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***Abstract*—**

***Index Terms*—**

I. INTRODUCTION

II. SYSTEM MODEL

A. Graph Model

C is set of cloud nodes. F is set of fog nodes. E is set of edge nodes. S is set of sensor nodes. R is set of resources in each computational node(cloud, fog or edge node).

$$C = \{v_1^c, v_2^c, ..., v_{|C|}^c\}, c \in C \tag{1a}$$
$$F = \{v_1^f, v_2^f, ..., v_{|F|}^f\}, f \in F \tag{1b}$$
$$E = \{v_1^e, v_2^e, ..., v_{|E|}^e\}, e \in E \tag{1c}$$
$$S = \{v_1^s, v_2^s, ..., v_{|S|}^s\}, s \in S \tag{1d}$$
$$R = \{CPU, RAM, Storage\}, r \in R \tag{1e}$$

σ_c^r is total capacity of resource $r \in R$ on node $c \in C$. and also σ_f^r and σ_e^r are total capcity of resource $r \in R$ on nodes $f \in F$ and $e \in E$ respectively.

T is set of tasks.

$$T = \{t_1, t_2, ..., t_{|T|}\} \tag{2a}$$

Each task expresses as follows:

$$t \in T => t = (w_t, \delta_t, N_t, f_t^r(\lambda_t)) \tag{3}$$

w_t shows computation workload of the task. δ_t is completion deadline of the task and N_t determines the maximum number of instaces of task $t \in T$.

π_c is unit price of processing in node $c \in C$ and also π_f and π_e are the ralated prices in nodes $f \in F$ and $e \in E$ respectively.

Transmition delays that show required time for trasmiting packets from sensors to each computational node are defined as follows:
$\tau_{s,c}^{tr}$ = trasmition delay between node $s \in S$ and $c \in C$
$\tau_{s,f}^{tr}$ = trasmition delay between node $s \in S$ and $f \in F$
$\tau_{s,e}^{tr}$ = trasmition delay between node $s \in S$ and $e \in E$

B. Variables

We define three integer variables for allocating tasks between nodes.

$$x_{t,c} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } c \in C \\ 0 & \text{o.w.} \end{cases} \tag{4a}$$

$$x_{t,f} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } f \in F \\ 0 & \text{o.w.} \end{cases} \tag{4b}$$

$$x_{t,e} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } e \in E \\ 0 & \text{o.w.} \end{cases} \tag{4c}$$

there are two continouse variables:

$$\lambda_{t,s} = \text{poisson rate of task } t \in T \text{ generated by node } s \in S \tag{5a}$$

$$0 \leq \beta_{t,s,c} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{5b}$$
$$0 \leq \beta_{t,s,f} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{5c}$$
$$0 \leq \beta_{t,s,e} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{5d}$$

$$\beta_{t,s,c} = \text{size of flow of task } t \in T \text{ from node } s \in S \text{ to node } c \in C \tag{5e}$$

$$\gamma_{t,s,c} = \begin{cases} 1 & \text{flow of task } t \in T \text{ from node } s \in S \text{ to node } c \in C \text{ exist} \\ \text{o.w.} & \end{cases} \tag{5f}$$

$$\gamma_{t,s,f} = \begin{cases} 1 & \text{flow of task } t \in T \text{ from node } s \in S \text{ to node } f \in F \text{ exis} \\ \text{o.w.} & \end{cases} \tag{5g}$$

$$\gamma_{t,s,e} = \begin{cases} 1 & \text{flow of task } t \in T \text{ from node } s \in S \text{ to node } e \in E \text{ exist} \\ \text{o.w.} & \end{cases} \tag{5h}$$

C. Constraints

$$\lambda_{t,c} = \sum_{s\in S} \beta_{t,s,c} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{6a}$$

$$\lambda_{t,f} = \sum_{s\in S} \beta_{t,s,f} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{6b}$$

$$\lambda_{t,e} = \sum_{s\in S} \beta_{t,s,e} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{6c}$$

$$\gamma_{t,s,c} \le x_{t,c} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{7a}$$

$$\gamma_{t,s,f} \le x_{t,f} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{7b}$$

$$\gamma_{t,s,e} \le x_{t,e} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{7c}$$

$$\gamma_{t,s,c} - 1 + \epsilon \le \beta_{t,s,c} \le \gamma_{t,s,c}\lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{8a}$$

$$\gamma_{t,s,f} - 1 + \epsilon \le \beta_{t,s,f} \le \gamma_{t,s,f}\lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{8b}$$

$$\gamma_{t,s,e} - 1 + \epsilon \le \beta_{t,s,e} \le \gamma_{t,s,e}\lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{8c}$$

$$\lambda_{t,s} = \sum_{e\in E} \beta_{t,s,e} + \sum_{f\in F} \beta_{t,s,f} + \sum_{c\in C} \beta_{t,s,c} \quad \forall t \in T, \forall s \in S \tag{9a}$$

$$\sum_{t\in T} x_{t,c} f_t^r(\lambda_{t,c}) \le \sigma_c^r \quad \forall r \in R, \forall c \in C \tag{10a}$$

$$x_{t,c} f_t^r(\lambda_{t,c}) = k_1^r x_{t,c}\lambda_{t,c} + k_2^r x_{t,c} \tag{10b}$$

$$\psi_{t,c} \triangleq x_{t,c}\lambda_{t,c} \Rightarrow 0 \le \psi_{t,c} \le \lambda_{t,c} \tag{10c}$$

$$Q(x_{t,c} - 1) + \lambda_{t,c} \le \psi_{t,c} \le x_{t,c} Q \tag{10d}$$

$$\begin{aligned} Q &= \max_{t\ inT, c\in C} \lambda_{t,c} \\ &= \max_{t\ inT, c\in C} \sum_{s\in S} \beta_{t,s,c} \\ &= \sum_{s\in S} \max_{t\ inT, c\in C} \beta_{t,s,c} \\ &= \sum_{s\in S} \lambda_{t,s} \end{aligned} \tag{10e}$$

$$0 \le \psi_{t,c} \le \lambda_{t,c} \tag{11a}$$

$$Q(x_{t,c} - 1) + \lambda_{t,c} \le \psi_{t,c} \le x_{t,c} Q \quad \forall t \in T, \forall c \in C \tag{11b}$$

$$0 \le \psi_{t,f} \le \lambda_{t,f} \tag{11c}$$

$$Q(x_{t,f} - 1) + \lambda_{t,f} \le \psi_{t,f} \le x_{t,f} Q \quad \forall t \in T, \forall f \in F \tag{11d}$$

$$0 \le \psi_{t,e} \le \lambda_{t,e} \tag{11e}$$

$$Q(x_{t,e} - 1) + \lambda_{t,e} \le \psi_{t,e} \le x_{t,e} Q \quad \forall t \in T, \forall e \in E \tag{11f}$$

$$\sum_{t\in T} k_1^r \psi_{t,c} + k_2^r x_{t,c} \le \sigma_c^r \quad \forall r \in R, \forall c \in C \tag{12a}$$

$$\sum_{t\in T} k_1^r \psi_{t,f} + k_2^r x_{t,f} \le \sigma_f^r \quad \forall r \in R, \forall f \in F \tag{12b}$$

$$\sum_{t\in T} k_1^r \psi_{t,e} + k_2^r x_{t,e} \le \sigma_e^r \quad \forall r \in R, \forall e \in E \tag{12c}$$

$$\tau_{t,c} = \tau_{t,s,c}^{tr} + \frac{1}{\mu_{t,c} - \lambda_{t,c}} \tag{13a}$$

We have:

$$\frac{1}{\mu_{t,c}} = \frac{w_t}{f_t^{cpu}(\lambda_{t,c})} \tag{13b}$$

$$f_t^{cpu}(\lambda_{t,c}) = k_1^{cpu}\lambda_{t,c} + k_2^{cpu} \tag{13c}$$

$$\begin{aligned} &\Rightarrow \gamma_{t,s,c}\tau_{t,c} = \gamma_{t,s,c}(\tau_{t,s,c}^{tr} + \frac{w_t}{(k_1^{cpu} - w_t)\lambda_{t,c} + k_2^{cpu}}) \\ &\le \delta_t \quad \forall t \in T, \forall s \in S, \forall c \in C \end{aligned} \tag{13d}$$

$$\begin{aligned} &\gamma_{t,s,c}\lambda_{t,c}(k_1^{cpu} - w_t)\tau_{t,s,c}^{tr} + \\ &\gamma_{t,s,c}k_2^{cpu}\tau_{t,s,c}^{tr} + w_t\gamma_{t,s,c} - k_2^{cpu}\delta_t \\ &- (k_1^{cpu} - w_t)\delta_t\lambda_{t,c} \le 0 \quad \forall t \in T, \forall s \in S, \forall c \in C \end{aligned} \tag{13e}$$

$$\phi_{t,s,c} = \gamma_{t,s,c}\lambda_{t,c} \tag{13f}$$

$$0 \le \phi_{t,s,c} \le \lambda_{t,c} \tag{13g}$$

$$Q(\gamma_{t,s,c} - 1) + \lambda_{t,c} \le \phi_{t,s,c} \le \gamma_{t,s,c} Q \tag{13h}$$

$$\begin{aligned} &\phi_{t,s,c}(k_1^{cpu} - w_t)\tau_{t,s,c}^{tr} + \\ &\gamma_{t,s,c}k_2^{cpu}\tau_{t,s,c}^{tr} + w_t\gamma_{t,s,c} - k_2^{cpu}\delta_t \\ &- (k_1^{cpu} - w_t)\delta_t\lambda_{t,c} \le 0 \end{aligned} \tag{14a}$$

$$0 \le \phi_{t,s,c} \le \lambda_{t,c} \tag{14b}$$

$$Q(\gamma_{t,s,c} - 1) + \lambda_{t,c} \le \phi_{t,s,c} \le \gamma_{t,s,c} Q \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{14c}$$

$$\begin{aligned} &\phi_{t,s,f}(k_1^{cpu} - w_t)\tau_{t,s,f}^{tr} + \\ &\gamma_{t,s,f}k_2^{cpu}\tau_{t,s,f}^{tr} + w_t\gamma_{t,s,f} - k_2^{cpu}\delta_t \\ &- (k_1^{cpu} - w_t)\delta_t\lambda_{t,f} \le 0 \end{aligned} \tag{14d}$$

$$0 \le \phi_{t,s,f} \le \lambda_{t,c} \tag{14e}$$

$$Q(\gamma_{t,s,f} - 1) + \lambda_{t,f} \le \phi_{t,s,f} \le \gamma_{t,s,f} Q \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{14f}$$

$$\begin{aligned} &\phi_{t,s,e}(k_1^{cpu} - w_t)\tau_{t,s,e}^{tr} + \\ &\gamma_{t,s,e}k_2^{cpu}\tau_{t,s,e}^{tr} + w_t\gamma_{t,s,e} - k_2^{cpu}\delta_t \\ &- (k_1^{cpu} - w_t)\delta_t\lambda_{t,e} \le 0 \end{aligned} \tag{14g}$$

$$0 \le \phi_{t,s,e} \le \lambda_{t,e} \tag{14h}$$

$$Q(\gamma_{t,s,e} - 1) + \lambda_{t,e} \le \phi_{t,s,e} \le \gamma_{t,s,e} Q \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{14i}$$

$$1 \le \sum_{e\in E} x_{t,e} + \sum_{f\in F} x_{t,f} + \sum_{c\in C} x_{t,c} \le N_t \quad \forall t \in T \tag{15a}$$

$$x_{t,c}(\lambda_{t,c} < \mu_{t,c}) => x_{t,c}(\lambda_{t,c} + \epsilon \leq \mu_{t,c}) \tag{16a}$$
$$x_{t,c}\lambda_{t,c} = \lambda_{t,c} \tag{16b}$$
$$=> \epsilon x_{t,c} - k_1^{cpu}\lambda_{t,c} - k_2^{cpu} + w_t\lambda_{t,c} \leq 0 \quad \forall t \in T, \forall c \in C \tag{16c}$$
$$\epsilon x_{t,f} - k_1^{cpu}\lambda_{t,f} - k_2^{cpu} + w_t\lambda_{t,f} \leq 0 \quad \forall t \in T, \forall f \in F \tag{16d}$$
$$\epsilon x_{t,e} - k_1^{cpu}\lambda_{t,e} - k_2^{cpu} + w_t\lambda_{t,e} \leq 0 \quad \forall t \in T, \forall e \in E \tag{16e}$$

$$\begin{aligned} \min \sum_{t\in T}\sum_{e\in E}(x_{t,e}\pi_e \sum_{r\in R} f_t^r(\lambda_{t,e})) \\ + \sum_{t\in T}\sum_{f\in F}(x_{t,f}\pi_f \sum_{r\in R} f_t^r(\lambda_{t,f})) \\ + \sum_{t\in T}\sum_{c\in C}(x_{t,c}\pi_c \sum_{r\in R} f_t^r(\lambda_{t,c})) \end{aligned} \tag{17a}$$

$$\begin{aligned} \min \sum_{t\in T}\sum_{e\in E} x_{t,e}\Gamma_{t,e} \\ + \sum_{t\in T}\sum_{f\in F} x_{t,f}\Gamma_{t,f} \\ + \sum_{t\in T}\sum_{c\in C} x_{t,c}\Gamma_{t,c} \end{aligned} \tag{17b}$$

$$\begin{aligned} \Gamma_{t,e} &= \pi_e((k_1^{cpu} + k_1^{ram} + k_1^{storage})\lambda_{t,e} \\ &+ k_2^{cpu} + k_2^{ram} + k_2^{storage}) \\ &= \pi_e(K_1\lambda_{t,e} + K_2) \end{aligned} \tag{17c}$$

$$\begin{aligned} x_{t,e}\Gamma_{t,e} &= K_1\pi_e x_{t,e}\lambda_{t,e} + K_2\pi_e x_{t,e} \\ &= K_1\pi_e\psi_{t,e} + K_2\pi_e x_{t,e} \end{aligned} \tag{17d}$$

$$\begin{aligned} \min \sum_{t\in T}\sum_{e\in E} K_1\pi_e\psi_{t,e} + K_2\pi_e x_{t,e} \\ \sum_{t\in T}\sum_{f\in F} K_1\pi_f\psi_{t,f} + K_2\pi_f x_{t,f} \\ \sum_{t\in T}\sum_{c\in C} K_1\pi_c\psi_{t,c} + K_2\pi_c x_{t,c} \end{aligned} \tag{18a}$$

$$\begin{aligned} L(\underline{\underline{x}}, \underline{\underline{\beta}}, \underline{\underline{\gamma}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) = \sum_{t\in T}\sum_{e\in E} x_{t,e}\Gamma_{t,e} \\ + \sum_{t\in T}\sum_{f\in F} x_{t,f}\Gamma_{t,f} + \sum_{t\in T}\sum_{c\in C} x_{t,c}\Gamma_{t,c} + \sum \end{aligned} \tag{19a}$$

D. Objective

$$\begin{aligned} p_k = \sum_{i=1}^{l_e} x_{k,i}^e C(v_i^e, t_k) \\ + \sum_{j=1}^{l_f} x_{k,j}^f C(v_j^f, t_k) \\ + \sum_{h=1}^{l_c} x_{k,h}^c C(v_h^c, t_k) \end{aligned} \tag{20a}$$

$$\min \sum_{k=1}^{l_t} p_k \tag{21a}$$

subject to: 9

E. Solution

We can reshape main problem as following:

$$\begin{aligned} &\min(\sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e \\ &+ \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c) \end{aligned} \tag{22a}$$

subject to:

$$\sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c \leq \delta_k \quad \forall k \in \{1, ..., l_t\}$$
$$\sum_{k=1}^{l_t} x_{k,h}^c w_k \leq c_h^c \qquad \forall h \in \{1, 2, ..., l_c\}$$
$$\sum_{k=1}^{l_t} x_{k,j}^f w_k \leq c_j^f \qquad \forall j \in \{1, 2, ..., l_f\}$$
$$\sum_{k=1}^{l_t} x_{k,i}^e w_k \leq c_i^e \qquad \forall i \in \{1, 2, ..., l_e\}$$
$$\sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c = 1 \qquad \forall k \in \{1, 2, ..., l_t\}$$

We define u^m for each computational agent m, that is a matrix with size $l_t * (l_e + l_f + l_c)$. It is the local copy of all variables in agent m. i.e. $u_{k,i}^{e,m}$ is the copy of variable $x_{k,i}^e$ in agent m for $m = 1, .., (l_m = l_e + l_f + l_c)$. So we should add new constraint $u^m = z \quad \forall m$ to main problem. We will use admm on this new constraint so:

$$L_p = \sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e + \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c \tag{23a}$$

$$+ \sum_{m=1}^{l_m} \nu^m * (u^m - z) + \sum_{m=1}^{l_m} \frac{\rho}{2}\|u^m - z\|^2$$

We can seperate augmented lagrangian for each computational agent m then:

$$L_p^m = \sum_{k=1}^{l_t} u_{k,m}^m C_{k,m} + \nu^m * (u^m - z) + \frac{\rho}{2}\|u^m - z\|^2 \tag{24a}$$

$$\forall m \in \{1, 2, ..., l_m\}$$

So we can write the algorithm as following:

for each iteration k (25a)

1. $u^{m,(k+1)} = arg\min L_p^m(u^m, z^{(k)}, \nu^{m,(k)}) =$

$$\sum_{k=1}^{l_t} u_{k,m}^m C_{k,m} + \nu^{m,(k)} * (u^m - z^{(k)}) + \frac{\rho}{2}\|u^m - z^{(k)}\|^2$$

subject to:

$$\sum_{k=1}^{l_t} u_{k,m}^m w_k \le c^m$$

$$\sum_{i=1}^{l_e} u_{k,i}^{e,m} + \sum_{j=1}^{l_f} u_{k,j}^{f,m} + \sum_{h=1}^{l_c} u_{k,h}^{c,m} = 1 \qquad \forall k \in \{1, 2, ..., l_t\}$$

$$\sum_{i=1}^{l_e} u_{k,i}^{e,m} \tau_{k,i}^e + \sum_{j=1}^{l_f} u_{k,j}^{f,m} \tau_{k,j}^f + \sum_{h=1}^{l_c} u_{k,h}^{c,m} \tau_{k,h}^c \le \delta_k \qquad \forall k \in \{1, 2, ..., l_t\}$$

2. $z^{(k+1)} = \bar{u}^{(k+1)} + \frac{1}{\rho}\bar{\nu}^{(k)}$
3. $\nu^{m,(k+1)} = \nu^{m,(k)} + \rho(u^{m,(k+1)} - z^{(k+1)})$

Algorithm 1

while not converged **do**
end while

REFERENCES