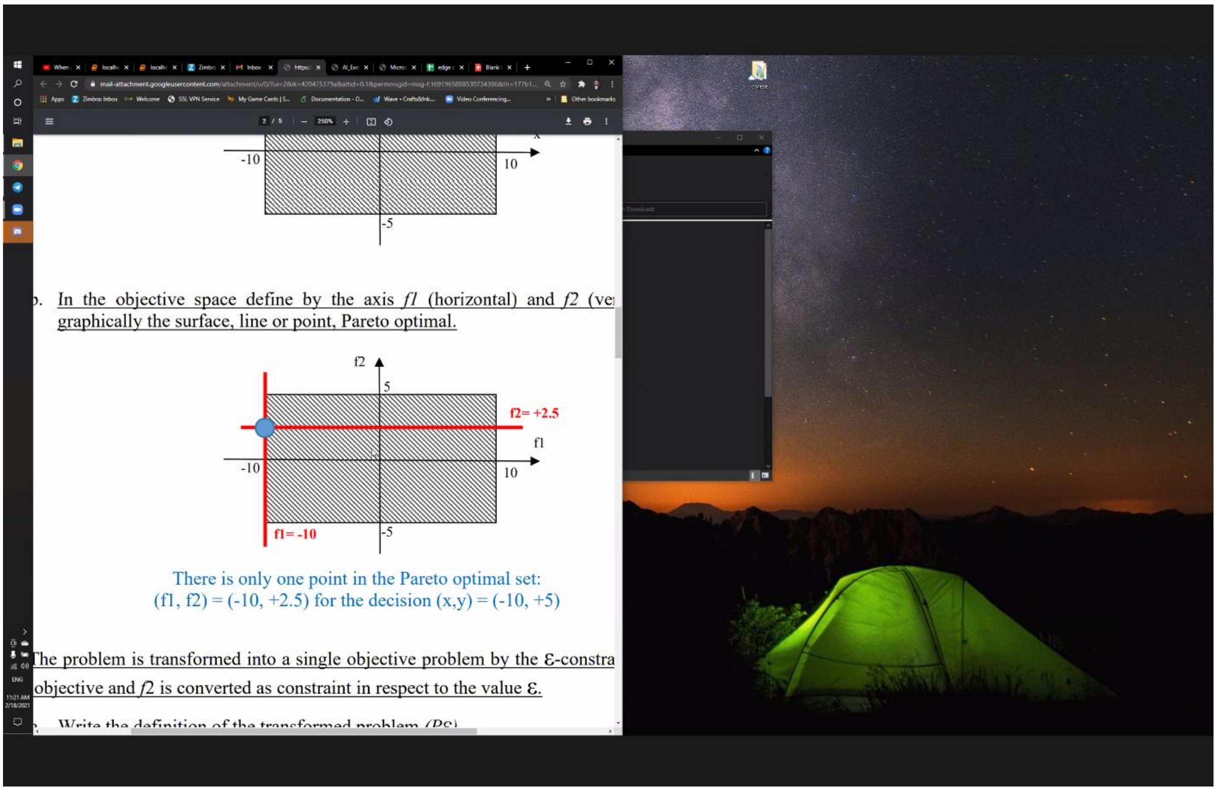
1. MULTICRITERIA DECISION

1- Let *(P)* the optimization problem:

a. In the decision space define by the axis *x* (horizontal) and *y* (vertical), represent graphically the area of all valid solutions *(x,y)*.

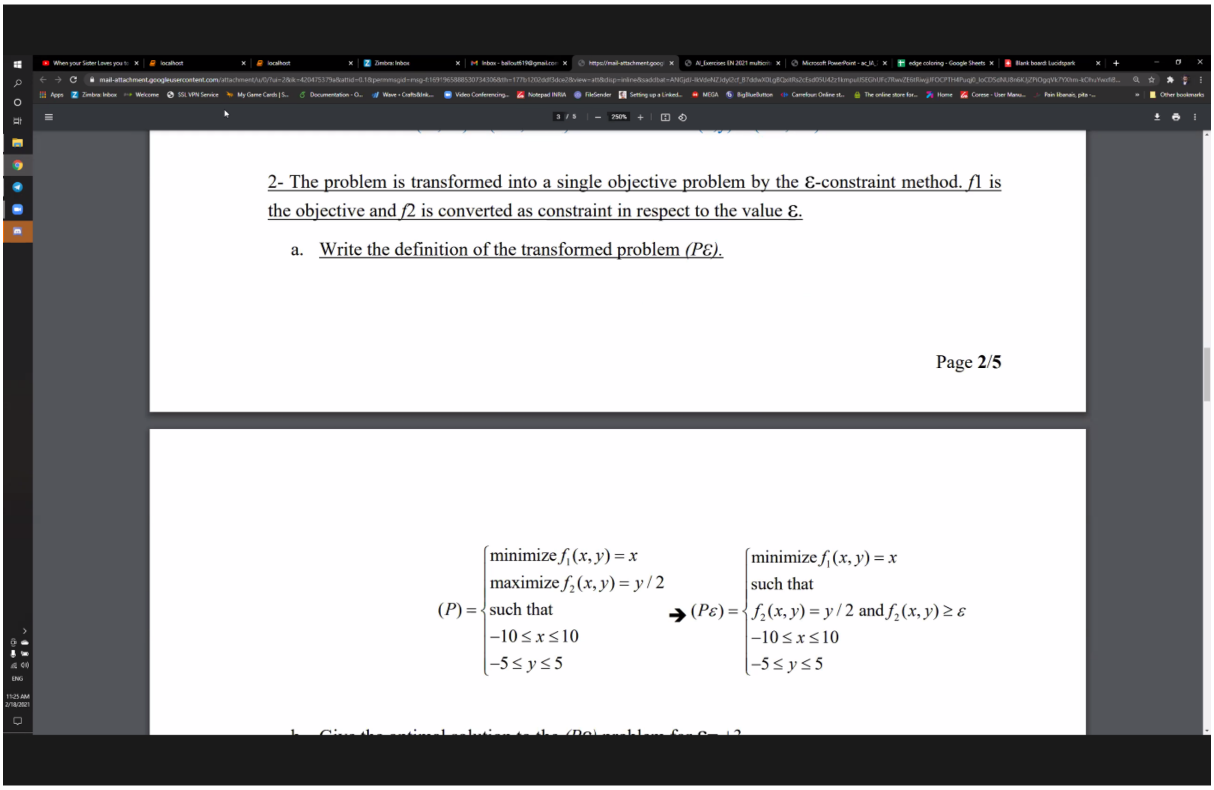
It's a rectangle

b. In the objective space define by the axis *f1* (horizontal) and *f2* (vertical), represent graphically the surface, line or point, Pareto optimal.



2- The problem is transformed into a single objective problem by the Ɛ-constraint method. *f*1 is the objective and *f*2 is converted as constraint in respect to the value Ɛ.

a. Write the definition of the transformed problem *(PƐ)*.



2- The problem is transformed into a single objective problem by the ε-constraint method. *f1* is the objective and *f2* is converted as constraint in respect to the value ε.

a. Write the definition of the transformed problem *(Pε)*.

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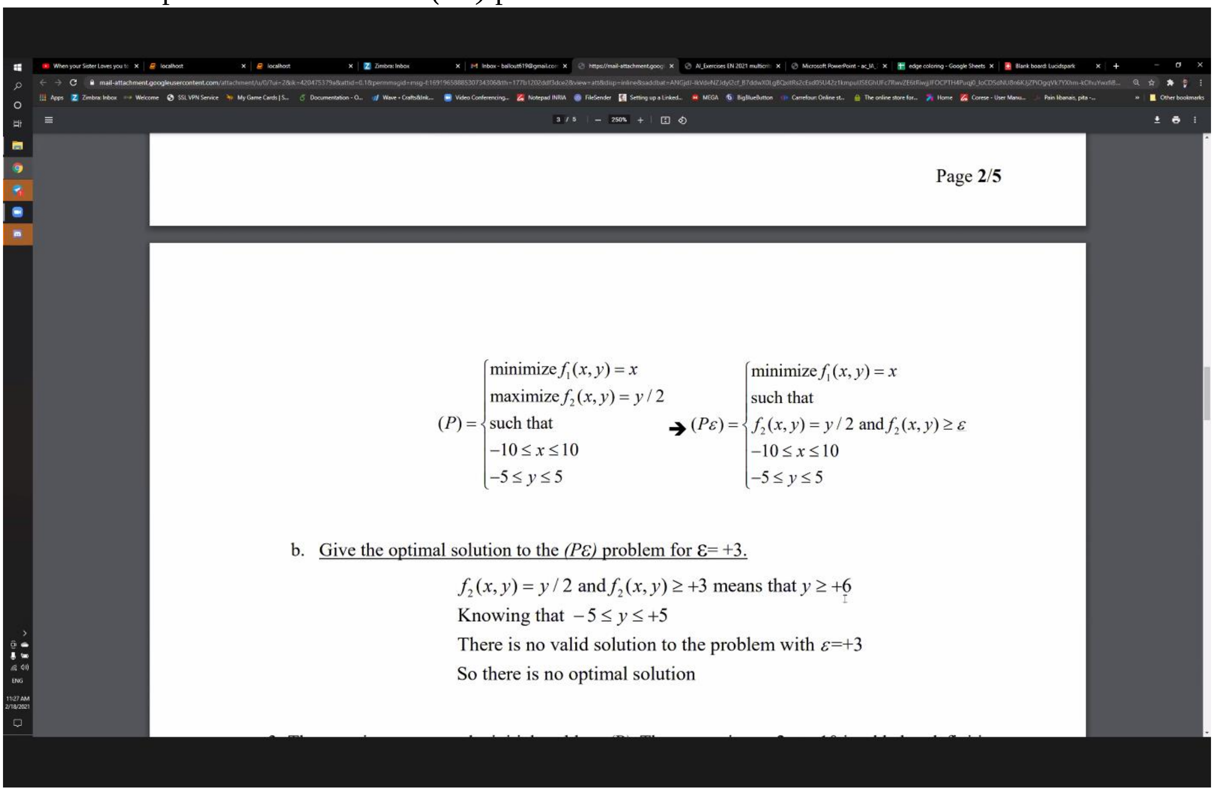
$$(P)=\begin{cases}\text{minimize } f_1(x,y)=x\\ \text{maximize } f_2(x,y)=y/2\\ \text{such that}\\ -10\le x\le 10\\ -5\le y\le 5\end{cases} \rightarrow (P\varepsilon)=\begin{cases}\text{minimize } f_1(x,y)=x\\ \text{such that}\\ f_2(x,y)=y/2 \text{ and } f_2(x,y)\ge\varepsilon\\ -10\le x\le 10\\ -5\le y\le 5\end{cases}$$

b. Give the optimal solution to the *(Pε)* problem for ε= +3.



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b. Give the optimal solution to the *(Pε)* problem for ε= +3.

$f_2(x,y)=y/2$ and $f_2(x,y)\ge +3$ means that $y\ge +6$

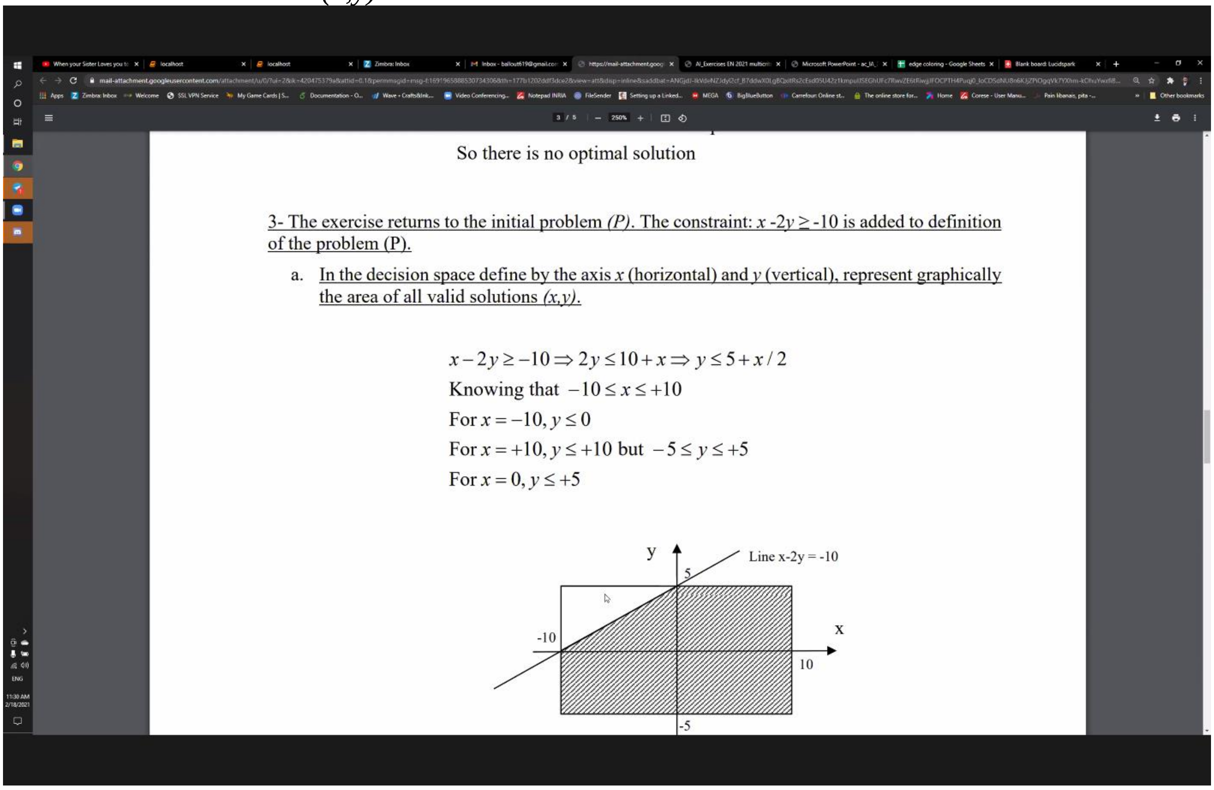
Knowing that $-5\le y\le +5$

There is no valid solution to the problem with ε=+3

So there is no optimal solution

3- The exercise returns to the initial problem *(P)*. The constraint: *x* -2*y* ≥ -10 is added to definition of the problem (P).

a. In the decision space define by the axis *x* (horizontal) and *y* (vertical), represent graphically the area of all valid solutions *(x,y)*.


So there is no optimal solution

3- The exercise returns to the initial problem *(P)*. The constraint: *x* -2*y* ≥ -10 is added to definition of the problem (P).

a. In the decision space define by the axis *x* (horizontal) and *y* (vertical), represent graphically the area of all valid solutions *(x,y)*.

$x-2y \geq -10 \Rightarrow 2y \leq 10+x \Rightarrow y \leq 5+x/2$

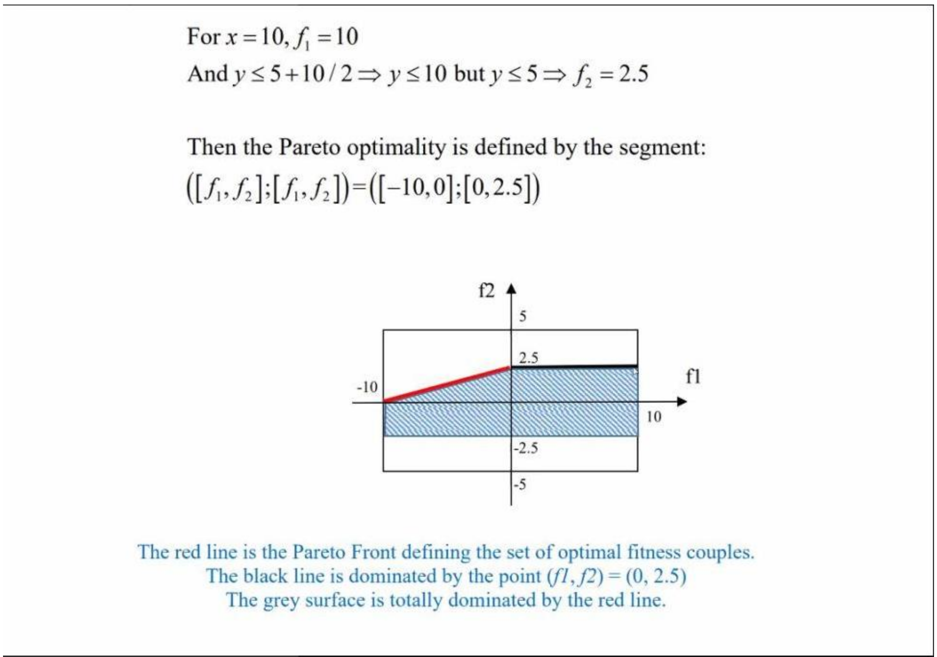
Knowing that $-10 \leq x \leq +10$

For $x=-10, y \leq 0$

For $x=+10, y \leq +10$ but $-5 \leq y \leq +5$

For $x=0, y \leq +5$

b. In the objective space define by the axis *f1* (horizontal) and *f2* (vertical), compute and represent graphically the surface, line or point, Pareto optimal.


For $x=10, f_1=10$

And $y \leq 5+10/2 \Rightarrow y \leq 10$ but $y \leq 5 \Rightarrow f_2 = 2.5$

Then the Pareto optimality is defined by the segment:

$([f_1,f_2];[f_1,f_2])=([-10,0];[0,2.5])$

The red line is the Pareto Front defining the set of optimal fitness couples.
The black line is dominated by the point (*f1*, *f2*) = (0, 2.5)
The grey surface is totally dominated by the red line.

1.2. Pareto – Selection in a Pareto Front

$$P_i = \frac{(TP - r_i)}{\sum_{j \in X} (TP - r_j)} \quad \textit{with } TP = \sum_{j \in X} r_j \quad \textit{and } r_j \text{ the rank of the solution } j$$

WAR (Weighted Average Ranking) function: each solution is first ranked according to its quality in each objective, then its overall rank is equal to the average of its ranks over all the objectives. The solutions are therefore ranked on the basis of the average rank by separating the objectives from each other.

	x1	*x2*	*x3*	*x4*	*x5*	*x6*	*x7*	*x8*	*x9*	*x10*
WAR rank	[illegible].5	4.5	3.5	7	5	10	7	8.5	3.5	2.5
Pi	0.104	0.102	0.104	0.096	0.101	0.090	0.096	0.093	0.104	0.106

Verify that the sum of $P_i = 1$ to check the computation.

Considering that only 4 solutions will be kept for convergence, which ones will be used by the algorithm?