# Linear regression

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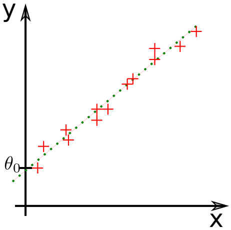
# Outline

# Data

- data :
  - scalar value : $x$
  - many scalar values : vector : $\mathbf{x} = [x_0 x_1 ... x_n]$
- we want to predict $y$ value
- Examples :
  - amount of rain from altitude in the Alps
  - price of an appartment from size in $m^2$
  - vote from age, sex, income, residence location, ...
  - risk of a disease from age, weight, result of blood analysis, ...

- Regression : determine value of $y$ with respect to $x$.
- Linear : the function is a line parameterised by $\theta = [\theta_0 \theta_1]$ :

$$y = \theta_0 + x * \theta_1$$

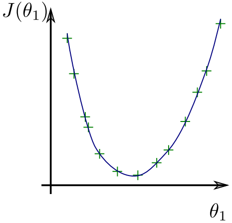



- Learning : determine $\theta_0$ et $\theta_1$
- Regression : compute $y = h_\theta(x) = \theta_0 + x * \theta_1$
- Cost function (or error) :

$$J(\theta) = \frac{1}{2m} \sum (h_\theta(x^i) - y^i)^2$$

# Learning : determine $\theta$

- Cost minimisation $J(\theta)$
  - Let us consider cases where $\theta_0 = 0$ : determine $\theta_1$ that minimises $J(\theta)$

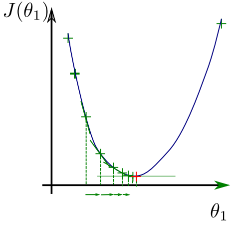



    - Exhaustive search ?
    - Gradient descent

# Gradient descent (one variable)

- Find extrema of $J$ : find the zeros of $\frac{dJ}{d\theta_1}$
- Iterative algorithm :
  - pick initial value of $\theta_1$
  - while $J(\theta_1)$ changes (stop when $\frac{dJ}{d\theta_1}(\theta_1) \simeq 0$ ) :
    - replace $\theta_1$ by $\theta_1 - \alpha \frac{dJ}{d\theta_1}$ ($\alpha > 0$, small)

- $\alpha$ : learning rate
- $\alpha$ has to be chosen carefully :
  - if too small : slow convergence
  - if too big : oscillations
  - $\Rightarrow$ plot $J(\theta)$



# Gradient descent (many variables)

- many scalar values : vector $\mathbf{x} = [x_1 ... x_n]$
- linear model : $y = \theta_0 + x_1\,\theta_1 + x_2\,\theta_2 + \ldots + x_n\,\theta_n$
- Find extrema of $J$ : find the zeros of $\frac{dJ}{d\theta}$
- Iterative algorithm :
    - pick initial value of $\theta = [\theta_0 ... \theta_n]$
    - while $J(\theta)$ changes (stop when $\frac{dJ}{d\theta}(\theta) \simeq 0$ ) :
        - replace each $\theta_i$ by $\theta_i - \alpha\,\frac{dJ}{d\theta_i}$ ($\alpha > 0$, small)

# Gradient descent : many data

- batch gradient descent
  - all training samples for each step
- stochastic gradient descent
  - one training sample for each step (need to shuffle training data)
- mini batch (b=10)
  - a set of b training samples for each step

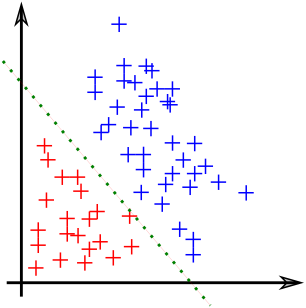
# Outline

# Context

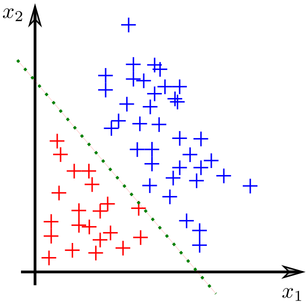
- Supervised learning = learn to predict an output when given an input vector
- We know the class / label $y$ for all training data $\mathbf{x}$
- Logistic regression is a classification method
  - Linear regression leads to values $h_\theta(x) \in \mathcal{R}$
  - the idea : values in [0 1] then thresholding at 0.5

# Logistic regression

- Data separation according to labels (0 or 1).
  - Linear separation : line, plane or hyperplane
  - Non-linear separation : polynomial or gaussian
- Notations :
  - data : $\mathbf{x} = [x_1 \;\; x_2 ...]$
  - labels : $y \in \{0, 1\}$
  - decision criteria $h_\theta$ parameterised by $\theta$
    - $\theta = [\theta_0 \;\; \theta_1 \;\; ...]$



# Linear separation



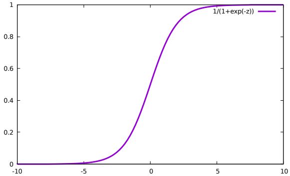

- Decision boundary :
  - line of equation : $\theta_0 + \theta_1 x_1 + \theta_2 x_2$
  - also written as : $\theta^T \mathbf{x} = 0$
- Decision :
  - if $\theta^T \mathbf{x} \geq 0$ then $y = 1$
  - if $\theta^T \mathbf{x} < 0$ then $y = 0$

# Logistic function

$$h_\theta(\mathbf{x}) = s(\theta^T \mathbf{x})$$

with :

$$s(z) = \frac{1}{1 + e^{-z}}$$



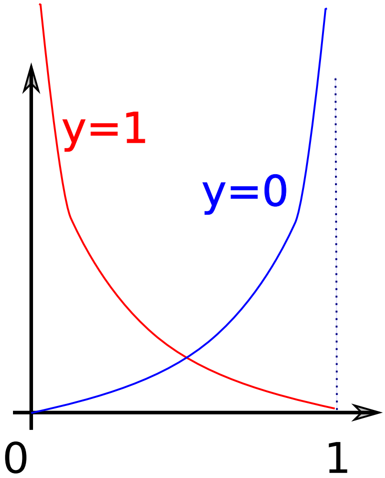

- The decision is :
  - if $h_\theta(\mathbf{x}) \geq 0.5$ then $y = 1$
  - if $h_\theta(\mathbf{x}) < 0.5$ then $y = 0$

# Logistic regression : learning

- data : $(\mathbf{x}^{(1)}, y^{(1)}), (\mathbf{x}^{(2)}, y^{(2)}), ..., (\mathbf{x}^{(m)}, y^{(m)})$
- $m$ data
- learning aims at finding $\theta$
- method :
  - error minimisation
  - gradient descent (or other minimisation method)

# Cost function

$$J = -\frac{1}{m}\sum_{i=1}^{m}(y^{(i)}\log(h_\theta(\mathbf{x^{(i)}})) + (1-y^{(i)})\log(1-h_\theta(\mathbf{x^{(i)}})))$$



# Gradient descent

For all components $\theta_j$ de $\theta$ :

$$\theta_j = \theta_j - \alpha \frac{\partial J}{\partial \theta_j}$$

with

$$\frac{\partial J}{\partial \theta_j} = \frac{1}{m} \sum_{i=1}^{m} (h_\theta(\mathbf{x^{(i)}}) - y^{(i)}) x_j^{(i)}$$

# Non linear logistic regression

- polynomial model :

$$\mathbf{x} = [x_0 x_1 \dots x_n x_0^2 x_1^2 \dots x_0 x_1 x_0 x_2 \dots$$

- under-fitting
  - add parameters
- over-fitting
  - reduce the number of parameters
- regularisation

# Regularisation

- in order to avoid over-fitting
- add $\|\theta\|$ to the cost :

$$J = \|\theta\| - C\frac{1}{m}\sum_{i=1}^{m}(y^{(i)}\log(h_\theta(\mathbf{x^{(i)}})) + (1-y^{(i)})\log(1-h_\theta(\mathbf{x^{(i)}})))$$

- norm : $\mathcal{L}_1$ (Lasso), $\mathcal{L}_2$ (Ridge), Elastic-Net ($\frac{1-\rho}{2}\|\theta\|_2^2 + \rho\|\theta\|_1$)
- many solvers :
  - Coordinate Descent (C++ LIBLINEAR library),
  - Stochastic Average Gradient (SAG) or variant SAGA : good for large dataset
  - Broyden–Fletcher–Goldfarb–Shanno algorithm (small data set only) : family of Newton algorithm
- more details on `https://scikit-learn.org/stable/modules/linear_model.html#logistic-regression`

# Logistic regression using OpenCV

From `https://docs.opencv.org/3.0-last-rst/modules/ml/doc/logistic_regression.html` :
A sample set of traininttg parameters for the Logistic Regression classifier can be initialized as follows :

- `LogisticRegression::Params params;`
- `PARAMS.ALPHA = 0.5;`
- `PARAMS.NUM_ITERS = 10000;`
- `PARAMS.NORM = LOGISTICREGRESSION::REG_L2;`
- `PARAMS.REGULARIZED = 1;`
- `PARAMS.TRAIN_METHOD = LOGISTICREGRESSION::MINI_BATCH;`
- `PARAMS.MINI_BATCH_SIZE = 10;`

# Logistic regression using `scikit-learn`

From `https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html` :

# Logistic regression using scikit-learn

Description at : https://scikit-learn.org/stable/modules/generated/sklearn.linear_model.LogisticRegression.html An toy example :

```
#logistic regression object creation
logisticRegr = LogisticRegression()
#learning
logisticRegr.fit(bows, labels)
#predicted labels computation
labelsPredicted = logisticRegr.predict(bows)
#score computation and display
score = logisticRegr.score(bows, labels)
print("train score = ", score)
#objet saving
with open('sauvegarde.logr', 'wb') as output:
   pickle.dump(logisticRegr, output, pickle.HIGHEST_PROTOCOL)
#object loading
with open('sauvegarde.logr',  'rb') as input:
   logisticRegr = pickle.load(input)
```

# Next week

From logistic regression to artificial neuron.