

# Paper Title\*

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**Abstract—**  
**Index Terms—**

## I. INTRODUCTION

## II. SYSTEM MODEL

### A. Graph Model

$C$  is set of cloud nodes.  $F$  is set of fog nodes.  $E$  is set of edge nodes.  $S$  is set of sensor nodes.  $R$  is set of resources in each computational node (cloud, fog or edge node).

$$C = \{v_1^c, v_2^c, \dots, v_{|C|}^c\}, c \in C \quad (1a)$$

$$F = \{v_1^f, v_2^f, \dots, v_{|F|}^f\}, f \in F \quad (1b)$$

$$E = \{v_1^e, v_2^e, \dots, v_{|E|}^e\}, e \in E \quad (1c)$$

$$S = \{v_1^s, v_2^s, \dots, v_{|S|}^s\}, s \in S \quad (1d)$$

$$R = \{CPU, RAM, Storage\}, r \in R \quad (1e)$$

$\sigma_c^r$  is total capacity of resource  $r \in R$  on node  $c \in C$ . and also  $\sigma_f^r$  and  $\sigma_e^r$  are total capacity of resource  $r \in R$  on nodes  $f \in F$  and  $e \in E$  respectively.

$T$  is set of tasks.

$$T = \{t_1, t_2, \dots, t_{|T|}\} \quad (2a)$$

Each task expresses as follows:

$$t \in T \Rightarrow t = (w_t, \delta_t, N_t, f_t^r(\lambda_t)) \quad (3)$$

$w_t$  shows computation workload of the task.  $\delta_t$  is completion deadline of the task and  $N_t$  determines the maximum number of instaces of task  $t \in T$ .

$\pi_c$  is unit price of processing in node  $c \in C$  and also  $\pi_f$  and  $\pi_e$  are the related prices in nodes  $f \in F$  and  $e \in E$  respectively.

Transmission delays that show required time for trasmiting packets from sensors to each computational node are defined as follows:

$\tau_{s,c}^{tr}$  = trasmission delay between node  $s \in S$  and  $c \in C$

$\tau_{s,f}^{tr}$  = trasmission delay between node  $s \in S$  and  $f \in F$

$\tau_{s,e}^{tr}$  = trasmission delay between node  $s \in S$  and  $e \in E$

### B. Variables

We define three integer variables for allocating tasks between nodes.

$$x_{t,c} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } c \in C \\ 0 & \text{o.w.} \end{cases} \quad (4a)$$

$$x_{t,f} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } f \in F \\ 0 & \text{o.w.} \end{cases} \quad (4b)$$

$$x_{t,e} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } e \in E \\ 0 & \text{o.w.} \end{cases} \quad (4c)$$

there are two continouse variables:

$$\lambda_{t,s} = \text{poisson rate of task } t \in T \text{ generated by node } s \in S \quad (5a)$$

$$0 \leq \beta_{t,s,c} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall c \in C \quad (5b)$$

$$0 \leq \beta_{t,s,f} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall f \in F \quad (5c)$$

$$0 \leq \beta_{t,s,e} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall e \in E \quad (5d)$$

$$\beta_{t,s,c} = \text{size of flow of task } t \in T \text{ from node } s \in S \text{ to node } c \in C \quad (5e)$$

$$\gamma_{t,s,c} = \begin{cases} 1 & \text{flow of task } t \in T \text{ from node } s \in S \text{ to node } c \in C \text{ exist} \\ 0 & \text{o.w.} \end{cases} \quad (5f)$$

$$\gamma_{t,s,f} = \begin{cases} 1 & \text{flow of task } t \in T \text{ from node } s \in S \text{ to node } f \in F \text{ exist} \\ 0 & \text{o.w.} \end{cases} \quad (5g)$$

$$\gamma_{t,s,e} = \begin{cases} 1 & \text{flow of task } t \in T \text{ from node } s \in S \text{ to node } e \in E \text{ exist} \\ 0 & \text{o.w.} \end{cases} \quad (5h)$$

### C. Constraints

$$\lambda_{t,c} = \sum_{s \in S} \beta_{t,s,c} \quad \forall t \in T, \forall s \in S, \forall c \in C \quad (6a)$$

$$\lambda_{t,f} = \sum_{s \in S} \beta_{t,s,f} \quad \forall t \in T, \forall s \in S, \forall f \in F \quad (6b)$$

$$\lambda_{t,e} = \sum_{s \in S} \beta_{t,s,e} \quad \forall t \in T, \forall s \in S, \forall e \in E \quad (6c)$$

$$\gamma_{t,s,c} \leq x_{t,c} \quad \forall t \in T, \forall s \in S, \forall c \in C \quad (7a)$$

$$\gamma_{t,s,f} \leq x_{t,f} \quad \forall t \in T, \forall s \in S, \forall f \in F \quad (7b)$$

$$\gamma_{t,s,e} \leq x_{t,e} \quad \forall t \in T, \forall s \in S, \forall e \in E \quad (7c)$$

$$\gamma_{t,s,c} - 1 + \epsilon \leq \beta_{t,s,c} \leq \gamma_{t,s,c} \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall c \in C \quad (8a)$$

$$\gamma_{t,s,f} - 1 + \epsilon \leq \beta_{t,s,f} \leq \gamma_{t,s,f} \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall f \in F \quad (8b)$$

$$\gamma_{t,s,e} - 1 + \epsilon \leq \beta_{t,s,e} \leq \gamma_{t,s,e} \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall e \in E \quad (8c)$$

$$\lambda_{t,s} = \sum_{e \in E} \beta_{t,s,e} + \sum_{f \in F} \beta_{t,s,f} + \sum_{c \in C} \beta_{t,s,c} \quad \forall t \in T, \forall s \in S \quad (9a)$$

$$\sum_{t \in T} x_{t,c} f_t^r(\lambda_{t,c}) \leq \sigma_c^r \quad \forall r \in R, \forall c \in C \quad (10a)$$

$$x_{t,c} f_t^r(\lambda_{t,c}) = k_1^r x_{t,c} \lambda_{t,c} + k_2^r x_{t,c} \quad (10b)$$

$$\psi_{t,c} \triangleq x_{t,c} \lambda_{t,c} \Rightarrow 0 \leq \psi_{t,c} \leq \lambda_{t,c} \quad (10c)$$

$$Q(x_{t,c} - 1) + \lambda_{t,c} \leq \psi_{t,c} \leq x_{t,c} Q \quad (10d)$$

$$\begin{aligned} Q &= \max_{t \in T, c \in C} \lambda_{t,c} \\ &= \max_{t \in T, c \in C} \sum_{s \in S} \beta_{t,s,c} \\ &= \sum_{s \in S} \max_{t \in T, c \in C} \beta_{t,s,c} \\ &= \sum_{s \in S} \lambda_{t,s} \end{aligned} \quad (10e)$$

$$0 \leq \psi_{t,c} \leq \lambda_{t,c} \quad (11a)$$

$$Q(x_{t,c} - 1) + \lambda_{t,c} \leq \psi_{t,c} \leq x_{t,c} Q \quad \forall t \in T, \forall c \in C \quad (11b)$$

$$0 \leq \psi_{t,f} \leq \lambda_{t,f} \quad (11c)$$

$$Q(x_{t,f} - 1) + \lambda_{t,f} \leq \psi_{t,f} \leq x_{t,f} Q \quad \forall t \in T, \forall f \in F \quad (11d)$$

$$0 \leq \psi_{t,e} \leq \lambda_{t,e} \quad (11e)$$

$$Q(x_{t,e} - 1) + \lambda_{t,e} \leq \psi_{t,e} \leq x_{t,e} Q \quad \forall t \in T, \forall e \in E \quad (11f)$$

$$\sum_{t \in T} k_1^r \psi_{t,c} + k_2^r x_{t,c} \leq \sigma_c^r \quad \forall r \in R, \forall c \in C \quad (12a)$$

$$\sum_{t \in T} k_1^r \psi_{t,f} + k_2^r x_{t,f} \leq \sigma_f^r \quad \forall r \in R, \forall f \in F \quad (12b)$$

$$\sum_{t \in T} k_1^r \psi_{t,e} + k_2^r x_{t,e} \leq \sigma_e^r \quad \forall r \in R, \forall e \in E \quad (12c)$$

$$\tau_{t,c} = \tau_{t,s,c}^{tr} + \frac{1}{\mu_{t,c} - \lambda_{t,c}} \quad (13a)$$

We have:

$$\frac{1}{\mu_{t,c}} = \frac{w_t}{f_t^{cpu}(\lambda_{t,c})} \quad (13b)$$

$$f_t^{cpu}(\lambda_{t,c}) = k_1^{cpu} \lambda_{t,c} + k_2^{cpu} \quad (13c)$$

$$\begin{aligned} &\Rightarrow \gamma_{t,s,c} \tau_{t,c} = \gamma_{t,s,c} (\tau_{t,s,c}^{tr} + \frac{w_t}{(k_1^{cpu} - w_t) \lambda_{t,c} + k_2^{cpu}}) \\ &\leq \delta_t \quad \forall t \in T, \forall s \in S, \forall c \in C \end{aligned} \quad (13d)$$

$$\begin{aligned} &\gamma_{t,s,c} \lambda_{t,c} (k_1^{cpu} - w_t) \tau_{t,s,c}^{tr} + \\ &\gamma_{t,s,c} k_2^{cpu} \tau_{t,s,c}^{tr} + w_t \gamma_{t,s,c} - k_2^{cpu} \delta_t \\ &- (k_1^{cpu} - w_t) \delta_t \lambda_{t,c} \leq 0 \quad \forall t \in T, \forall s \in S, \forall c \in C \end{aligned} \quad (13e)$$

$$\phi_{t,s,c} = \gamma_{t,s,c} \lambda_{t,c} \quad (13f)$$

$$0 \leq \phi_{t,s,c} \leq \lambda_{t,c} \quad (13g)$$

$$Q(\gamma_{t,s,c} - 1) + \lambda_{t,c} \leq \phi_{t,s,c} \leq \gamma_{t,s,c} Q \quad (13h)$$

$$\begin{aligned} &\phi_{t,s,c} (k_1^{cpu} - w_t) \tau_{t,s,c}^{tr} + \\ &\gamma_{t,s,c} k_2^{cpu} \tau_{t,s,c}^{tr} + w_t \gamma_{t,s,c} - k_2^{cpu} \delta_t \\ &- (k_1^{cpu} - w_t) \delta_t \lambda_{t,c} \leq 0 \end{aligned} \quad (14a)$$

$$0 \leq \phi_{t,s,c} \leq \lambda_{t,c} \quad (14b)$$

$$Q(\gamma_{t,s,c} - 1) + \lambda_{t,c} \leq \phi_{t,s,c} \leq \gamma_{t,s,c} Q \quad \forall t \in T, \forall s \in S, \forall c \in C \quad (14c)$$

$$\begin{aligned} &\phi_{t,s,f} (k_1^{cpu} - w_t) \tau_{t,s,f}^{tr} + \\ &\gamma_{t,s,f} k_2^{cpu} \tau_{t,s,f}^{tr} + w_t \gamma_{t,s,f} - k_2^{cpu} \delta_t \\ &- (k_1^{cpu} - w_t) \delta_t \lambda_{t,f} \leq 0 \end{aligned} \quad (14d)$$

$$0 \leq \phi_{t,s,f} \leq \lambda_{t,c} \quad (14e)$$

$$Q(\gamma_{t,s,f} - 1) + \lambda_{t,f} \leq \phi_{t,s,f} \leq \gamma_{t,s,f} Q \quad \forall t \in T, \forall s \in S, \forall f \in F \quad (14f)$$

$$\begin{aligned} &\phi_{t,s,e} (k_1^{cpu} - w_t) \tau_{t,s,e}^{tr} + \\ &\gamma_{t,s,e} k_2^{cpu} \tau_{t,s,e}^{tr} + w_t \gamma_{t,s,e} - k_2^{cpu} \delta_t \\ &- (k_1^{cpu} - w_t) \delta_t \lambda_{t,e} \leq 0 \end{aligned} \quad (14g)$$

$$0 \leq \phi_{t,s,e} \leq \lambda_{t,e} \quad (14h)$$

$$Q(\gamma_{t,s,e} - 1) + \lambda_{t,e} \leq \phi_{t,s,e} \leq \gamma_{t,s,e} Q \quad \forall t \in T, \forall s \in S, \forall e \in E \quad (14i)$$

$$1 \leq \sum_{e \in E} x_{t,e} + \sum_{f \in F} x_{t,f} + \sum_{c \in C} x_{t,c} \leq N_t \quad \forall t \in T \quad (15a)$$

$$x_{t,c}(\lambda_{t,c} < \mu_{t,c}) \Rightarrow x_{t,c}(\lambda_{t,c} + \epsilon \leq \mu_{t,c}) \quad (16a)$$

$$x_{t,c}\lambda_{t,c} = \lambda_{t,c} \quad (16b)$$

$$\Rightarrow \epsilon x_{t,c} - k_1^{cpu}\lambda_{t,c} - k_2^{cpu} + w_t\lambda_{t,c} \leq 0 \quad \forall t \in T, \forall c \in C \quad (16c)$$

$$\epsilon x_{t,f} - k_1^{cpu}\lambda_{t,f} - k_2^{cpu} + w_t\lambda_{t,f} \leq 0 \quad \forall t \in T, \forall f \in F \quad (16d)$$

$$\epsilon x_{t,e} - k_1^{cpu}\lambda_{t,e} - k_2^{cpu} + w_t\lambda_{t,e} \leq 0 \quad \forall t \in T, \forall e \in E \quad (16e)$$

$$\begin{aligned} & \min \sum_{t \in T} \sum_{e \in E} (x_{t,e} \pi_e \sum_{r \in R} f_t^r(\lambda_{t,e})) \\ & + \sum_{t \in T} \sum_{f \in F} (x_{t,f} \pi_f \sum_{r \in R} f_t^r(\lambda_{t,f})) \\ & + \sum_{t \in T} \sum_{c \in C} (x_{t,c} \pi_c \sum_{r \in R} f_t^r(\lambda_{t,c})) \end{aligned} \quad (17a)$$

$$\begin{aligned} & \min \sum_{t \in T} \sum_{e \in E} x_{t,e} \Gamma_{t,e} \\ & + \sum_{t \in T} \sum_{f \in F} x_{t,f} \Gamma_{t,f} \\ & + \sum_{t \in T} \sum_{c \in C} x_{t,c} \Gamma_{t,c} \end{aligned} \quad (17b)$$

$$\begin{aligned} \Gamma_{t,e} &= \pi_e((k_1^{cpu} + k_1^{ram} + k_1^{storage})\lambda_{t,e} \\ & + k_2^{cpu} + k_2^{ram} + k_2^{storage}) \\ &= \pi_e(K_1\lambda_{t,e} + K_2) \end{aligned} \quad (17c)$$

$$\begin{aligned} x_{t,e} \Gamma_{t,e} &= K_1 \pi_e x_{t,e} \lambda_{t,e} + K_2 \pi_e x_{t,e} \\ &= K_1 \pi_e \psi_{t,e} + K_2 \pi_e x_{t,e} \end{aligned} \quad (17d)$$

$$\begin{aligned} & \min \sum_{t \in T} \sum_{e \in E} K_1 \pi_e \psi_{t,e} + K_2 \pi_e x_{t,e} \\ & \sum_{t \in T} \sum_{f \in F} K_1 \pi_f \psi_{t,f} + K_2 \pi_f x_{t,f} \\ & \sum_{t \in T} \sum_{c \in C} K_1 \pi_c \psi_{t,c} + K_2 \pi_c x_{t,c} \end{aligned} \quad (18a)$$

$$\begin{aligned} L(\underline{x}, \underline{\beta}, \underline{\gamma}, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) &= \sum_{t \in T} \sum_{e \in E} x_{t,e} \Gamma_{t,e} \\ & + \sum_{t \in T} \sum_{f \in F} x_{t,f} \Gamma_{t,f} + \sum_{t \in T} \sum_{c \in C} x_{t,c} \Gamma_{t,c} \\ & + \sum_{t \in T} \eta_{1,t} (1 - \sum_{e \in E} x_{t,e} + \sum_{f \in F} x_{t,f} + \sum_{c \in C} x_{t,c}) \\ & + \sum_{t \in T} \eta_{2,t} (\sum_{e \in E} x_{t,e} + \sum_{f \in F} x_{t,f} + \sum_{c \in C} x_{t,c} - N_t) \\ & + \sum_{t \in T} \sum_{s \in S} \nu_{t,s} (\lambda_{t,s} - \sum_{e \in E} \beta_{t,s,e} + \sum_{f \in F} \beta_{t,s,f} + \sum_{c \in C} \beta_{t,s,c}) \end{aligned} \quad (19a)$$

$$\begin{aligned} L &= \sum_{e \in E} \sum_{t \in T} (\sum_{s \in S} (\nu_{t,s} \beta_{t,s,e} + \frac{\nu_{t,s} \lambda_{t,s}}{3|E|}) \\ & + x_{t,e} (\Gamma_{t,e} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t \eta_{2,t}}{3|E|}) \\ & + \sum_{f \in F} \sum_{t \in T} (\sum_{s \in S} (\nu_{t,s} \beta_{t,s,f} + \frac{\nu_{t,s} \lambda_{t,s}}{3|F|}) \\ & + x_{t,f} (\Gamma_{t,f} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t \eta_{2,t}}{3|F|}) \\ & + \sum_{c \in C} \sum_{t \in T} (\sum_{s \in S} (\nu_{t,s} \beta_{t,s,c} + \frac{\nu_{t,s} \lambda_{t,s}}{3|C|}) \\ & + x_{t,c} (\Gamma_{t,c} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t \eta_{2,t}}{3|C|}) \end{aligned} \quad (20a)$$

$$L = \sum_{e \in E} L_e + \sum_{f \in F} L_f + \sum_{c \in C} L_c \quad (20b)$$

$$\begin{aligned} g(\underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) &= \inf_{\underline{x}, \underline{\beta}, \underline{\gamma}} L(\underline{x}, \underline{\beta}, \underline{\gamma}, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\ &= \sum_{e \in E} \inf_{\underline{x}_e, \underline{\beta}_e, \underline{\gamma}_e} L_e(\underline{x}_e, \underline{\beta}_e, \underline{\gamma}_e, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\ &+ \sum_{f \in F} \inf_{\underline{x}_f, \underline{\beta}_f, \underline{\gamma}_f} L_f(\underline{x}_f, \underline{\beta}_f, \underline{\gamma}_f, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\ &+ \sum_{c \in C} \inf_{\underline{x}_c, \underline{\beta}_c, \underline{\gamma}_c} L_c(\underline{x}_c, \underline{\beta}_c, \underline{\gamma}_c, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\ &= \sum_{e \in E} g_e(\underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\ &+ \sum_{f \in F} g_f(\underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\ &+ \sum_{c \in C} g_c(\underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \end{aligned} \quad (21a)$$

$$\underline{x}_e^{(k+1)}, \underline{\beta}_e^{(k+1)}, \underline{\gamma}_e^{(k+1)} = \arg \min_{\underline{x}_e, \underline{\beta}_e, \underline{\gamma}_e} L_e(\underline{x}_e, \underline{\beta}_e, \underline{\gamma}_e, \underline{\eta}_1^{(k)}, \underline{\eta}_2^{(k)}, \underline{\nu}^{(k)}) \quad (22a)$$

$$\underline{x}_f^{(k+1)}, \underline{\beta}_f^{(k+1)}, \underline{\gamma}_f^{(k+1)} = \arg \min_{\underline{x}_f, \underline{\beta}_f, \underline{\gamma}_f} L_f(\underline{x}_f, \underline{\beta}_f, \underline{\gamma}_f, \underline{\eta}_1^{(k)}, \underline{\eta}_2^{(k)}, \underline{\nu}^{(k)}) \quad (22b)$$

$$\underline{x}_c^{(k+1)}, \underline{\beta}_c^{(k+1)}, \underline{\gamma}_c^{(k+1)} = \arg \min_{\underline{x}_c, \underline{\beta}_c, \underline{\gamma}_c} L_c(\underline{x}_c, \underline{\beta}_c, \underline{\gamma}_c, \underline{\eta}_1^{(k)}, \underline{\eta}_2^{(k)}, \underline{\nu}^{(k)}) \quad (22c)$$

$$\eta_{1,t}^{(k+1)} = \eta_{1,t}^{(k)} + \alpha^{(k)} (1 - \sum_{e \in E} x_{t,e}^{(k+1)} + \sum_{f \in F} x_{t,f}^{(k+1)} + \sum_{c \in C} x_{t,c}^{(k+1)}) \quad (23a)$$

$$\eta_{2,t}^{(k+1)} = \eta_{2,t}^{(k)} + \alpha^{(k)} (\sum_{e \in E} x_{t,e}^{(k+1)} + \sum_{f \in F} x_{t,f}^{(k+1)} + \sum_{c \in C} x_{t,c}^{(k+1)} - N_t) \quad (23b)$$

$$\nu_{t,s}^{(k+1)} = \nu_{t,s}^{(k)} + \alpha^{(k)} (\lambda_{t,s}^{(k+1)} - \sum_{e \in E} \beta_{t,s,e}^{(k+1)} + \sum_{f \in F} \beta_{t,s,f}^{(k+1)} + \sum_{c \in C} \beta_{t,s,c}^{(k+1)}) \quad (23c)$$

#### D. Objective

$$p_k = \sum_{i=1}^{l_e} x_{k,i}^e C(v_i^e, t_k) + \sum_{j=1}^{l_f} x_{k,j}^f C(v_j^f, t_k) + \sum_{h=1}^{l_c} x_{k,h}^c C(v_h^c, t_k) \quad (24a)$$

$$\min \sum_{k=1}^{l_t} p_k \quad \text{subject to: } 9 \quad (25a)$$

#### E. Solution

We can reshape main problem as following:

$$\min \left( \sum_{i=1}^{l_e} \sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e + \sum_{j=1}^{l_f} \sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c} \sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c \right)$$

subject to:

$$\begin{aligned} \sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c &\leq \delta_k \quad \forall k \in \{1, \dots, l_t\} \\ \sum_{k=1}^{l_t} x_{k,h}^c w_k &\leq c_h^c \quad \forall h \in \{1, 2, \dots, l_c\} \\ \sum_{k=1}^{l_t} x_{k,j}^f w_k &\leq c_j^f \quad \forall j \in \{1, 2, \dots, l_f\} \\ \sum_{k=1}^{l_t} x_{k,i}^e w_k &\leq c_i^e \quad \forall i \in \{1, 2, \dots, l_e\} \\ \sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c &= 1 \quad \forall k \in \{1, 2, \dots, l_t\} \end{aligned}$$

We define  $u^m$  for each computational agent  $m$ , that is a matrix with size  $l_t * (l_e + l_f + l_c)$ . It is the local copy of all variables in agent  $m$ . i.e.  $u_{k,i}^{e,m}$  is the copy of variable  $x_{k,i}^e$  in agent  $m$  for  $m = 1, \dots, (l_m = l_e + l_f + l_c)$ . So we should add new constraint  $u^m = z \quad \forall m$  to main problem. We will use admm on this new constraint so:

$$L_p = \sum_{i=1}^{l_e} \sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e + \sum_{j=1}^{l_f} \sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c} \sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c + \sum_{m=1}^{l_m} \nu^m * (u^m - z) + \sum_{m=1}^{l_m} \frac{\rho}{2} \|u^m - z\|^2 \quad (27a)$$

We can separate augmented lagrangian for each computational agent  $m$  then:

$$L_p^m = \sum_{k=1}^{l_t} u_{k,m}^m C_{k,m} + \nu^m * (u^m - z) + \frac{\rho}{2} \|u^m - z\|^2 \quad (28a)$$

$$\forall m \in \{1, 2, \dots, l_m\}$$

So we can write the algorithm as following:

for each iteration  $k$  (29a)

$$1. \quad u^{m,(k+1)} = \arg \min L_p^m(u^m, z^{(k)}, \nu^{m,(k)}) =$$

$$\sum_{k=1}^{l_t} u_{k,m}^m C_{k,m} + \nu^{m,(k)} * (u^m - z^{(k)}) + \frac{\rho}{2} \|u^m - z^{(k)}\|^2$$

subject to:

$$\begin{aligned} \sum_{k=1}^{l_t} u_{k,m}^m w_k &\leq c^m \\ \sum_{i=1}^{l_e} u_{k,i}^{e,m} + \sum_{j=1}^{l_f} u_{k,j}^{f,m} + \sum_{h=1}^{l_c} u_{k,h}^{c,m} &= 1 \quad \forall k \in \{1, 2, \dots, l_t\} \\ \sum_{i=1}^{l_e} u_{k,i}^{e,m} \tau_{k,i}^e + \sum_{j=1}^{l_f} u_{k,j}^{f,m} \tau_{k,j}^f + \sum_{h=1}^{l_c} u_{k,h}^{c,m} \tau_{k,h}^c &\leq \delta_k \quad \forall k \in \{1, 2, \dots, l_t\} \\ 2. \quad z^{(k+1)} &= \bar{u}^{(k+1)} + \frac{1}{\rho} \bar{v}^{(k)} \\ 3. \quad \nu^{m,(k+1)} &= \nu^{m,(k)} + \rho(u^{m,(k+1)} - z^{(k+1)}) \end{aligned}$$

#### F. Solution 2

lagrangian of main problem is as following

$$L(x^e, x^f, x^c, \lambda, \nu) = \sum_{i=1}^{l_e} \sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e + \sum_{j=1}^{l_f} \sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c} \sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c \quad (30a)$$

$$\begin{aligned} &+ \sum_{k=1}^{l_t} \lambda_k \left( \sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c - \delta_k \right) \\ &+ \sum_{k=1}^{l_t} \nu_k \left( \sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c - 1 \right) \end{aligned}$$

So we can decompose the lagrangian as follows

$$L(x^e, x^f, x^c, \lambda, \nu) = \sum_{i=1}^{l_e} \sum_{k=1}^{l_t} \left( x_{k,i}^e C_{k,i}^e + \lambda_k x_{k,i}^e \tau_{k,i}^e + \nu_k x_{k,i}^e - \frac{\lambda_k \delta_k + \nu_k}{3l_e} \right) + \sum_{j=1}^{l_f} \sum_{k=1}^{l_t} \left( x_{k,j}^f C_{k,j}^f + \lambda_k x_{k,j}^f \tau_{k,j}^f + \nu_k x_{k,j}^f - \frac{\lambda_k \delta_k + \nu_k}{3l_f} \right) + \sum_{h=1}^{l_c} \sum_{k=1}^{l_t} \left( x_{k,h}^c C_{k,h}^c + \lambda_k x_{k,h}^c \tau_{k,h}^c + \nu_k x_{k,h}^c - \frac{\lambda_k \delta_k + \nu_k}{3l_c} \right) \quad (31a)$$

$$\begin{aligned}
L(x^e, x^f, x^c, \lambda, \nu) &= \sum_{i=1}^{l_e} L_i^e(x_i^e, \lambda, \nu) \\
&+ \sum_{j=1}^{l_f} L_j^f(x_j^f, \lambda, \nu) \\
&+ \sum_{h=1}^{l_c} L_h^c(x_h^c, \lambda, \nu)
\end{aligned} \tag{32a}$$

$$\begin{aligned}
g(\lambda, \nu) &= \inf_{x^e, x^f, x^c} L(x^e, x^f, x^c, \lambda, \nu) \\
&= \sum_{i=1}^{l_e} \inf_{x_i^e} L_i^e(x_i^e, \lambda, \nu) \\
&+ \sum_{j=1}^{l_f} \inf_{x_j^f} L_j^f(x_j^f, \lambda, \nu) \\
&+ \sum_{h=1}^{l_c} \inf_{x_h^c} L_h^c(x_h^c, \lambda, \nu) \\
&= \sum_{i=1}^{l_e} g_i^e(\lambda, \nu) \\
&+ \sum_{j=1}^{l_f} g_j^f(\lambda, \nu) \\
&+ \sum_{h=1}^{l_c} g_h^c(\lambda, \nu)
\end{aligned} \tag{33a}$$

$$\lambda_k^+ = \lambda_k^- + \alpha \left( \sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c - \delta_k \right) \tag{34a}$$

$$\nu_k^+ = \nu_k^- + \alpha \left( \sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c - 1 \right) \tag{34b}$$

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**Algorithm 1**


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**while** not converged **do**

1. update  $x^c, x^f, x^e$  using (19a)

2. update lagrangian multipliers using (20a) and (21a)

$\alpha = \frac{\alpha_0}{k}$  for each iteration  $k$

**end while**

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REFERENCES