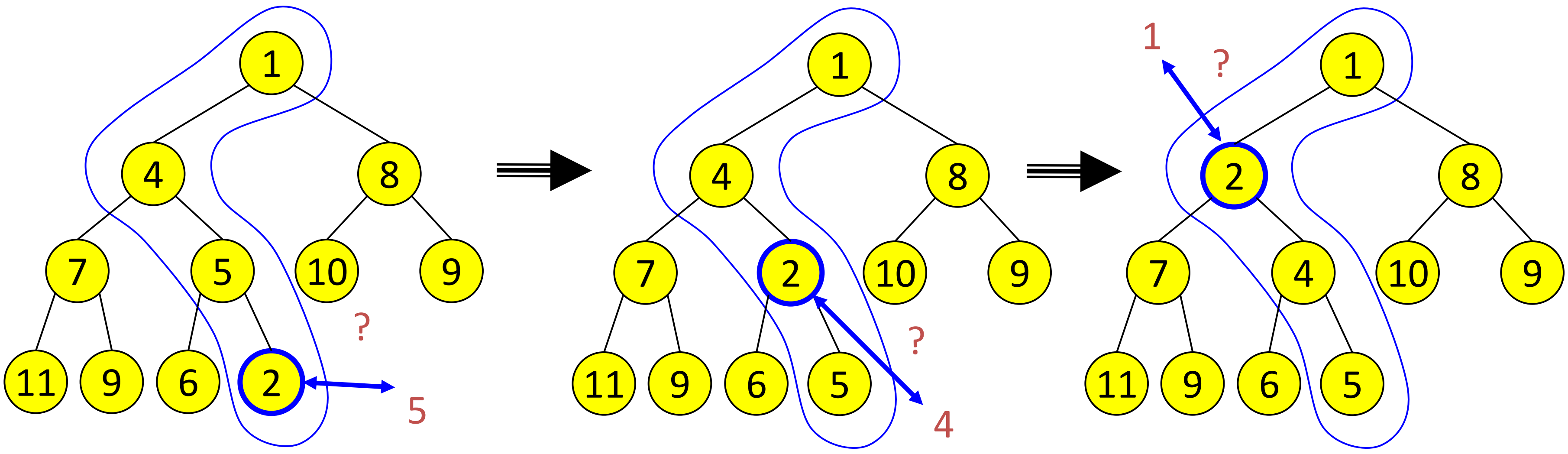
Insert: Maintain the Structure Property

- There is only one valid tree shape after we add one more node

- So put our new data there and then focus on restoring the heap property



Maintain the heap property



Percolate up:

- Put new data in new location
- If parent larger, swap with parent, and continue
- Done if parent ≤ item or reached root