

Analytical and Empirical Comparisons of Voltage Unbalance Definitions

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Abstract—Growing penetrations of distributed energy resources (DERs) increase the power injection variability in distribution systems, which can result in power quality issues such as voltage unbalance. Realistic distribution systems typically have some degree of unbalance, with voltage magnitudes and angle offsets differing between the three phases. To measure unbalance, organizations such as IEC, NEMA, and IEEE define phase unbalance in their power quality standards. However, the definitions in the different standards are not consistent, and voltages that are considered acceptable by one standard may violate good practices defined by another standard. Furthermore, the definitions are based on availability of different measurements and have differing mathematical characteristics. To address this issue, this paper provides analytical comparisons of the most common voltage unbalance definitions. The analytical relationships suggest that it is possible to approximately bound the true voltage unbalance (which depends on the magnitude and relative phase angle) by measuring line-to-line voltage magnitudes, whereas line-to-ground voltages neglect all information about phase angle offsets. The relationships are validated using empirical experiments.

I. INTRODUCTION

Increasing penetrations of distributed energy resources (DERs) are resulting in greater variability of the net load in distribution systems. This challenges distribution system operators' ability to maintain acceptable power quality and may exaggerate problems such as voltage unbalance, which is the focus of this paper. Balanced three-phase voltages have identical phase voltage magnitudes and 120 degree offsets between the phases, whereas unbalanced voltages have different voltage magnitudes and phase offsets. In practice, all distribution systems have some amount of voltage unbalance due to, e.g., unbalanced loads. Unbalanced operating conditions can damage power system equipment, such as three-phase induction motors [1]. Accordingly, motor manufacturers recommend derating three-phase induction machines in unbalanced conditions in order to avoid overheating the windings [2], which leads to under utilization of these machines. Unbalances also increase network losses [3], resulting in inefficient operation of distribution systems. Distribution system operators strive to maintain balanced voltages by equally distributing load demands on all three phases of distribution feeders [2]. Variability associated with DERs challenges traditional approaches for balancing the net load between phases, since the changes in the per-phase loading may be more significant over time.

Various organizations representing different power engineering communities have proposed power quality standards regarding voltage unbalance. The voltage unbalance definitions used by these communities are based on different

measurements and criteria. The IEC standard [4] uses a definition based on the positive and negative sequences from the symmetrical component transformation [5], which requires measurements of both voltage magnitudes and relative phase angles. Standards from NEMA [6] and IEEE [7] define phase unbalance using line-to-line and line-to-ground voltage magnitudes, respectively.

Various voltage unbalance definitions have been analyzed in existing literature. In [8] and [9], three definitions of voltage unbalance (IEEE 1159, IEEE 141-1993, and NEMA) are numerically compared. While the overall conclusion of these papers is that voltage unbalance definitions agree reasonably well for unbalances below 5%, the numerical comparisons show that there are still differences. This implies that operating points which satisfy one power quality standard might violate other standards, which motivates a more thorough analysis of the relationships among various voltage unbalance definitions. Several approximations for the IEC definition based only on line-to-line voltage magnitude measurements (thus avoiding angle measurements) are analyzed in [10] and [11]. However, the resulting expressions are highly non-linear and no clear bounds on the errors are provided.

This paper takes a different approach by comparing the unbalance definitions from an analytical perspective. This enables the derivation of approximate analytical bounds on the difference between the definitions, such that we can assess whether the voltage unbalance as defined by one standard will satisfy unbalance limits imposed by different standard. In summary, the main contributions of this paper are 1) derivations of approximate relationships between the voltage unbalance definitions, 2) empirical evaluations characterizing these relationships' accuracy, and 3) assessment of the conditions under which these relationships are most inaccurate.

The rest of the paper is structured as follows. Section II introduces various voltage unbalance definitions. Section III provides analytical analyses and numerical experiments that characterize voltage unbalances according to these definitions. Section IV concludes the paper.

II. VOLTAGE UNBALANCE DEFINITIONS

Voltage unbalance arises from asymmetric operation leading to phase voltages with unequal magnitudes, phase shifts that are not equal to 120 degrees, or a combination of both [5]. This section presents three common definitions for voltage unbalance from IEC [4], NEMA [6], and IEEE [7]. Other

unbalance definition based solely on line-to-line voltage magnitudes are used by instrument manufacturers for power quality measurements as an approximation to the IEC definition. These approximations are not discussed in this paper as they do not represent fundamentally different definitions, but are rather approximations of one of the considered definitions.

We use the following notation to represent phasors: $\mathbf{V} = V\angle\theta$, where \mathbf{V} denotes the voltage phasor as a complex number, V is the voltage magnitude, and $\angle\theta$ is the phase angle. Denote the complex conjugate of \mathbf{V} as $\bar{\mathbf{V}}$. Let $j = \sqrt{-1}$.

A. IEC definition (VUF)

Unbalanced voltages can be described using symmetrical components. The definition of the Voltage Unbalance Factor (VUF) frequently considered to be the “true” definition of voltage unbalance is based on the relative magnitudes of the negative and positive sequence voltages. The VUF definition adopted by the IEC [4] is

$$\begin{aligned} VUF [\%] &= \frac{|\mathbf{V}_n|}{|\mathbf{V}_p|} \times 100, \\ \text{where } \mathbf{V}_p &= \frac{\mathbf{V}_a + \mathbf{a} \cdot \mathbf{V}_b + \mathbf{a}^2 \cdot \mathbf{V}_c}{3}, \\ \mathbf{V}_n &= \frac{\mathbf{V}_a + \mathbf{a}^2 \cdot \mathbf{V}_b + \mathbf{a} \cdot \mathbf{V}_c}{3}. \end{aligned} \quad (1)$$

In (1), \mathbf{V}_p and \mathbf{V}_n are the positive and negative sequence voltage phasors, respectively; $\mathbf{a} = 1\angle 120^\circ$; and \mathbf{V}_a , \mathbf{V}_b , and \mathbf{V}_c are the three-phase unbalanced line-to-ground voltage phasors. The two sequence components pose some practical difficulties associated with their use of complex algebra [8] and the need to track both the voltage magnitudes and phase angles for each phase. One of the major challenges in using the IEC definition is the inability to directly measure the sequence components using installed RMS meters as they provide no information about the relative phase angles of the voltages [9]. For low- and medium-voltage systems, IEC standard 61000-2-2 [4] requires that the voltage unbalances, as defined in (1), are less than 2%.

B. NEMA definition (LVUR)

Motors make up a large portion of the loads connected to distribution systems. Motor manufacturers use the NEMA definition of voltage unbalance [6], which is also referred to as the line voltage unbalance rate (LVUR). The NEMA definition calculates unbalance using the magnitudes of line-to-line voltages V_{ab} , V_{bc} and V_{ca} :

$$\begin{aligned} LVUR [\%] &= \frac{\Delta V_L^{max}}{V_{avg}} \times 100, \\ \text{where } V_{avg} &= \frac{V_{ab} + V_{bc} + V_{ca}}{3}, \\ \Delta V_L^{max} &= \max\{|V_{ab} - V_{avg}|, |V_{bc} - V_{avg}|, |V_{ca} - V_{avg}|\}. \end{aligned} \quad (2)$$

Per NEMA MG-1 [6] and ANSI C84.1 [12], power systems must be operated such that the maximum voltage unbalance, as defined in (2), does not exceed 3% under no-load conditions.

C. IEEE definition (PVUR)

The IEEE definition is a recommended guideline to measure unbalance for the electrical design of industrial facilities [7]. It is commonly referred to as phase voltage unbalance rate (PVUR) and described using the line-to-ground voltage magnitudes V_a , V_b and V_c :

$$\begin{aligned} PVUR [\%] &= \frac{\Delta V_P^{max}}{V_{avg}} \times 100, \\ \text{where } V_{avg} &= \frac{V_a + V_b + V_c}{3}, \\ \Delta V_P^{max} &= \max\{|V_a - V_{avg}|, |V_b - V_{avg}|, |V_c - V_{avg}|\}. \end{aligned} \quad (3)$$

Motors are affected by phase-voltage unbalances that exceed 2% and are most likely to overheat when operating near full load [7]. The difference between the NEMA and IEEE definitions is the use of line-to-ground voltages versus line-to-line voltages, respectively. By ignoring the phase angle information, the IEEE definition does not detect voltage unbalance that is due to phase angle asymmetries.

III. COMPARISON OF VOLTAGE UNBALANCE DEFINITIONS

In order to analyze the different definitions, voltage unbalance can be divided into three categories:

- (a) Voltage magnitude unbalance,
- (b) Voltage angle unbalance,
- (c) Voltage magnitude and angle unbalance.

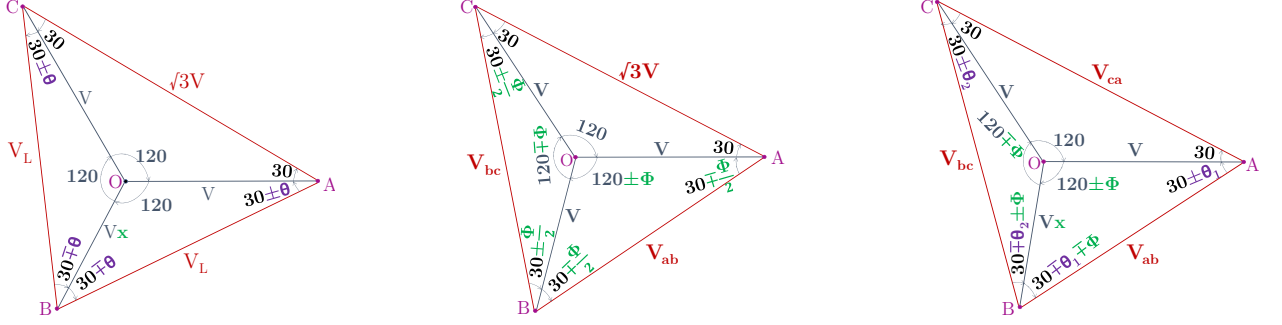
Fig. 1 on the next page illustrates the voltage triangle with unbalance in the arbitrarily selected phase B for each of these categories. With balanced voltage magnitude indicated by V , the magnitude unbalance is denoted using the multiplicative factor x and the angle unbalance is denoted using ϕ . The angle θ in Fig. 1 is determined by x and ϕ . For each category of unbalance, this section derives analytical expressions using certain approximations to characterize the relationships among the definitions. These expressions are validated using numerical simulations. The comparisons focus on accuracy with respect to the VUF in (1) as defined by the IEC.

A. Voltage magnitude unbalance in Phase B

This section investigates the voltage magnitude unbalance case (i.e., $\phi = 0^\circ$). We first consider a voltage magnitude unbalance in one of the three phases as shown in Fig. 1(a). Two line-to-line voltages V_L are affected by the unbalance of the phase- B -to-ground voltage magnitude, while the third line-to-line voltage remains the same as in the balanced case with a magnitude of $\sqrt{3}V$. The line-to-ground and line-to-line voltage phasors are

$$\begin{aligned} \mathbf{V}_a &= V\angle 0^\circ, & \mathbf{V}_{ab} &= V_L\angle(30^\circ \pm \theta), \\ \mathbf{V}_b &= Vx\angle -120^\circ, & \mathbf{V}_{bc} &= V_L\angle -(90^\circ \pm \theta), \\ \mathbf{V}_c &= V\angle 120^\circ, & \mathbf{V}_{ca} &= \sqrt{3}V\angle 150^\circ. \end{aligned}$$

We next provide analytical expressions for the three voltage unbalance definitions in this case.



(a) Voltage magnitude unbalance

(b) Voltage angle unbalance

(c) Voltage magnitude and angle unbalance

Fig. 1: Voltage triangles for three cases of unbalance in phase B

1) *IEC definition (VUF)*: The magnitudes of the positive and negative symmetrical components V_p and V_n are

$$V_p = \frac{|V\angle 0^\circ + Vx\angle 0^\circ + V\angle 0^\circ|}{3} = \frac{V \cdot (x+2)}{3}, \quad (4a)$$

$$V_n = \frac{|V\angle 0^\circ + Vx\angle 120^\circ + V\angle -120^\circ|}{3} = \frac{V \cdot (1-x) |1 - j\sqrt{3}|}{3 \cdot 2} = \frac{V \cdot |x-1|}{3}. \quad (4b)$$

The voltage unbalance according to the IEC definition is then

$$VUF = \frac{|x-1|}{x+2}. \quad (5)$$

2) *NEMA definition (LVUR)*: The LVUR voltage unbalance definition used by NEMA depends on the line-to-line voltage V_L . To express V_L in terms of x , the sine triangle rule is applied to the triangle $\triangle OAB$ in Fig. 1(a):

$$\frac{V_L}{\sin 120^\circ} = \frac{V}{\sin(30^\circ \mp \theta)} = \frac{Vx}{\sin(30^\circ \pm \theta)}. \quad (6)$$

Starting from (6), we use standard trigonometric identities for the sum of angles and the small angle approximations $\sin \theta \approx \theta$ and $\cos \theta \approx 1$ to solve for x and V_L . We first obtain the following expression for x ,

$$x = \frac{\cos \theta \pm \sqrt{3} \sin \theta}{\cos \theta \mp \sqrt{3} \sin \theta} \approx \frac{1 \pm \sqrt{3}\theta}{1 \mp \sqrt{3}\theta},$$

which yields the following expression for V_L ,

$$V_L = \sqrt{3}V \left(\frac{x+1}{2 \cos \theta} \right) \approx \sqrt{3}V \left(\frac{x+1}{2} \right). \quad (7)$$

The average line-to-line voltage V_{Lavg} and maximum deviation ΔV_L^{max} are

$$V_{Lavg} = \frac{V_L + V_L + \sqrt{3}V}{3} \approx \sqrt{3}V \left(\frac{x+2}{3} \right), \quad (8a)$$

$$\Delta V_L^{max} = |\sqrt{3}V - V_{Lavg}| \approx \sqrt{3}V \left(\frac{|x-1|}{3} \right), \quad (8b)$$

which allows us to obtain the following expression for LVUR:

$$LVUR \approx \frac{|x-1|}{x+2}. \quad (9)$$

3) *IEEE definition (PVUR)*: The PVUR definition (3) is computed using the average voltage magnitude, V_{Pavg} ,

$$V_{Pavg} = \frac{V + V + Vx}{3} = \frac{V \cdot (x+2)}{3},$$

and the deviation from the average, ΔV_P^{max} ,

$$\Delta V_P^{max} = \left| Vx - \frac{V \cdot (x+2)}{3} \right| = \frac{2V \cdot |x-1|}{3}.$$

This gives rise to the following expression for PVUR,

$$PVUR = \frac{2 \cdot |x-1|}{x+2}. \quad (10)$$

4) *Comparisons*: Based on (5), (9), and (10), we observe the following relationships:

$$LVUR \approx VUF = 2 \cdot PVUR. \quad (11)$$

Note that the calculations of LVUR involve small angle approximations, whereas VUF and PVUR are exact calculations.

Another interesting observation is that for a given deviation $|x-1|$ in voltage magnitude, the expressions for ΔV_P^{max} , ΔV_L^{max} and V_n are invariant to over-voltage ($x > 1$) or under-voltage ($x < 1$) conditions. Conversely, the average voltages V_{Pavg} and V_{Lavg} as well as V_p are directly proportional to x . Hence, the unbalance is greater for under-voltage cases compared to the over-voltage cases, which is consistent with the observations in [10]. However, although not evident in (9) due to the small angle approximation, the true maximum voltage deviation in the line-to-line voltages ΔV_L^{max} actually depends on the angle θ . Therefore, LVUR will not be exactly the same as VUF, and specifically will be larger or smaller than VUF for under-voltage and over-voltage conditions, respectively.

5) *Numerical Validation*: To validate the expressions derived above, we compute the voltage unbalances that result from varying the phase-B voltage magnitude. The multiplicative factor x is varied from 0.8 to 1.2 in steps of 0.01 to investigate both under-voltage and over-voltage conditions. The upper part of Fig. 2 illustrates the variation of VUF (on the y-axis) with respect to both PVUR and LVUR (on the x-axis) for voltage magnitude unbalance in phase B only. The dashed

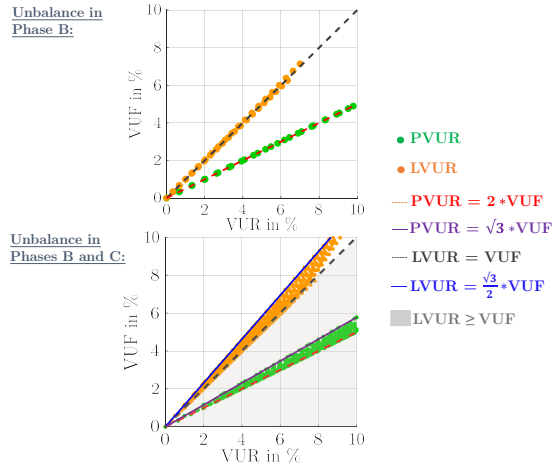


Fig. 2: Voltage magnitude unbalance for $0.80 \leq x \leq 1.20$

black line represents the situations where VUF and LVUR have the same value, while the dashed red line represents the situation where PVUR is a factor of 2 larger than VUF. The orange and green dots correspond to values obtained for LVUR and PVUR, respectively.

Consistent with our analytical results, we observe that PVUR is exactly twice as large as VUF. The values for VUF and LVUR are almost identical, although there are minor variations due to the inaccuracy of the small angle approximation. For the range of x values considered, the LVUR values are within 2% of the VUF values.

B. Voltage magnitude unbalance in two phases

Due to the complexity of the expressions, we only include a high-level discussion of the case with voltage magnitude unbalances in both phases B and C . We express the voltage unbalance in a similar way as the voltage unbalance for just phase B , except that we consider two deviations in the magnitudes, i.e., $V_b = x_1 V$ and $V_c = x_2 V$.

Similar to the case with magnitude deviation in only one phase, the denominators of all unbalance definitions simplify to the same expressions when we have magnitude deviations in two phases. To assess the relative magnitudes of the voltage unbalance definitions for a given voltage unbalance, we therefore only need to assess the differences in the numerators.

1) *IEC definition (VUF)*: The numerator for VUF in (1) is the magnitude of the negative sequence voltage V_n :

$$|V_n| = \frac{V}{3} \sqrt{x_1^2 + x_2^2 + x_1 + x_2 + x_1 x_2}. \quad (12)$$

2) *NEMA and IEEE definitions (LVUR and PVUR)*: The numerators for LVUR and PVUR in (2) and (3), respectively, depend on which voltage magnitude is further away from the average. We hence need to consider three cases,

$$\Delta V_L^{max} \approx \frac{\Delta V_P^{max}}{2} = \frac{V}{3} \max\{|y_1|, |y_2|, |y_3|\}, \quad (13)$$

$$\begin{aligned} \text{where } y_1 &= x_1 - 2x_2 + 1, & y_2 &= x_1 + x_2 - 2, \\ y_3 &= -2x_1 + x_2 + 1 \end{aligned}$$

3) *Comparison*: The denominator for LVUR is still approximately equivalent to the denominator for PVUR, which suggests the following relationship between LVUR and PVUR:

$$\frac{LVUR}{PVUR} \approx \frac{\Delta V_L^{max}}{\Delta V_P^{max}/2} \approx 2 \Rightarrow LVUR \approx 2 \cdot PVUR. \quad (14)$$

When comparing LVUR and PVUR with VUF, we observe that there is not a one-to-one mapping because we need to consider the three different cases corresponding to whether y_1 , y_2 , or y_3 is the largest. Since both $|V_n|$ and ΔV_L^{max} are non-negative, we consider the square of the ratio between VUF and LVUR, i.e.,

$$\left(\frac{VUF}{LVUR} \right)^2 \approx \frac{|V_n|^2}{(\Delta V_L^{max})^2} \approx \frac{x_1^2 + x_2^2 + x_1 + x_2 + x_1 x_2}{(\max\{|y_1|, |y_2|, |y_3|\})^2}.$$

By identifying the largest among $|y_1|$, $|y_2|$ and $|y_3|$, simplifying the expressions, and taking the square root of both sides, we obtain an upper bound on the ratio of VUF to LVUR,

$$\frac{|V_n|}{\Delta V_L^{max}} \leq \frac{2}{\sqrt{3}} \Rightarrow VUF \leq \left(\frac{2}{\sqrt{3}} \right) \cdot LVUR. \quad (15)$$

The definitions of y_1 , y_2 , and y_3 imply that the unbalance ratio seen in (15) with unbalance in two phases is always larger than the situation where either $x_1 = 1$ or $x_2 = 1$. This implies that (11) is a lower bound. Combining this observation with (15) yields the following relationship between LVUR and VUF:

$$LVUR \leq VUF \leq \left(\frac{2}{\sqrt{3}} \right) \cdot LVUR \quad (16a)$$

By considering (14), we extend this to PVUR:

$$\left(\frac{1}{2} \right) \cdot PVUR \leq VUF \leq \left(\frac{1}{\sqrt{3}} \right) \cdot PVUR. \quad (16b)$$

4) *Numerical Validation*: Similar to the case with unbalance in only one phase, we run experiments where we let x_1 and x_2 vary from 0.8 to 1.2 in steps of 0.01. The results are shown in the lower part of Fig. 2. The figure uses a similar legend as the upper part, with the orange points and green points representing the values for LVUR and PVUR relative to VUF, respectively. In addition to the black and red lines that are used to identify the situation where $LVUR = VUF$ and $PVUR = 2 \cdot VUF$ respectively, we also add lines to represent the upper bounds $VUF = \left(\frac{2}{\sqrt{3}} \right) \cdot LVUR$ (solid blue line) and $VUF = \left(\frac{1}{\sqrt{3}} \right) \cdot PVUR$ (solid purple line). Almost all points fall within the predicted range.

However, LVUR is greater than VUF for approximately 17% of the considered points. These points fall below the dashed black line in the grey shaded region as shown in Fig. 2. This situation corresponds to cases where two of the line-to-line voltages were significantly higher than the third line-to-line voltage, i.e., extreme cases of under-voltage in two phases and over-voltage in the third phase, resulting in inaccuracy of the small angle approximation. However, the errors in our bounds are small: within the ranges of voltage magnitudes corresponding to $0.8 \leq x_1, x_2 \leq 1.2$, we never see violations larger than 2% relative to the bounds in (16).

C. Voltage angle unbalance

We next consider the single voltage angle unbalance case with angle displacement ϕ and balanced voltage magnitude V as shown in Fig. 1(b). The voltage phasors in this case are

$$\begin{aligned} \mathbf{V}_a &= V\angle 0^\circ, & \mathbf{V}_{ab} &= V_{ab}\angle(30^\circ \mp \phi/2), \\ \mathbf{V}_b &= V\angle-(120^\circ \pm \phi), & \mathbf{V}_{bc} &= V_{bc}\angle-(90^\circ \pm \phi/2), \\ \mathbf{V}_c &= V\angle 120^\circ, & \mathbf{V}_{ca} &= \sqrt{3}V\angle 150^\circ. \end{aligned}$$

1) *IEC definition (VUF)*: To derive the expression for VUF, we use the small angle approximation to find the magnitudes of the sequence voltages, V_p and V_n :

$$V_p \approx \frac{V}{3}\sqrt{9 + \phi^2} \approx V, \quad V_n \approx V \cdot \frac{\phi}{3} \Rightarrow VUF \approx \frac{\phi}{3}. \quad (17)$$

2) *NEMA definition (LVUR)*: To determine LVUR, we use the sine triangle rule and the small angle approximation for ϕ to obtain the line-to-line voltage magnitudes, V_{ab} and V_{bc} :

$$V_{ab} \approx V(\sqrt{3} \pm \frac{\phi}{2}), \quad V_{bc} \approx V(\sqrt{3} \mp \frac{\phi}{2}). \quad (18)$$

We then derive an approximation of the average line-to-line voltage V_{Lavg} and the maximum deviation ΔV_L^{max} , which provides an approximate expression for LVUR:

$$V_{Lavg} \approx \sqrt{3}V, \quad \Delta V_L^{max} \approx V \frac{\phi}{2} \Rightarrow LVUR \approx \frac{\sqrt{3}}{2} \frac{\phi}{3}. \quad (19)$$

3) *IEEE definition (PVUR)*: Since all the line-to-ground voltage magnitudes are balanced, ΔV_P^{max} is zero. Hence, PVUR is also equal to zero for any unbalance that only affects the voltage angles:

$$PVUR = 0. \quad (20)$$

4) *Comparison*: Based on the analytical expressions, we observe that PVUR does not provide any information about phase angle unbalance. The approximate relationship between LVUR and VUF is

$$LVUR \approx \left(\frac{\sqrt{3}}{2} \right) \cdot VUF. \quad (21)$$

5) *Numerical Validation*: These relationships are verified by numerical experiments. With one of the phase angles selected as a reference, the phase angle displacement in the other two phases are varied in the range $-20^\circ \leq \phi \leq 20^\circ$ in steps of 1° . The results of these computations are shown in the upper part of Fig. 3 for unbalance in one phase, arbitrarily selected as phase B . The experiment confirms our the analytical results, with $PVUR = 0$ regardless of the angle unbalance and $LVUR \approx (\frac{\sqrt{3}}{2}) \cdot VUF$.

D. Voltage angle unbalance in Phase B and C

To extend the above analysis, we assess how voltage angle unbalance in two phases affects the relationships among the definitions. We introduce two phase angle deviations, ϕ_1 and ϕ_2 for phases B and C , respectively, and perform a similar numerical study, with each phase angle deviation varying in the range $-20^\circ \leq \phi_1, \phi_2 \leq 20^\circ$ in steps of 1° . The

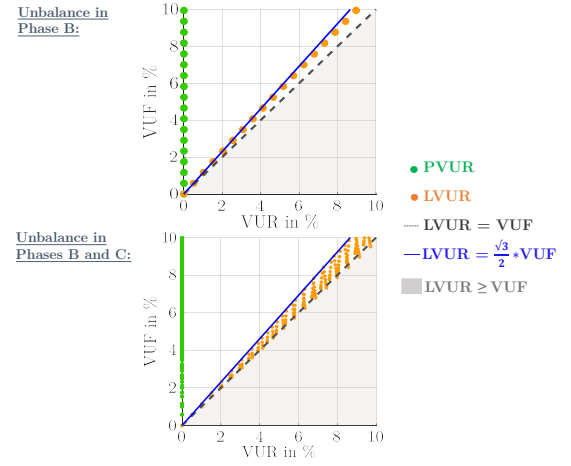


Fig. 3: Voltage angle unbalance for $-20^\circ \leq \phi \leq 20^\circ$

results are shown in the lower part of Fig. 3. Similar to the voltage magnitude unbalance case, we get some variation in the relation between LVUR and VUF when we introduce phase angle unbalance in both phases. However, VUF is still effectively bounded by the same relations as in (16),

$$LVUR \leq VUF \leq \left(\frac{2}{\sqrt{3}} \right) \cdot LVUR. \quad (22)$$

E. Voltage magnitude and angle unbalance

Finally, we consider the general case where the unbalance involves asymmetries in both the voltage magnitudes and angles. We first investigate the case with both magnitude and angle unbalance in one of the line-to-ground voltages, i.e., the voltage in the arbitrarily selected phase B has an unbalanced voltage magnitude Vx and the angle displacement defined by ϕ as shown in Fig. 1(c) and expressed below:

$$\begin{aligned} \mathbf{V}_a &= V\angle 0^\circ, & \mathbf{V}_{ab} &= V_{ab}\angle(30^\circ \mp \theta_1), \\ \mathbf{V}_b &= Vx\angle-(120^\circ \pm \phi), & \mathbf{V}_{bc} &= V_{bc}\angle-(90^\circ \pm \theta_2), \\ \mathbf{V}_c &= V\angle 120^\circ, & \mathbf{V}_{ca} &= \sqrt{3}V\angle 150^\circ. \end{aligned}$$

Applying the sine triangle rule and the small angle approximation for ϕ , we can express the line-to-line voltages magnitude and angle deviations as

$$\begin{aligned} V_{ab} &= \frac{V \cdot (\sqrt{3} \mp \phi)(1+x)}{(2 \mp \sqrt{3}\phi)}, & V_{bc} &= \frac{V \cdot (\sqrt{3} \pm \phi)(1+x)}{(2 \pm \sqrt{3}\phi)}, \\ \theta_1 &= \frac{x-1}{\sqrt{3}(x+1)} \mp \frac{\phi x}{x+1}, & \theta_2 &= \frac{x-1}{\sqrt{3}(x+1)} \pm \frac{\phi x}{x+1}. \end{aligned}$$

1) *IEC definition (VUF)*: For the VUF calculation, the sequence component magnitudes V_p and V_n are

$$V_p = \frac{V \cdot (x+2)}{3}, \quad V_n = \frac{V \cdot \sqrt{(1+\phi^2)x^2 - 2x + 1}}{3},$$

which leads to the following expression for VUF:

$$VUF = \frac{\sqrt{(1+\phi^2)x^2 - 2x + 1}}{x+2}. \quad (24)$$

Note that these expressions depend on both the voltage magnitude and phase angle deviations.

2) *NEMA definition (LVUR)*: For LVUR, the expression for the average line-to-line voltage, V_{Lavg} , simplifies to the same expression as in the voltage magnitude unbalance case (8a). However, computing the maximum deviation, ΔV_L^{max} , involves taking the maximum over three different cases:

$$LVUR = \frac{\max\{|y_1|, |y_2|, |y_3|\}}{x+2}, \quad (25)$$

$$\text{where } y_1 = x - 1 \text{ and } y_2, y_3 = \frac{1-x}{2} \mp \sqrt{3}\phi \left(\frac{1+x}{4} \right).$$

Here, y_1 dominates if the angle unbalance is negligible. In this case, the voltage unbalance expressions for LVUR and VUF are similar to (5) and (9).

3) *IEEE definition (PVUR)*: Since PVUR ignores angle information, the expression for PVUR is the same as in the case with only voltage magnitude unbalance, i.e., (10).

4) *Numerical Comparison*: To compare the definitions in the presence of both magnitude and angle asymmetries, we again perform a numerical experiment. We choose V_a as reference voltage phasor at $1\angle 0^\circ$ and calculate the voltage unbalance according to each definition as we vary the deviations in both the voltage magnitudes (from 0.8 to 1.2 p.u. in steps of 0.01 p.u.) and angles (from -20° to 20° in steps of 1°). The results are shown in Fig. 4. The upper part of this figure shows the case with deviations only in phase B and the lower part shows the case with unbalances in both phases B and C .

As previously discussed, PVUR does not consider voltage angle unbalances. Hence, Fig. 4 shows no clear relationship between PVUR and VUF, i.e., there is a large variation of PVUR corresponding to any given value of VUF.

The relationship between LVUR and VUF is similar to the previous cases, yielding the following bounds:

$$LVUR \leq VUF \leq \left(\frac{2}{\sqrt{3}} \right) \cdot LVUR. \quad (26)$$

The few points where $VUF < LVUR$ correspond to the extreme condition of under-voltage in two phases and over-

voltage in the third phase. For the ranges of voltage magnitude deviations $0.8 \leq x_1, x_2 \leq 1.2$ and phase angle deviations $-20^\circ \leq \phi_1, \phi_2 \leq 20^\circ$, we never observe violations greater than 5% relative to the bounds in (26).

IV. CONCLUSION

This paper has analyzed the relationship between the three most prevalent definitions of voltage unbalance. The “true” voltage unbalance definition VUF used in the IEC standard captures both voltage magnitude and angle unbalance, but can be difficult to use due to challenges in obtaining measurements of both voltage magnitudes and phase angles. Definitions from NEMA and IEEE based on the line-to-line voltages (LVUR) and line-to-ground voltages (PVUR), respectively, are frequent substitutes used in the design of electrical machines and installations.

We have derived and empirically demonstrated approximate relationships among these definitions. In summary, PVUR does not provide any information about phase angle unbalance and has no clear relationship with VUF. Conversely, LVUR provides information about unbalances in both voltage magnitudes and phases. Using the small angle approximation, we approximately bound the relationships between the IEC definition (VUF) and the NEMA definition (LVUR) as $LVUR \leq VUF \leq \left(\frac{2}{\sqrt{3}} \right) \cdot LVUR$. For a reasonable ranges of voltage magnitude and angle deviations (0.8 to 1.2 p.u. voltage magnitude and -20° to 20°), the violations of these bounds are less than 5% (relative to the bounds).

Our ongoing work is investigating how these definitions can be formulated in order to provide the most tractable constraints in optimization problems that consider voltage unbalances.

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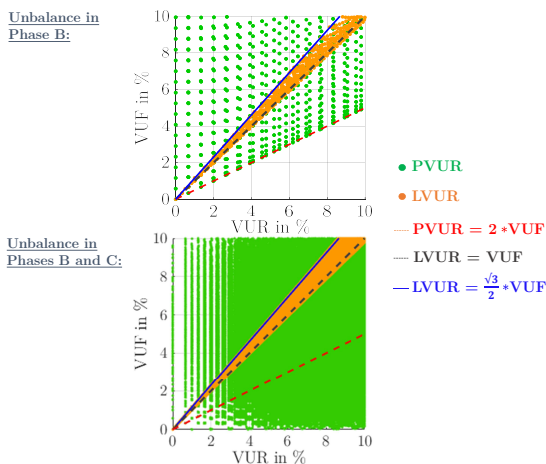


Fig. 4: Voltage magnitude and angle unbalances for $0.8 \leq x \leq 1.2$ and $-20^\circ \leq \phi \leq 20^\circ$.