Exercises

Table of Materials

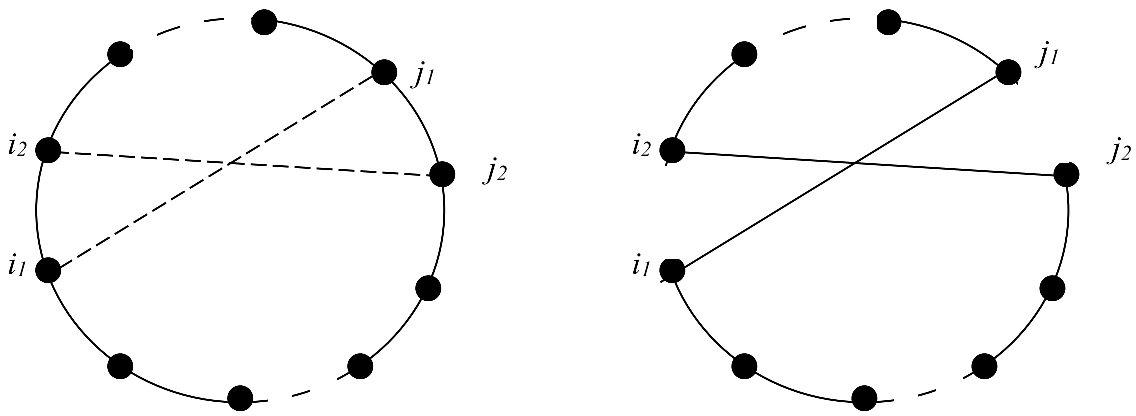
1. NEIGHBORHOOD

1.1. Neighborhoods for the Travelling Salesman Problem

Consider a graph oriented $G=(N, E)$ and a cost matrix C such as $\forall(i,j)\in E,\ c_{ij}\neq 0$ (i.e. each city can be connected to all the others). From a Hamiltonian cycle (oriented cycle that passes one and only once through each node of G), several transformations of a cycle can be defined.

Give the neighborhood of the following transformations, knowing that the nodes i_1, i_2 and i_3 follow each other in the cycle, as well j_1, j_2 and j_3, and that the nodes noted i and j are all different. $n=|N|$ is the number of graph nodes.

1. 2-OPT neighborhood (or 2-permutation): 2 cities are swapped; choose 2 non-adjacent edges (no nodes in common) in a Hamiltonian cycle and replace them with the 2 edges that rebuilds a Hamiltonian cycle.



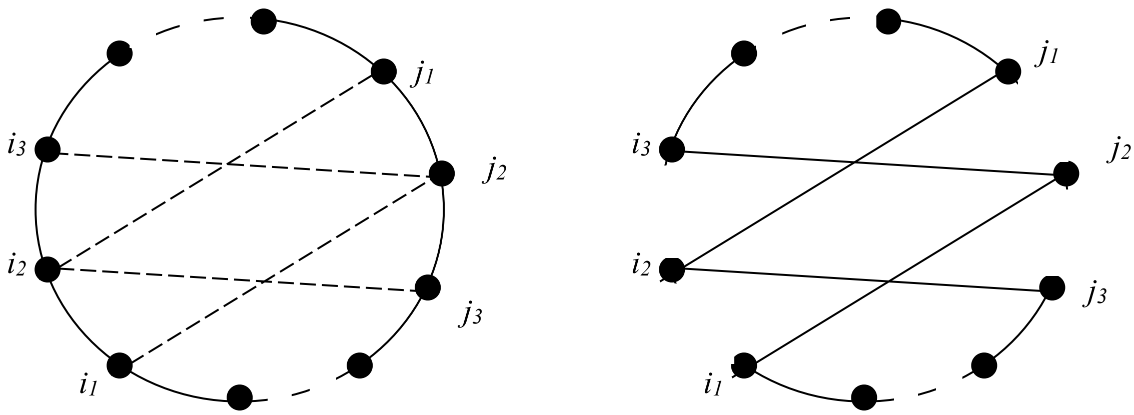
2-OPT: the edges (i_1,i_2) and (j_1,j_2) are replaced by the edges (i_1,j_1) and (i_2,j_2).

What is the mathematical formulation of the neighborhood size?

To test your hypothesis:
for n=100 the number of neighbors of any solution is 4850

What is the effect of the transformation on the cycle?

2. Two-node exchange: 2 cities are swapped; choose 2 non-adjacent nodes i_2 and j_2 (no adjacent nodes in common) in a Hamiltonian cycle, swap and reconnect them to the respective neighbors j_1,j_3 and i_1,i_3 to rebuild a Hamiltonian cycle.



Two-node exchange: the edges (i_1,i_2),(i_2,i_3),(j_1,j_2) and (j_2,j_3) are replaced by the edges (i_1,j_2),(j_2,i_3),(j_1,i_2) and (i_2,j_3).

What is the mathematical formulation of the neighborhood size?

To test your hypothesis:
for n=100 the number of neighbors of any solution is 4750

What is the effect of the transformation on the cycle?

2. PROBLEM MODELING

2.1. Modelling of timetable

A congress is organized with 3 rooms in which will be held 11 sessions rated *(A, B, C, ..., J, K*). Each session lasts 1/2 day. The congress lasts two days. Several sessions can be held in parallel in different rooms. The organization must distribute the sessions in the halls considering the constraints of temporal incompatibility between certain sessions i.e. they cannot be held at the same time. These incompatibilities are described by the following n-tuple:

Incompatibilities: *AJ, JI, IE, EC, CF, FG, DH, BD, KE, BIHG, AGE, BHK, ABCH, DFJ*

For example, *AJ* indicates that sessions *A* and *J* cannot be held at the same time and *DFJ* indicates that any couple from *DFJ* cannot be held at the same time.

1. Show that the problem is a graph coloring problem with 4 colors. Answer the following questions:
    a. What are the nodes of the graph? (0.5)
    b. In what condition does an edge connect two nodes? (0.5)
    c. What does a color represent? What does it mean to color a node? (0.5)
    d. Why we have 4 colors? (0.5)
    e. How is defined the constraint of the number of rooms for the coloring of a graph? (0.5)

2. Added are the following constraints: the session *E* must take place before the session *J* and the sessions *D* and *F* before the session *K*.

    a. How can these constraints be incorporated into the coloration of graphs at the edges?
    b. How can these constraints be incorporated into the coloring of the graph at the level of the colors?

3. Give a matrix representation of the graph with the constraints of sections 1 and 2. How many constraints are there?

The Adjacent Matrix A of the graph G=(N, E) is s.t. $a_{ij}=1$ if $(i,j)\in E$ else $a_{ij}=0$ with $a_{ij}=a_{ji}=1$ if i and j are incompatible ((i,j) is an edge), and $a_{ij}=1$ and $a_{ji}=0$ if i precedes j ((i,j) is an arc).
NB:

i and *j* incompatible means *i* may take place before *j* or *j* may take place before *i*
i precedes *j* means *i* must take place before *j*

Write a table representation of the matrix.

4. Give a vector representation to write down the solutions to the problem.

There are many ways to do it. Give one way at minimum.

5. Propose an algorithm that tries to solve this problem and give a solution by processing this algorithm.

Consider two cases: a constructive algorithm and a neighborhood algorithm.

3. TABOU SEARCH

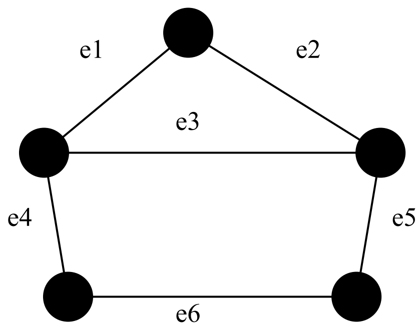
3.1. Edge coloring

Let the problem of coloring the edges of a graph $G=(N, E)$, with N the nodes and E the edges, with k colors such that two edges having a common node must have different colors. Let $e=\{i,j\}$ an edge where $\{i,j\}\in N^2$ defines the end nodes of the edge e and $c(e)$ the color assigned to the edge e.

a. How many solutions has this problem (the size of the search space)?

b. Formulate the fitness function that counts the number of invalid assignments in a solution s. Use $c(e)$ to note the color of the edge e. What should be the optimal value of this fitness?

c. Let the graph to color with 3 colors (R Red, G Green, Y Yellow):



The initial solution S0 sets all edges to the value R. We define a heuristic that selects one edge to change its color. The selected edge and its new color are those for which *m(e,l)*, element of the *M matrix* given below, is minimal; in case of a tie on the edges, the smallest index is chosen, and in case of a tie on the colors, one chooses in order R, then G, then Y.

The *M matrix,* called the cost difference matrix, is written as *M(|E|,k),* with $|E|$ lines and k columns, with:

$$\forall e \in E, \forall l \in \{1,\ldots,k\}, m(e,l) = A - B$$

$$A = \left(\sum_{\substack{e' \in E \;\; , e' \neq e \\ st\, \{i,j\} \cap \{i',j'\} \neq \emptyset}} c(e') == l \right)$$

$$B = \left(\sum_{\substack{e' \in E \;\; , e' \neq e \\ st\, \{i,j\} \cap \{i',j'\} \neq \emptyset}} c(e') == c(e) \right)$$

Using the given specification, what will be the graph coloration with this heuristic?
To find it, you need to process the algorithm starting from S0 then moving to S1, then S2, etc.
Use the following structure initiated for S0 to process it:

	Initial solution *S0*=(R,R,R,R,R))			
	R	G	Y	**B**
e1	0	-3	-3	**-3**
e2	0	-3	-3	**-3**
e3	0	**-4**	-4	**-4**
e4	0	-3	-3	**-3**
e5	0	-3	-3	**-3**
e6	0	-2	-2	**-2**