محمدرضا کشاورزی - 810192409 HW#5 - تمرین سری آنتن

۱۵ $\frac{۱۰۵}{۱۱۰}$ + $\frac{۵}{۲۰}$

① broadside array $\Rightarrow$ Max @ $\theta_m = 90°$

$HPBW = 40°$, θ_n is elevation angle of the half power point.

$$2|\theta_m - \theta_n| = 40° \Rightarrow |\theta_m - \theta_n| = 20° \Rightarrow \theta_n = 70° \text{ or } 110°$$

broad side array $\Rightarrow \beta = 0 \Rightarrow \theta_n = \frac{\pi}{2} - \sin^{-1}\left(\frac{\lambda}{2\pi d}\left(-\beta \pm \frac{2.782}{N}\right)\right)$

$$\Rightarrow \sin^{-1}\left(\frac{\lambda}{2\pi d}\frac{2.782}{N}\right) = 20 \Rightarrow \frac{2}{\pi} \times \frac{2.782}{N} = \sin 20 = 0.342 \Rightarrow N = 5.17$$

$$\Rightarrow \boxed{N = 5}$$

$$D_0 = 2\left(1 + \frac{L}{d}\right)\frac{d}{\lambda} = 2N\left(\frac{d}{\lambda}\right) = 2 \times 5 \times \frac{1}{4} = 2.5$$

برا محاسبه دقیق داریم:

$$U_0 = \frac{1}{Nkd}\int_{-Nk\frac{d}{2}}^{Nk\frac{d}{2}} \left(\frac{\sin Z}{Z}\right)^2 dZ \quad , \quad D_0 = \frac{U_{max}}{U_0} = \frac{1}{U_0}$$

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② Array Factor $= AF_x AF_y = \frac{1}{2}\frac{\sin \psi_x}{\sin \frac{\psi_x}{2}} \cdot \frac{1}{2}\frac{\sin \psi_y}{\sin \frac{\psi_y}{2}}$, $\left| \begin{array}{l} \psi_x = kd\sin\theta\cos\varphi \\ \psi_y = kd\sin\theta\sin\varphi \end{array}\right.$

Element Factor $= \frac{2}{\pi}\frac{\cos\left(\frac{\pi}{2}\sin\theta\cos\varphi\right)}{\sqrt{1-\sin^2\theta\cos^2\varphi}}$

$\Rightarrow$ الگوی تشعشعی نرمالیزه شده $= \frac{2}{\pi}\frac{\cos(\pi/2 \sin\theta\cos\varphi)}{\sqrt{1-\sin^2\theta\cos^2\varphi}} \cdot \frac{1}{4}\frac{\sin\psi_x}{\sin\frac{\psi_x}{2}}\frac{\sin\psi_y}{\sin\frac{\psi_y}{2}}$

$d = \frac{\lambda}{2}$, $\varphi = 0$ (صفحه xz) $\Rightarrow AF = \frac{1}{2}\frac{\sin(\pi\sin\theta)}{\sin(\frac{\pi}{2}\sin\theta)}$, $EF = \frac{2}{\pi}\frac{\cos(\frac{\pi}{2}\sin\theta)}{\cos\theta}$

$\Rightarrow$ الگوی تشعشعی $= AF \cdot EF$

$d = \frac{\lambda}{2}$, $\varphi = \frac{\pi}{2}$ (صفحه yz) $\Rightarrow AF = \frac{1}{2}\frac{\sin(\pi\sin\theta)}{\sin(\frac{\pi}{2}\sin\theta)}$, $EF = \frac{2}{\pi}$

$\Rightarrow$ الگوی تشعشعی $= AF \cdot EF$

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③ $2M + 1 = 5 \Rightarrow M = 2$ ۲۰

روش ۱: $AF_5 = \sum_{n=1}^{M+1} a_n \cos[2(n-1)u]$, $u = \frac{\pi d}{\lambda}\cos\theta$

روش ۲: $AF_5 = a_1 + a_2\cos 2u + a_3\cos 4u$

3 مر/ : $30 = 20\log R_0 \Rightarrow R_0 = 31.62$

$$Z_0 = \frac{1}{2}\left[\left(R_0 + \sqrt{R_0^2 - 1}\right)^{\frac{1}{P}} + \left(R_0 - \sqrt{R_0^2-1}\right)^{\frac{1}{P}} \right] \qquad P = N-1$$

$$= \frac{1}{2}\left[\left(31.62 + \sqrt{31.62^2-1}\right)^{\frac{1}{4}} + \left(31.62 - \sqrt{31.62^2-1}\right)^{\frac{1}{4}} \right] = 1.587$$

4 مر/ : $\cos U = \frac{Z}{Z_0}$

5 مر/ : $$AF_5 = a_1 - a_2 + a_3 + Z^2\left[(2a_2 - 8a_3)/Z_0^2\right] + Z^4 \frac{a_3}{Z_0^4} = T_4(Z)$$

$$= 1 - 8Z^2 + 8Z^4$$

$$\Rightarrow \begin{cases} \frac{a_3}{Z_0^4} = 8 \Rightarrow a_3 = 50.75 \\ (2a_2 - 8a_3)/Z_0^2 = -8 \Rightarrow a_2 = 192.93 \\ a_1 - a_2 + a_3 = 1 \Rightarrow a_1 = 142.18 \end{cases} \Rightarrow \begin{cases} a_1 = 0.737 \\ a_2 = 1 \\ a_3 = 0.263 \end{cases} \Rightarrow$$

$$\boxed{AF_5 = 0.737 + \cos 2u + 0.263\cos 4u}$$

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(4) $I(z') = \frac{1}{2\pi}\int_{-\infty}^{\infty} SF(\eta)e^{-jz'\eta}\,d\eta$ Jo

$\eta = k\cos\theta - k_z \xrightarrow{k_z=0} \eta = k\cos\theta \rightarrow \cos\theta = \frac{\eta}{k} \Longrightarrow -k < \eta < k$

$$\Rightarrow I(z') = \frac{1}{2\pi}\int_{-k}^{k} \sin\left(\frac{\pi\eta}{k}\right)e^{-jz'\eta}\,d\eta = \frac{1}{2\pi}\int_{-k}^{k}\frac{e^{j\frac{\pi\eta}{k}} - e^{-j\frac{\pi\eta}{k}}}{2j}e^{-jz'\eta}\,d\eta$$

$$= \frac{1}{4\pi j}\left[\frac{1}{j\left(\frac{\pi}{k} - z'\right)} e^{j\left(\frac{\pi}{k}-z'\right)}\Big|_{-k}^{k} + \frac{1}{j\left(\frac{\pi}{k}+z'\right)} e^{-j\left(\frac{\pi}{k}+z'\right)}\Big|_{-k}^{k} \right]$$

$$= \frac{1}{4\pi j}\left[\frac{1}{j\left(\frac{\pi}{k}-z'\right)} 2\cos kz' + \frac{1}{j\left(\frac{\pi}{k}+z'\right)} 2\cos kz' \right] = -\frac{1}{4\pi} 2\cos kz' \frac{\frac{2\pi}{k}}{\left(\frac{\pi}{k}\right)^2 - z'^2}$$

$$\Rightarrow I(z') = \frac{\cos kz'}{z'^2 - \left(\frac{\pi}{k}\right)^2}$$

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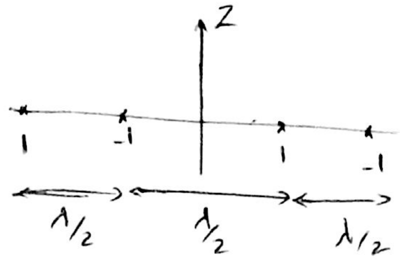
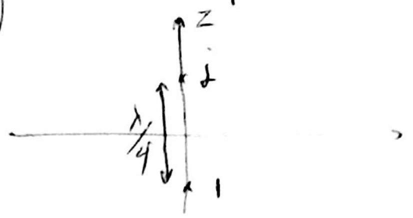
آرایه‌ی صفحه‌ای را به دو آرایه‌ی زیر تفکیک می‌کنیم.





$$AF = AF_x \, AF_y$$

$$AF_x = e^{-j\frac{3}{2}\psi} - e^{-j\frac{\psi}{2}} + e^{j\frac{\psi}{2}} - e^{j\frac{3}{2}\psi} = 2j\sin\frac{\psi}{2} - 2j\sin\frac{3}{2}\psi = 2j\left(\sin\frac{\psi}{2} - \sin\frac{3}{2}\psi\right)$$

$$\psi = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2}\sin\theta\cos\varphi = \pi\sin\theta\cos\varphi$$

$$AF_y = e^{-j\frac{\psi_y}{2}} + e^{j\frac{\pi}{2}} e^{j\frac{\psi_y}{2}} = e^{j\pi/4}\left[e^{-j\frac{\pi}{4}} e^{-j\frac{\psi_y}{2}} + e^{j\pi/4} e^{j\frac{\psi_y}{2}}\right]$$

$$= e^{j\pi/4} \times 2\cos\left(\frac{\pi}{4} + \frac{\psi_y}{2}\right)$$

$$\psi_y = \frac{2\pi}{\lambda} \times \frac{\lambda}{4}\cos\theta = \frac{\pi}{2}\cos\theta$$

$\varphi = 0 \implies \psi = \pi\sin\theta$

(صفحه xy)

$\varphi = \pi/2 \implies \psi = 0 \implies AF_x = 0 \implies AF = 0$

(صفحه xz)

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