

Lab#1: Proof by Induction, algorithm analysis, recursion

Part 3: Running time complexity

Give an analysis of the running time

```
    sum = 0;
a   for( i = 1; i < n; i++ )
b       for( j = 1; j < i * i; j++ )
c           if( j % i == 0 )
d               for( k = 0; k < j; k++ )
                    sum++;
```

If we disregard the "if" we have:

$$\sum_{i=1}^{n-1} \sum_{j=1}^{i^2} \sum_{k=1}^{j-1}.$$

The loop **a** will be $\theta(n)$, the loop **b** will be $\theta(n^2)$ and the loop **d** should be $\theta(n^2)$, but the loop **d** will only be executed i times for each i because of the if(**c**) (verify the multiples of i), so instead of be $\theta(n^2)$ will take $\theta(n)$ running time.

So it will be: $\theta(n) * \theta(n^2) * \theta(n)$, that means the complexity it is $\theta(n^4)$.

Part 4: Complexity growth

An algorithm takes 0.5 ms for input size 100. How long will it take for input size 500 if the running time is the following (assume low-order terms are negligible):

a) **Linear**

$$100 \rightarrow 0,5$$

$$500 \rightarrow X$$

$$X = \frac{500 * 0,5}{100} = 2,5 \text{ ms}$$

b) $\theta(N \log N)$

$$100 \Rightarrow 100 \log 100 \rightarrow 0.5$$

$$500 \Rightarrow 500 \log 500 \rightarrow X$$

$$X = \frac{0.5 * 500 \log 500}{100 \log 100} \approx 3.37$$

c) **quadratic**

$$100^2 \rightarrow 0,5$$

$$500^2 \rightarrow X$$

$$X = \frac{500^2 * 0,5}{100^2} = 12,5 \text{ ms}$$

d) **cubic**

$$100^3 \rightarrow 0,5$$

$$500^3 \rightarrow X$$

$$X = \frac{500^3 * 0,5}{100^3} = 62,5 \text{ ms}$$