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Abstract—
Index Terms—

I. INTRODUCTION

II. SYSTEM MODEL

A. Graph Model

C is set of cloud nodes. F is set of fog nodes. E is set of edge nodes. S is set of sensor nodes. R is set of resources in each computational node (cloud, fog or edge node).

$$C = \{v_1^c, v_2^c, \dots, v_{|C|}^c\}, c \in C \quad (1a)$$

$$F = \{v_1^f, v_2^f, \dots, v_{|F|}^f\}, f \in F \quad (1b)$$

$$E = \{v_1^e, v_2^e, \dots, v_{|E|}^e\}, e \in E \quad (1c)$$

$$S = \{v_1^s, v_2^s, \dots, v_{|S|}^s\}, s \in S \quad (1d)$$

$$R = \{CPU, RAM, Storage\}, r \in R \quad (1e)$$

σ_c^r is total capacity of resource $r \in R$ on node $c \in C$. and also σ_f^r and σ_e^r are total capacity of resource $r \in R$ on nodes $f \in F$ and $e \in E$ respectively.

T is set of tasks.

$$T = \{t_1, t_2, \dots, t_{|T|}\} \quad (2a)$$

Each task expresses as follows:

$$t \in T \Rightarrow t = (w_t, \delta_t, N_t, f_t^r(\lambda_t)) \quad (3)$$

w_t shows computation workload of the task. δ_t is completion deadline of the task and N_t determines the maximum number of instances of task $t \in T$.

π_c is unit price of processing in node $c \in C$ and also π_f and π_e are the related prices in nodes $f \in F$ and $e \in E$ respectively.

Transmission delays that show required time for transmitting packets from sensors to each computational node are defined as follows:

$\tau_{s,c}^{tr}$ = transmission delay between node $s \in S$ and $c \in C$

$\tau_{s,f}^{tr}$ = transmission delay between node $s \in S$ and $f \in F$

$\tau_{s,e}^{tr}$ = transmission delay between node $s \in S$ and $e \in E$

B. Variables

We define three integer variables for allocating tasks between nodes.

$$x_{t,c} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } c \in C \\ 0 & \text{o.w.} \end{cases} \quad (4a)$$

$$x_{t,f} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } f \in F \\ 0 & \text{o.w.} \end{cases} \quad (4b)$$

$$x_{t,e} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } e \in E \\ 0 & \text{o.w.} \end{cases} \quad (4c)$$

there are two continuous variables:

$$\lambda_{t,s} = \text{poisson rate of task } t \in T \text{ generated by node } s \in S \quad (5a)$$

$$0 \leq \beta_{t,s,c} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall c \in C \quad (5b)$$

$$0 \leq \beta_{t,s,f} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall f \in F \quad (5c)$$

$$0 \leq \beta_{t,s,e} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall e \in E \quad (5d)$$

$$\beta_{t,s,c} = \text{size of flow of task } t \in T \text{ from node } s \in S \text{ to node } c \in C \quad (5e)$$

C. Constraints

$$\lambda_{t,c} = \sum_{s \in S} \beta_{t,s,c} \quad \forall t \in T, \forall c \in C \quad (6a)$$

$$\lambda_{t,f} = \sum_{s \in S} \beta_{t,s,f} \quad \forall t \in T, \forall f \in F \quad (6b)$$

$$\lambda_{t,e} = \sum_{s \in S} \beta_{t,s,e} \quad \forall t \in T, \forall e \in E \quad (6c)$$

$$\frac{\lambda_{t,c}}{\sum_{s \in S} \lambda_{t,s}} \leq x_{t,c} \quad \forall t \in T, \forall c \in C \quad (7a)$$

$$\frac{\lambda_{t,f}}{\sum_{s \in S} \lambda_{t,s}} \leq x_{t,f} \quad \forall t \in T, \forall f \in F \quad (7b)$$

$$\frac{\lambda_{t,e}}{\sum_{s \in S} \lambda_{t,s}} \leq x_{t,e} \quad \forall t \in T, \forall e \in E \quad (7c)$$

$$\lambda_{t,s} = \sum_{e \in E} \beta_{t,s,e} + \sum_{f \in F} \beta_{t,s,f} + \sum_{c \in C} \beta_{t,s,c} \quad \forall t \in T, \forall s \in S \quad (8a)$$

$$\lambda_{t,s} \leq \sum_{e \in E} \beta_{t,s,e} + \sum_{f \in F} \beta_{t,s,f} + \sum_{c \in C} \beta_{t,s,c} \leq \lambda_{t,s} + \epsilon \quad \forall t \in T, \forall s \in S \quad (8b)$$

$$\sum_{t \in T} x_{t,c} f_t^r(\lambda_{t,c}) \leq \sigma_c^r \quad \forall r \in R, \forall c \in C \quad (9a)$$

$$x_{t,c} f_t^r(\lambda_{t,c}) = k_1^r x_{t,c} \lambda_{t,c} + k_2^r x_{t,c} \quad (9b)$$

$$\psi_{t,c} \triangleq x_{t,c} \lambda_{t,c} \Rightarrow 0 \leq \psi_{t,c} \leq \lambda_{t,c} \quad (9c)$$

$$Q(x_{t,c} - 1) + \lambda_{t,c} \leq \psi_{t,c} \leq x_{t,c} Q \quad (9d)$$

$$Q = \max_{t \in T, c \in C} \lambda_{t,c}$$

$$= \max_{t \in T, c \in C} \sum_{s \in S} \beta_{t,s,c}$$

$$= \sum_{s \in S} \max_{t \in T, c \in C} \beta_{t,s,c}$$

$$= \sum_{s \in S} \lambda_{t,s} \quad (9e)$$

$$0 \leq \psi_{t,c} \leq \lambda_{t,c} \quad (10a)$$

$$Q(x_{t,c} - 1) + \lambda_{t,c} \leq \psi_{t,c} \leq x_{t,c} Q \quad \forall t \in T, \forall c \in C \quad (10b)$$

$$0 \leq \psi_{t,f} \leq \lambda_{t,f} \quad (10c)$$

$$Q(x_{t,f} - 1) + \lambda_{t,f} \leq \psi_{t,f} \leq x_{t,f} Q \quad \forall t \in T, \forall f \in F \quad (10d)$$

$$0 \leq \psi_{t,e} \leq \lambda_{t,e} \quad (10e)$$

$$Q(x_{t,e} - 1) + \lambda_{t,e} \leq \psi_{t,e} \leq x_{t,e} Q \quad \forall t \in T, \forall e \in E \quad (10f)$$

$$\sum_{t \in T} k_1^r \psi_{t,c} + k_2^r x_{t,c} \leq \sigma_c^r \quad \forall r \in R, \forall c \in C \quad (11a)$$

$$\sum_{t \in T} k_1^r \psi_{t,f} + k_2^r x_{t,f} \leq \sigma_f^r \quad \forall r \in R, \forall f \in F \quad (11b)$$

$$\sum_{t \in T} k_1^r \psi_{t,e} + k_2^r x_{t,e} \leq \sigma_e^r \quad \forall r \in R, \forall e \in E \quad (11c)$$

$$\tau_{t,c} = \tau_{t,s,c}^{tr} + \frac{1}{\mu_{t,c} - \lambda_{t,c}} \quad (12a)$$

We have:

$$\frac{1}{\mu_{t,c}} = \frac{w_t}{f_t^{cpu}(\lambda_{t,c})} \quad (12b)$$

$$f_t^{cpu}(\lambda_{t,c}) = k_1^{cpu} \lambda_{t,c} + k_2^{cpu} \quad (12c)$$

$$\begin{aligned} &\Rightarrow x_{t,c} \tau_{t,c} = x_{t,c} (\tau_{t,s,c}^{tr} + \frac{w_t}{(k_1^{cpu} - w_t) \lambda_{t,c} + k_2^{cpu}}) \\ &\leq \delta_t \quad \forall t \in T, \forall s \in S, \forall c \in C \end{aligned} \quad (12d)$$

$$\begin{aligned} &x_{t,c} \lambda_{t,c} (k_1^{cpu} - w_t) \tau_{t,s,c}^{tr} + \\ &x_{t,c} k_2^{cpu} \tau_{t,s,c}^{tr} + w_t x_{t,c} - k_2^{cpu} \delta_t \\ &- (k_1^{cpu} - w_t) \delta_t \lambda_{t,c} \leq 0 \quad \forall t \in T, \forall s \in S, \forall c \in C \end{aligned} \quad (12e)$$

$$\begin{aligned} &\psi_{t,s,c} (k_1^{cpu} - w_t) \tau_{t,s,c}^{tr} + \\ &x_{t,c} k_2^{cpu} \tau_{t,s,c}^{tr} + w_t x_{t,c} - k_2^{cpu} \delta_t \\ &- (k_1^{cpu} - w_t) \delta_t \lambda_{t,c} \leq 0 \end{aligned} \quad (13a)$$

$$\begin{aligned} &\psi_{t,s,f} (k_1^{cpu} - w_t) \tau_{t,s,f}^{tr} + \\ &x_{t,f} k_2^{cpu} \tau_{t,s,f}^{tr} + w_t x_{t,f} - k_2^{cpu} \delta_t \\ &- (k_1^{cpu} - w_t) \delta_t \lambda_{t,f} \leq 0 \end{aligned} \quad (13b)$$

$$\begin{aligned} &\psi_{t,s,e} (k_1^{cpu} - w_t) \tau_{t,s,e}^{tr} + \\ &x_{t,e} k_2^{cpu} \tau_{t,s,e}^{tr} + w_t x_{t,e} - k_2^{cpu} \delta_t \\ &- (k_1^{cpu} - w_t) \delta_t \lambda_{t,e} \leq 0 \end{aligned} \quad (13c)$$

$$1 \leq \sum_{e \in E} x_{t,e} + \sum_{f \in F} x_{t,f} + \sum_{c \in C} x_{t,c} \leq N_t \quad \forall t \in T \quad (14a)$$

$$x_{t,c} (\lambda_{t,c} < \mu_{t,c}) \Rightarrow x_{t,c} (\lambda_{t,c} + \epsilon \leq \mu_{t,c}) \quad (15a)$$

$$x_{t,c} \lambda_{t,c} = \lambda_{t,c} \quad (15b)$$

$$\Rightarrow \epsilon x_{t,c} - k_1^{cpu} \lambda_{t,c} - k_2^{cpu} + w_t \lambda_{t,c} \leq 0 \quad \forall t \in T, \forall c \in C \quad (15c)$$

$$\epsilon x_{t,f} - k_1^{cpu} \lambda_{t,f} - k_2^{cpu} + w_t \lambda_{t,f} \leq 0 \quad \forall t \in T, \forall f \in F \quad (15d)$$

$$\epsilon x_{t,e} - k_1^{cpu} \lambda_{t,e} - k_2^{cpu} + w_t \lambda_{t,e} \leq 0 \quad \forall t \in T, \forall e \in E \quad (15e)$$

$$\begin{aligned} &\min \sum_{t \in T} \sum_{e \in E} (x_{t,e} \pi_e \sum_{r \in R} f_t^r(\lambda_{t,e})) \\ &+ \sum_{t \in T} \sum_{f \in F} (x_{t,f} \pi_f \sum_{r \in R} f_t^r(\lambda_{t,f})) \\ &+ \sum_{t \in T} \sum_{c \in C} (x_{t,c} \pi_c \sum_{r \in R} f_t^r(\lambda_{t,c})) \end{aligned} \quad (16a)$$

$$\begin{aligned} &\min \sum_{t \in T} \sum_{e \in E} x_{t,e} \Gamma_{t,e} \\ &+ \sum_{t \in T} \sum_{f \in F} x_{t,f} \Gamma_{t,f} \\ &+ \sum_{t \in T} \sum_{c \in C} x_{t,c} \Gamma_{t,c} \end{aligned} \quad (16b)$$

$$\begin{aligned} &\Gamma_{t,e} = \pi_e ((k_1^{cpu} + k_1^{ram} + k_1^{storage}) \lambda_{t,e} \\ &+ k_2^{cpu} + k_2^{ram} + k_2^{storage}) \\ &= \pi_e (K_1 \lambda_{t,e} + K_2) \end{aligned} \quad (16c)$$

$$\begin{aligned} &x_{t,e} \Gamma_{t,e} = K_1 \pi_e x_{t,e} \lambda_{t,e} + K_2 \pi_e x_{t,e} \\ &= K_1 \pi_e \psi_{t,e} + K_2 \pi_e x_{t,e} \end{aligned} \quad (16d)$$

$$\begin{aligned}
& \min \sum_{t \in T} \sum_{e \in E} K_1 \pi_e \psi_{t,e} + K_2 \pi_e x_{t,e} \\
& \sum_{t \in T} \sum_{f \in F} K_1 \pi_f \psi_{t,f} + K_2 \pi_f x_{t,f} \\
& \sum_{t \in T} \sum_{c \in C} K_1 \pi_c \psi_{t,c} + K_2 \pi_c x_{t,c}
\end{aligned} \tag{17a}$$

$$\begin{aligned}
L(\underline{x}, \underline{\beta}, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) &= \sum_{t \in T} \sum_{e \in E} x_{t,e} \Gamma_{t,e} \\
&+ \sum_{t \in T} \sum_{f \in F} x_{t,f} \Gamma_{t,f} + \sum_{t \in T} \sum_{c \in C} x_{t,c} \Gamma_{t,c} \\
&+ \sum_{t \in T} \eta_{1,t} (1 - \sum_{e \in E} x_{t,e} + \sum_{f \in F} x_{t,f} + \sum_{c \in C} x_{t,c}) \\
&+ \sum_{t \in T} \eta_{2,t} (\sum_{e \in E} x_{t,e} + \sum_{f \in F} x_{t,f} + \sum_{c \in C} x_{t,c} - N_t) \\
&+ \sum_{t \in T} \sum_{s \in S} \nu_{t,s} (\lambda_{t,s} - \sum_{e \in E} \beta_{t,s,e} + \sum_{f \in F} \beta_{t,s,f} + \sum_{c \in C} \beta_{t,s,c})
\end{aligned} \tag{18a}$$

$$\begin{aligned}
L &= \sum_{e \in E} \sum_{t \in T} (\sum_{s \in S} (\nu_{t,s} \beta_{t,s,e} + \frac{\nu_{t,s} \lambda_{t,s}}{3|E|}) \\
&+ x_{t,e} (\Gamma_{t,e} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t \eta_{2,t}}{3|E|}) \\
&+ \sum_{f \in F} \sum_{t \in T} (\sum_{s \in S} (\nu_{t,s} \beta_{t,s,f} + \frac{\nu_{t,s} \lambda_{t,s}}{3|F|}) \\
&+ x_{t,f} (\Gamma_{t,f} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t \eta_{2,t}}{3|F|}) \\
&+ \sum_{c \in C} \sum_{t \in T} (\sum_{s \in S} (\nu_{t,s} \beta_{t,s,c} + \frac{\nu_{t,s} \lambda_{t,s}}{3|C|}) \\
&+ x_{t,c} (\Gamma_{t,c} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t \eta_{2,t}}{3|C|})
\end{aligned} \tag{19a}$$

$$L = \sum_{e \in E} L_e + \sum_{f \in F} L_f + \sum_{c \in C} L_c \tag{19b}$$

$$\begin{aligned}
g(\underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) &= \inf_{\underline{x}, \underline{\beta}} L(\underline{x}, \underline{\beta}, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\
&= \sum_{e \in E} \inf_{\underline{x}_e, \underline{\beta}_e} L_e(\underline{x}_e, \underline{\beta}_e, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\
&+ \sum_{f \in F} \inf_{\underline{x}_f, \underline{\beta}_f} L_f(\underline{x}_f, \underline{\beta}_f, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\
&+ \sum_{c \in C} \inf_{\underline{x}_c, \underline{\beta}_c} L_c(\underline{x}_c, \underline{\beta}_c, \underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\
&= \sum_{e \in E} g_e(\underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\
&+ \sum_{f \in F} g_f(\underline{\eta}_1, \underline{\eta}_2, \underline{\nu}) \\
&+ \sum_{c \in C} g_c(\underline{\eta}_1, \underline{\eta}_2, \underline{\nu})
\end{aligned} \tag{20a}$$

$$\underline{x}_e^{(k+1)}, \underline{\beta}_e^{(k+1)} = \arg \min_{\underline{x}_e, \underline{\beta}_e} L_e(\underline{x}_e, \underline{\beta}_e, \underline{\eta}_1^{(k)}, \underline{\eta}_2^{(k)}, \underline{\nu}^{(k)}) \tag{21a}$$

$$\underline{x}_f^{(k+1)}, \underline{\beta}_f^{(k+1)} = \arg \min_{\underline{x}_f, \underline{\beta}_f} L_f(\underline{x}_f, \underline{\beta}_f, \underline{\eta}_1^{(k)}, \underline{\eta}_2^{(k)}, \underline{\nu}^{(k)}) \tag{21b}$$

$$\underline{x}_c^{(k+1)}, \underline{\beta}_c^{(k+1)} = \arg \min_{\underline{x}_c, \underline{\beta}_c} L_c(\underline{x}_c, \underline{\beta}_c, \underline{\eta}_1^{(k)}, \underline{\eta}_2^{(k)}, \underline{\nu}^{(k)}) \tag{21c}$$

$$\eta_{1,t}^{(k+1)} = \eta_{1,t}^{(k)} + \alpha^{(k)} (1 - \sum_{e \in E} x_{t,e}^{(k+1)} + \sum_{f \in F} x_{t,f}^{(k+1)} + \sum_{c \in C} x_{t,c}^{(k+1)}) \tag{22a}$$

$$\eta_{2,t}^{(k+1)} = \eta_{2,t}^{(k)} + \alpha^{(k)} (\sum_{e \in E} x_{t,e}^{(k+1)} + \sum_{f \in F} x_{t,f}^{(k+1)} + \sum_{c \in C} x_{t,c}^{(k+1)} - N_t) \tag{22b}$$

$$\nu_{t,s}^{(k+1)} = \nu_{t,s}^{(k)} + \alpha^{(k)} (\lambda_{t,s} - \sum_{e \in E} \beta_{t,s,e}^{(k+1)} + \sum_{f \in F} \beta_{t,s,f}^{(k+1)} + \sum_{c \in C} \beta_{t,s,c}^{(k+1)}) \tag{22c}$$

D. Objective

$$\begin{aligned}
p_k &= \sum_{i=1}^{l_e} x_{k,i}^e C(v_i^e, t_k) \\
&+ \sum_{j=1}^{l_f} x_{k,j}^f C(v_j^f, t_k) \\
&+ \sum_{h=1}^{l_c} x_{k,h}^c C(v_h^c, t_k)
\end{aligned} \tag{23a}$$

$$\begin{aligned}
& \min \sum_{k=1}^{l_t} p_k \\
& \text{subject to: } 9
\end{aligned} \tag{24a}$$

E. Solution

We can reshape main problem as following:

$$\begin{aligned}
 \min & \left(\sum_{i=1}^{l_e} \sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e + \sum_{j=1}^{l_f} \sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c} \sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c \right) \\
 \text{subject to:} \\
 & \sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c \leq \delta_k \quad \forall k \in \{1, \dots, l_t\} \\
 & \sum_{k=1}^{l_t} x_{k,h}^c w_k \leq c_h^c \quad \forall h \in \{1, 2, \dots, l_c\} \\
 & \sum_{k=1}^{l_t} x_{k,j}^f w_k \leq c_j^f \quad \forall j \in \{1, 2, \dots, l_f\} \\
 & \sum_{k=1}^{l_t} x_{k,i}^e w_k \leq c_i^e \quad \forall i \in \{1, 2, \dots, l_e\} \\
 & \sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c = 1 \quad \forall k \in \{1, 2, \dots, l_t\}
 \end{aligned} \tag{25a}$$

We define u^m for each computational agent m , that is a matrix with size $l_t * (l_e + l_f + l_c)$. It is the local copy of all variables in agent m . i.e. $u_{k,i}^{e,m}$ is the copy of variable $x_{k,i}^e$ in agent m for $m = 1, \dots, (l_m = l_e + l_f + l_c)$. So we should add new constraint $u^m = z \quad \forall m$ to main problem. We will use admm on this new constraint so:

$$\begin{aligned}
 L_p &= \sum_{i=1}^{l_e} \sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e + \sum_{j=1}^{l_f} \sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c} \sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c \\
 &+ \sum_{m=1}^{l_m} \nu^m * (u^m - z) + \sum_{m=1}^{l_m} \frac{\rho}{2} \|u^m - z\|^2
 \end{aligned} \tag{26a}$$

We can separate augmented lagrangian for each computational agent m then:

$$\begin{aligned}
 L_p^m &= \sum_{k=1}^{l_t} u_{k,m}^m C_{k,m} + \nu^m * (u^m - z) + \frac{\rho}{2} \|u^m - z\|^2 \\
 &\forall m \in \{1, 2, \dots, l_m\}
 \end{aligned} \tag{27a}$$

So we can write the algorithm as following:

for each iteration k (28a)

$$\begin{aligned}
 1. \quad & u^{m,(k+1)} = \arg \min L_p^m(u^m, z^{(k)}, \nu^{m,(k)}) = \\
 & \sum_{k=1}^{l_t} u_{k,m}^m C_{k,m} + \nu^{m,(k)} * (u^m - z^{(k)}) + \frac{\rho}{2} \|u^m - z^{(k)}\|^2 \\
 \text{subject to:} \\
 & \sum_{k=1}^{l_t} u_{k,m}^m w_k \leq c^m \\
 & \sum_{i=1}^{l_e} u_{k,i}^{e,m} + \sum_{j=1}^{l_f} u_{k,j}^{f,m} + \sum_{h=1}^{l_c} u_{k,h}^{c,m} = 1 \quad \forall k \in \{1, 2, \dots, l_t\} \\
 & \sum_{i=1}^{l_e} u_{k,i}^{e,m} \tau_{k,i}^e + \sum_{j=1}^{l_f} u_{k,j}^{f,m} \tau_{k,j}^f + \sum_{h=1}^{l_c} u_{k,h}^{c,m} \tau_{k,h}^c \leq \delta_k \quad \forall k \in \{1, 2, \dots, l_t\} \\
 2. \quad & z^{(k+1)} = \bar{u}^{(k+1)} + \frac{1}{\rho} \bar{\nu}^{(k)} \\
 3. \quad & \nu^{m,(k+1)} = \nu^{m,(k)} + \rho(u^{m,(k+1)} - z^{(k+1)})
 \end{aligned}$$

F. Solution 2

lagrangian of main problem is as following

$$\begin{aligned}
 L(x^e, x^f, x^c, \lambda, \nu) &= \sum_{i=1}^{l_e} \sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e + \sum_{j=1}^{l_f} \sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c} \sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c \\
 &+ \sum_{k=1}^{l_t} \lambda_k \left(\sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c - \delta_k \right) \\
 &+ \sum_{k=1}^{l_t} \nu_k \left(\sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c - 1 \right)
 \end{aligned} \tag{29a}$$

So we can decompose the lagrangian as follows

$$\begin{aligned}
 L(x^e, x^f, x^c, \lambda, \nu) &= \\
 & \sum_{i=1}^{l_e} \sum_{k=1}^{l_t} \left(x_{k,i}^e C_{k,i}^e + \lambda_k x_{k,i}^e \tau_{k,i}^e + \nu_k x_{k,i}^e - \frac{\lambda_k \delta_k + \nu_k}{3l_e} \right) \\
 &+ \sum_{j=1}^{l_f} \sum_{k=1}^{l_t} \left(x_{k,j}^f C_{k,j}^f + \lambda_k x_{k,j}^f \tau_{k,j}^f + \nu_k x_{k,j}^f - \frac{\lambda_k \delta_k + \nu_k}{3l_f} \right) \\
 &+ \sum_{h=1}^{l_c} \sum_{k=1}^{l_t} \left(x_{k,h}^c C_{k,h}^c + \lambda_k x_{k,h}^c \tau_{k,h}^c + \nu_k x_{k,h}^c - \frac{\lambda_k \delta_k + \nu_k}{3l_c} \right)
 \end{aligned} \tag{30a}$$

$$\begin{aligned}
L(x^e, x^f, x^c, \lambda, \nu) &= \sum_{i=1}^{l_e} L_i^e(x_i^e, \lambda, \nu) \\
&+ \sum_{j=1}^{l_f} L_j^f(x_j^f, \lambda, \nu) \\
&+ \sum_{h=1}^{l_c} L_h^c(x_h^c, \lambda, \nu)
\end{aligned} \tag{31a}$$

$$\begin{aligned}
g(\lambda, \nu) &= \inf_{x^e, x^f, x^c} L(x^e, x^f, x^c, \lambda, \nu) \\
&= \sum_{i=1}^{l_e} \inf_{x_i^e} L_i^e(x_i^e, \lambda, \nu) \\
&+ \sum_{j=1}^{l_f} \inf_{x_j^f} L_j^f(x_j^f, \lambda, \nu) \\
&+ \sum_{h=1}^{l_c} \inf_{x_h^c} L_h^c(x_h^c, \lambda, \nu) \\
&= \sum_{i=1}^{l_e} g_i^e(\lambda, \nu) \\
&+ \sum_{j=1}^{l_f} g_j^f(\lambda, \nu) \\
&+ \sum_{h=1}^{l_c} g_h^c(\lambda, \nu)
\end{aligned} \tag{32a}$$

$$\lambda_k^+ = \lambda_k^- + \alpha \left(\sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c - \delta_k \right) \tag{33a}$$

$$\nu_k^+ = \nu_k^- + \alpha \left(\sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c - 1 \right) \tag{33b}$$

REFERENCES

Algorithm Test Algorithm

```

1: for  $n = 1 : L_v$  do
2:   Determine the set of states  $Z_n$ 
3:    $RemovedStates =$ 
4:   for  $j \in Z_n$  do
5:     Determine the index of computational node  $l$  and
       the index of task  $t$  and the index of part  $u$ 
6:      $XP_n^j = Z_{n-1}$ 
7:     if  $j \neq 0$  then
8:       for  $i \in XP_n^j$  do
9:         if  $ServerResource < 0$  then
10:           $XP_n^j = XP_n^j - \{i\}$ 
11:        end if
12:      end for
13:    end if
14:    if  $XP_n^j \neq \emptyset$  then
15:      for  $i \in XP_n^j$  do
16:        Calculate  $T_{n-1,n}^{i,j}$ 
17:      end for
18:      Calculate  $I_n^j$  and  $\Lambda_n^j$ 
19:      Calculate  $\phi_n^j$ 
20:      if  $j \neq 0$  then
21:         $ServerResource < 0$ 
22:      end if
23:    else
24:       $RemovedStates = RemovedStates + \{j\}$ 
25:    end if
26:  end for
27:  if  $RemovedStates = Z_n$  then
28:    Set  $H = \sum_{m=1}^t N_m$  and  $ResourceIndicator = 0$ 
29:    Determine the Viterbi path  $P$  with  $H$ 
30:    break
31:  else
32:    Remove all states in the  $RemovedStates$  from the
        $Z_n$ 
33:  end if
34: end for
35: if  $ResourceIndicator = 1$  then
36:   Determine the Viterbi path  $P$  with  $H$ 
37: end if
38: Determine the task scheduling using Viterbi path  $P$ 

```
