Paper Title*

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***Abstract*—**
***Index Terms*—**

I. Introduction

II. System Model

A. Graph Model

C is set of cloud nodes. F is set of fog nodes. E is set of edge nodes. S is set of sensor nodes. R is set of resources in each computational node(cloud, fog or edge node).

$$C = \{v_1^c, v_2^c, ..., v_{|C|}^c\}, c \in C \tag{1a}$$
$$F = \{v_1^f, v_2^f, ..., v_{|F|}^f\}, f \in F \tag{1b}$$
$$E = \{v_1^e, v_2^e, ..., v_{|E|}^e\}, e \in E \tag{1c}$$
$$S = \{v_1^s, v_2^s, ..., v_{|S|}^s\}, s \in S \tag{1d}$$
$$R = \{CPU, RAM, Storage\}, r \in R \tag{1e}$$

σ_c^r is total capacity of resource $r \in R$ on node $c \in C$. and also σ_f^r and σ_e^r are total capcity of resource $r \in R$ on nodes $f \in F$ and $e \in E$ respectively.

T is set of tasks.

$$T = \{t_1, t_2, ..., t_{|T|}\} \tag{2a}$$

Each task expresses as follows:

$$t \in T => t = (w_t, \delta_t, N_t, f_t^r(\lambda_t)) \tag{3}$$

w_t shows computation workload of the task. δ_t is completion deadline of the task and N_t determines the maximum number of instaces of task $t \in T$.

π_c is unit price of processing in node $c \in C$ and also π_f and π_e are the ralated prices in nodes $f \in F$ and $e \in E$ respectively.

Transmition delays that show required time for trasmiting packets from sensors to each computational node are defined as follows:
$\tau_{s,c}^{tr}$ = trasmition delay between node $s \in S$ and $c \in C$
$\tau_{s,f}^{tr}$ = trasmition delay between node $s \in S$ and $f \in F$
$\tau_{s,e}^{tr}$ = trasmition delay between node $s \in S$ and $e \in E$

B. Variables

We define three integer variables for allocating tasks between nodes.

$$x_{t,c} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } c \in C \\ 0 & \text{o.w.} \end{cases} \tag{4a}$$

$$x_{t,f} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } f \in F \\ 0 & \text{o.w.} \end{cases} \tag{4b}$$

$$x_{t,e} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } e \in E \\ 0 & \text{o.w.} \end{cases} \tag{4c}$$

there are two continouse variables:

$$\lambda_{t,s} = \text{poisson rate of task } t \in T \text{ generated by node } s \in S \tag{5a}$$

$$0 \le \beta_{t,s,c} \le \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{5b}$$
$$0 \le \beta_{t,s,f} \le \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{5c}$$
$$0 \le \beta_{t,s,e} \le \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{5d}$$

$$\beta_{t,s,c} = \text{size of flow of task } t \in T \text{ from node } s \in S \text{ to node } c \in C \tag{5e}$$

$$\gamma_{t,s,c} = \begin{cases} 1 & \text{flow of task } t \in T \text{ from node } s \in S \text{ to node } c \in C \text{ exist} \\ \text{o.w.} \end{cases} \tag{5f}$$

$$\gamma_{t,s,f} = \begin{cases} 1 & \text{flow of task } t \in T \text{ from node } s \in S \text{ to node } f \in F \text{ exis} \\ \text{o.w.} \end{cases} \tag{5g}$$

$$\gamma_{t,s,e} = \begin{cases} 1 & \text{flow of task } t \in T \text{ from node } s \in S \text{ to node } e \in E \text{ exist} \\ \text{o.w.} \end{cases} \tag{5h}$$

C. Constraints

$$\lambda_{t,c} = \sum_{s\in S} \beta_{t,s,c} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{6a}$$

$$\lambda_{t,f} = \sum_{s\in S} \beta_{t,s,f} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{6b}$$

$$\lambda_{t,e} = \sum_{s\in S} \beta_{t,s,e} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{6c}$$

$$\gamma_{t,s,c} \leq x_{t,c} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{7a}$$

$$\gamma_{t,s,f} \leq x_{t,f} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{7b}$$

$$\gamma_{t,s,e} \leq x_{t,e} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{7c}$$

$$\gamma_{t,s,c} - 1 + \epsilon \leq \beta_{t,s,c} \leq \gamma_{t,s,c}\lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{8a}$$

$$\gamma_{t,s,f} - 1 + \epsilon \leq \beta_{t,s,f} \leq \gamma_{t,s,f}\lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{8b}$$

$$\gamma_{t,s,e} - 1 + \epsilon \leq \beta_{t,s,e} \leq \gamma_{t,s,e}\lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{8c}$$

$$\lambda_{t,s} = \sum_{e\in E} \beta_{t,s,e} + \sum_{f\in F} \beta_{t,s,f} + \sum_{c\in C} \beta_{t,s,c} \quad \forall t \in T, \forall s \in S \tag{9a}$$

$$\sum_{t\in T} x_{t,c} f_t^r(\lambda_{t,c}) \leq \sigma_c^r \quad \forall r \in R, \forall c \in C \tag{10a}$$

$$x_{t,c} f_t^r(\lambda_{t,c}) = k_1^r x_{t,c}\lambda_{t,c} + k_2^r x_{t,c} \tag{10b}$$

$$\psi_{t,c} \triangleq x_{t,c}\lambda_{t,c} \Rightarrow 0 \leq \psi_{t,c} \leq \lambda_{t,c} \tag{10c}$$

$$Q(x_{t,c} - 1) + \lambda_{t,c} \leq \psi_{t,c} \leq x_{t,c} Q \tag{10d}$$

$$\begin{aligned} Q &= \max_{t\in T, c\in C} \lambda_{t,c} \\ &= \max_{t\in T, c\in C} \sum_{s\in S} \beta_{t,s,c} \\ &= \sum_{s\in S} \max_{t\in T, c\in C} \beta_{t,s,c} \\ &= \sum_{s\in S} \lambda_{t,s} \end{aligned} \tag{10e}$$

$$0 \leq \psi_{t,c} \leq \lambda_{t,c} \tag{11a}$$

$$Q(x_{t,c} - 1) + \lambda_{t,c} \leq \psi_{t,c} \leq x_{t,c} Q \quad \forall t \in T, \forall c \in C \tag{11b}$$

$$0 \leq \psi_{t,f} \leq \lambda_{t,f} \tag{11c}$$

$$Q(x_{t,f} - 1) + \lambda_{t,f} \leq \psi_{t,f} \leq x_{t,f} Q \quad \forall t \in T, \forall f \in F \tag{11d}$$

$$0 \leq \psi_{t,e} \leq \lambda_{t,e} \tag{11e}$$

$$Q(x_{t,e} - 1) + \lambda_{t,e} \leq \psi_{t,e} \leq x_{t,e} Q \quad \forall t \in T, \forall e \in E \tag{11f}$$

$$\sum_{t\in T} k_1^r \psi_{t,c} + k_2^r x_{t,c} \leq \sigma_c^r \quad \forall r \in R, \forall c \in C \tag{12a}$$

$$\sum_{t\in T} k_1^r \psi_{t,f} + k_2^r x_{t,f} \leq \sigma_f^r \quad \forall r \in R, \forall f \in F \tag{12b}$$

$$\sum_{t\in T} k_1^r \psi_{t,e} + k_2^r x_{t,e} \leq \sigma_e^r \quad \forall r \in R, \forall e \in E \tag{12c}$$

$$\tau_{t,c} = \tau_{t,s,c}^{tr} + \frac{1}{\mu_{t,c} - \lambda_{t,c}} \tag{13a}$$

We have:

$$\frac{1}{\mu_{t,c}} = \frac{w_t}{f_t^{cpu}(\lambda_{t,c})} \tag{13b}$$

$$f_t^{cpu}(\lambda_{t,c}) = k_1^{cpu}\lambda_{t,c} + k_2^{cpu} \tag{13c}$$

$$\begin{aligned} &\Rightarrow \gamma_{t,s,c}\tau_{t,c} = \gamma_{t,s,c}\left(\tau_{t,s,c}^{tr} + \frac{w_t}{(k_1^{cpu} - w_t)\lambda_{t,c} + k_2^{cpu}}\right) \\ &\leq \delta_t \quad \forall t \in T, \forall s \in S, \forall c \in C \end{aligned} \tag{13d}$$

$$\begin{aligned} &\gamma_{t,s,c}\lambda_{t,c}(k_1^{cpu} - w_t)\tau_{t,s,c}^{tr} + \\ &\gamma_{t,s,c}k_2^{cpu}\tau_{t,s,c}^{tr} + w_t\gamma_{t,s,c} - k_2^{cpu}\delta_t \\ &- (k_1^{cpu} - w_t)\delta_t\lambda_{t,c} \leq 0 \quad \forall t \in T, \forall s \in S, \forall c \in C \end{aligned} \tag{13e}$$

$$\phi_{t,s,c} = \gamma_{t,s,c}\lambda_{t,c} \tag{13f}$$

$$0 \leq \phi_{t,s,c} \leq \lambda_{t,c} \tag{13g}$$

$$Q(\gamma_{t,s,c} - 1) + \lambda_{t,c} \leq \phi_{t,s,c} \leq \gamma_{t,s,c} Q \tag{13h}$$

$$\begin{aligned} &\phi_{t,s,c}(k_1^{cpu} - w_t)\tau_{t,s,c}^{tr} + \\ &\gamma_{t,s,c}k_2^{cpu}\tau_{t,s,c}^{tr} + w_t\gamma_{t,s,c} - k_2^{cpu}\delta_t \\ &- (k_1^{cpu} - w_t)\delta_t\lambda_{t,c} \leq 0 \end{aligned} \tag{14a}$$

$$0 \leq \phi_{t,s,c} \leq \lambda_{t,c} \tag{14b}$$

$$Q(\gamma_{t,s,c} - 1) + \lambda_{t,c} \leq \phi_{t,s,c} \leq \gamma_{t,s,c} Q \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{14c}$$

$$\begin{aligned} &\phi_{t,s,f}(k_1^{cpu} - w_t)\tau_{t,s,f}^{tr} + \\ &\gamma_{t,s,f}k_2^{cpu}\tau_{t,s,f}^{tr} + w_t\gamma_{t,s,f} - k_2^{cpu}\delta_t \\ &- (k_1^{cpu} - w_t)\delta_t\lambda_{t,f} \leq 0 \end{aligned} \tag{14d}$$

$$0 \leq \phi_{t,s,f} \leq \lambda_{t,c} \tag{14e}$$

$$Q(\gamma_{t,s,f} - 1) + \lambda_{t,f} \leq \phi_{t,s,f} \leq \gamma_{t,s,f} Q \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{14f}$$

$$\begin{aligned} &\phi_{t,s,e}(k_1^{cpu} - w_t)\tau_{t,s,e}^{tr} + \\ &\gamma_{t,s,e}k_2^{cpu}\tau_{t,s,e}^{tr} + w_t\gamma_{t,s,e} - k_2^{cpu}\delta_t \\ &- (k_1^{cpu} - w_t)\delta_t\lambda_{t,e} \leq 0 \end{aligned} \tag{14g}$$

$$0 \leq \phi_{t,s,e} \leq \lambda_{t,e} \tag{14h}$$

$$Q(\gamma_{t,s,e} - 1) + \lambda_{t,e} \leq \phi_{t,s,e} \leq \gamma_{t,s,e} Q \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{14i}$$

$$1 \leq \sum_{e\in E} x_{t,e} + \sum_{f\in F} x_{t,f} + \sum_{c\in C} x_{t,c} \leq N_t \quad \forall t \in T \tag{15a}$$

$$x_{t,c}(\lambda_{t,c} < \mu_{t,c}) => x_{t,c}(\lambda_{t,c} + \epsilon \le \mu_{t,c}) \tag{16a}$$

$$x_{t,c}\lambda_{t,c} = \lambda_{t,c} \tag{16b}$$

$$=> \epsilon x_{t,c} - k_1^{cpu}\lambda_{t,c} - k_2^{cpu} + w_t\lambda_{t,c} \le 0 \quad \forall t \in T, \forall c \in C \tag{16c}$$

$$\epsilon x_{t,f} - k_1^{cpu}\lambda_{t,f} - k_2^{cpu} + w_t\lambda_{t,f} \le 0 \quad \forall t \in T, \forall f \in F \tag{16d}$$

$$\epsilon x_{t,e} - k_1^{cpu}\lambda_{t,e} - k_2^{cpu} + w_t\lambda_{t,e} \le 0 \quad \forall t \in T, \forall e \in E \tag{16e}$$

$$\begin{aligned} \min \sum_{t\in T}\sum_{e\in E}(x_{t,e}\pi_e \sum_{r\in R} f_t^r(\lambda_{t,e})) \\ + \sum_{t\in T}\sum_{f\in F}(x_{t,f}\pi_f \sum_{r\in R} f_t^r(\lambda_{t,f})) \\ + \sum_{t\in T}\sum_{c\in C}(x_{t,c}\pi_c \sum_{r\in R} f_t^r(\lambda_{t,c})) \end{aligned} \tag{17a}$$

$$\begin{aligned} \min \sum_{t\in T}\sum_{e\in E} x_{t,e}\Gamma_{t,e} \\ + \sum_{t\in T}\sum_{f\in F} x_{t,f}\Gamma_{t,f} \\ + \sum_{t\in T}\sum_{c\in C} x_{t,c}\Gamma_{t,c} \end{aligned} \tag{17b}$$

$$\begin{aligned} \Gamma_{t,e} &= \pi_e((k_1^{cpu} + k_1^{ram} + k_1^{storage})\lambda_{t,e} \\ &+ k_2^{cpu} + k_2^{ram} + k_2^{storage}) \\ &= \pi_e(K_1\lambda_{t,e} + K_2) \end{aligned} \tag{17c}$$

$$\begin{aligned} x_{t,e}\Gamma_{t,e} &= K_1\pi_e x_{t,e}\lambda_{t,e} + K_2\pi_e x_{t,e} \\ &= K_1\pi_e\psi_{t,e} + K_2\pi_e x_{t,e} \end{aligned} \tag{17d}$$

$$\begin{aligned} \min \sum_{t\in T}\sum_{e\in E} K_1\pi_e\psi_{t,e} + K_2\pi_e x_{t,e} \\ \sum_{t\in T}\sum_{f\in F} K_1\pi_f\psi_{t,f} + K_2\pi_f x_{t,f} \\ \sum_{t\in T}\sum_{c\in C} K_1\pi_c\psi_{t,c} + K_2\pi_c x_{t,c} \end{aligned} \tag{18a}$$

$$\begin{aligned} L(\underline{\underline{x}}, \underline{\underline{\beta}}, \underline{\underline{\gamma}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) &= \sum_{t\in T}\sum_{e\in E} x_{t,e}\Gamma_{t,e} \\ &+ \sum_{t\in T}\sum_{f\in F} x_{t,f}\Gamma_{t,f} + \sum_{t\in T}\sum_{c\in C} x_{t,c}\Gamma_{t,c} \\ &+ \sum_{t\in T}\eta_{1,t}(1 - \sum_{e\in E} x_{t,e} + \sum_{f\in F} x_{t,f} + \sum_{c\in C} x_{t,c}) \\ &+ \sum_{t\in T}\eta_{2,t}(\sum_{e\in E} x_{t,e} + \sum_{f\in F} x_{t,f} + \sum_{c\in C} x_{t,c} - N_t) \\ &+ \sum_{t\in T}\sum_{s\in S}\nu_{t,s}(\lambda_{t,s} - \sum_{e\in E}\beta_{t,s,e} + \sum_{f\in F}\beta_{t,s,f} + \sum_{c\in C}\beta_{t,s,c}) \end{aligned} \tag{19a}$$

$$\begin{aligned} L &= \sum_{e\in E}\sum_{t\in T}(\sum_{s\in S}(\nu_{t,s}\beta_{t,s,e} + \frac{\nu_{t,s}\lambda_{t,s}}{3|E|}) \\ &+ x_{t,e}(\Gamma_{t,e} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t\eta_{2,t}}{3|E|}) \\ &+ \sum_{f\in F}\sum_{t\in T}(\sum_{s\in S}(\nu_{t,s}\beta_{t,s,f} + \frac{\nu_{t,s}\lambda_{t,s}}{3|F|}) \\ &+ x_{t,f}(\Gamma_{t,f} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t\eta_{2,t}}{3|F|}) \\ &+ \sum_{c\in C}\sum_{t\in T}(\sum_{s\in S}(\nu_{t,s}\beta_{t,s,c} + \frac{\nu_{t,s}\lambda_{t,s}}{3|C|}) \\ &+ x_{t,c}(\Gamma_{t,c} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t\eta_{2,t}}{3|C|}) \end{aligned} \tag{20a}$$

$$L = \sum_{e\in E} L_e + \sum_{f\in F} L_f + \sum_{c\in C} L_c \tag{20b}$$

$$\begin{aligned} g(\underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) &= \inf_{\underline{\underline{x}}, \underline{\underline{\beta}}, \underline{\underline{\gamma}}} L(\underline{\underline{x}}, \underline{\underline{\beta}}, \underline{\underline{\gamma}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\ &= \sum_{e\in E} \inf_{\underline{\underline{x_e}}, \underline{\underline{\beta_e}}, \underline{\underline{\gamma_e}}} L_e(\underline{\underline{x_e}}, \underline{\underline{\beta_e}}, \underline{\underline{\gamma_e}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\ &+ \sum_{f\in F} \inf_{\underline{\underline{x_f}}, \underline{\underline{\beta_f}}, \underline{\underline{\gamma_f}}} L_f(\underline{\underline{x_f}}, \underline{\underline{\beta_f}}, \underline{\underline{\gamma_f}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\ &+ \sum_{c\in C} \inf_{\underline{\underline{x_c}}, \underline{\underline{\beta_c}}, \underline{\underline{\gamma_c}}} L_c(\underline{\underline{x_c}}, \underline{\underline{\beta_c}}, \underline{\underline{\gamma_c}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\ &= \sum_{e\in E} g_e(\underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\ &+ \sum_{f\in F} g_f(\underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\ &+ \sum_{c\in C} g_c(\underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \end{aligned} \tag{21a}$$

$$\underline{\underline{x_e}}^{(k+1)}, \underline{\underline{\beta_e}}^{(k+1)}, \underline{\underline{\gamma_e}}^{(k+1)} = arg \min_{\underline{\underline{x_e}}, \underline{\underline{\beta_e}}, \underline{\underline{\gamma_e}}} L_e(\underline{\underline{x_e}}, \underline{\underline{\beta_e}}, \underline{\underline{\gamma_e}}, \underline{\eta_1}^{(k)}, \underline{\eta_2}^{(k)}, \underline{\underline{\nu}}^{(k)}) \tag{22a}$$

$$\underline{\underline{x_f}}^{(k+1)}, \underline{\underline{\beta_f}}^{(k+1)}, \underline{\underline{\gamma_f}}^{(k+1)} = arg \min_{\underline{\underline{x_f}}, \underline{\underline{\beta_f}}, \underline{\underline{\gamma_f}}} L_f(\underline{\underline{x_f}}, \underline{\underline{\beta_f}}, \underline{\underline{\gamma_f}}, \underline{\eta_1}^{(k)}, \underline{\eta_2}^{(k)}, \underline{\underline{\nu}}^{(k)}) \tag{22b}$$

$$\underline{\underline{x_c}}^{(k+1)}, \underline{\underline{\beta_c}}^{(k+1)}, \underline{\underline{\gamma_c}}^{(k+1)} = arg \min_{\underline{\underline{x_c}}, \underline{\underline{\beta_c}}, \underline{\underline{\gamma_c}}} L_c(\underline{\underline{x_c}}, \underline{\underline{\beta_c}}, \underline{\underline{\gamma_c}}, \underline{\eta_1}^{(k)}, \underline{\eta_2}^{(k)}, \underline{\underline{\nu}}^{(k)}) \tag{22c}$$

$$\eta_{1,t}^{(k+1)} = \eta_{1,t}^{(k)} + \alpha^{(k)}(1 - \sum_{e\in E} x_{t,e}^{(k+1)} + \sum_{f\in F} x_{t,f}^{(k+1)} + \sum_{c\in C} x_{t,c}^{(k+1)}) \tag{23a}$$

$$\eta_{2,t}^{(k+1)} = \eta_{2,t}^{(k)} + \alpha^{(k)}(\sum_{e\in E} x_{t,e}^{(k+1)} + \sum_{f\in F} x_{t,f}^{(k+1)} + \sum_{c\in C} x_{t,c}^{(k+1)} - N_t) \tag{23b}$$

$$\nu_{t,s}^{(k+1)} = \nu_{t,s}^{(k)} + \alpha^{(k)}(\lambda_{t,s}^{(k+1)} - \sum_{e\in E} \beta_{t,s,e}^{(k+1)} + \sum_{f\in F} \beta_{t,s,f}^{(k+1)} + \sum_{c\in C} \beta_{t,s,c}^{(k+1)}) \tag{23c}$$

D. Objective

$$p_k = \sum_{i=1}^{l_e} x^e_{k,i} C(v^e_i, t_k) + \sum_{j=1}^{l_f} x^f_{k,j} C(v^f_j, t_k) + \sum_{h=1}^{l_c} x^c_{k,h} C(v^c_h, t_k) \tag{24a}$$

$$\min \sum_{k=1}^{l_t} p_k \tag{25a}$$

subject to: 9

E. Solution

We can reshape main problem as following:

$$\min(\sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x^e_{k,i} C^e_{k,i} + \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x^f_{k,j} C^f_{k,j} + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x^c_{k,h} C^c_{k,h}) \tag{26a}$$

subject to:

$$\sum_{i=1}^{l_e} x^e_{k,i}\tau^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j}\tau^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h}\tau^c_{k,h} \le \delta_k \quad \forall k \in \{1, ..., l_t\}$$

$$\sum_{k=1}^{l_t} x^c_{k,h} w_k \le c^c_h \qquad \forall h \in \{1, 2, ..., l_c\}$$

$$\sum_{k=1}^{l_t} x^f_{k,j} w_k \le c^f_j \qquad \forall j \in \{1, 2, ..., l_f\}$$

$$\sum_{k=1}^{l_t} x^e_{k,i} w_k \le c^e_i \qquad \forall i \in \{1, 2, ..., l_e\}$$

$$\sum_{i=1}^{l_e} x^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h} = 1 \qquad \forall k \in \{1, 2, ..., l_t\}$$

We define u^m for each computational agent m, that is a matrix with size $l_t * (l_e + l_f + l_c)$. It is the local copy of all variables in agent m. i.e. $u^{e,m}_{k,i}$ is the copy of variable $x^e_{k,i}$ in agent m for $m = 1, .., (l_m = l_e + l_f + l_c)$. So we should add new constraint $u^m = z \quad \forall m$ to main problem. We will use admm on this new constraint so:

$$L_p = \sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x^e_{k,i} C^e_{k,i} + \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x^f_{k,j} C^f_{k,j} + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x^c_{k,h} C^c_{k,h} + \sum_{m=1}^{l_m} \nu^m * (u^m - z) + \sum_{m=1}^{l_m} \frac{\rho}{2}\|u^m - z\|^2 \tag{27a}$$

We can seperate augmented lagrangian for each computational agent m then:

$$L^m_p = \sum_{k=1}^{l_t} u^m_{k,m} C_{k,m} + \nu^m * (u^m - z) + \frac{\rho}{2}\|u^m - z\|^2 \tag{28a}$$

$$\forall m \in \{1, 2, ..., l_m\}$$

So we can write the algorithm as following:

for each iteration k (29a)

1. $u^{m,(k+1)} = arg\min L^m_p(u^m, z^{(k)}, \nu^{m,(k)}) = \sum_{k=1}^{l_t} u^m_{k,m} C_{k,m} + \nu^{m,(k)} * (u^m - z^{(k)}) + \frac{\rho}{2}\|u^m - z^{(k)}\|^2$

subject to:

$$\sum_{k=1}^{l_t} u^m_{k,m} w_k \le c^m$$

$$\sum_{i=1}^{l_e} u^{e,m}_{k,i} + \sum_{j=1}^{l_f} u^{f,m}_{k,j} + \sum_{h=1}^{l_c} u^{c,m}_{k,h} = 1 \qquad \forall k \in \{1, 2, ..., l_t\}$$

$$\sum_{i=1}^{l_e} u^{e,m}_{k,i}\tau^e_{k,i} + \sum_{j=1}^{l_f} u^{f,m}_{k,j}\tau^f_{k,j} + \sum_{h=1}^{l_c} u^{c,m}_{k,h}\tau^c_{k,h} \le \delta_k \qquad \forall k \in \{1, 2, ..., l_t\}$$

2. $z^{(k+1)} = \bar{u}^{(k+1)} + \frac{1}{\rho}\bar{\nu}^{(k)}$
3. $\nu^{m,(k+1)} = \nu^{m,(k)} + \rho(u^{m,(k+1)} - z^{(k+1)})$

F. Solution 2

lagrangian of main problem is as following

$$L(x^e, x^f, x^c, \lambda, \nu) = \sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x^e_{k,i} C^e_{k,i} + \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x^f_{k,j} C^f_{k,j} + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x^c_{k,h} C \tag{30a}$$

$$+ \sum_{k=1}^{l_t} \lambda_k (\sum_{i=1}^{l_e} x^e_{k,i}\tau^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j}\tau^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h}\tau^c_{k,h} - \delta_k)$$

$$+ \sum_{k=1}^{l_t} \nu_k (\sum_{i=1}^{l_e} x^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h} - 1)$$

So we can decompose the lagrangian as follows

$$L(x^e, x^f, x^c, \lambda, \nu) = \tag{31a}$$

$$\sum_{i=1}^{l_e}\sum_{k=1}^{l_t} (x^e_{k,i} C^e_{k,i} + \lambda_k x^e_{k,i}\tau^e_{k,i} + \nu_k x^e_{k,i} - \frac{\lambda_k \delta_k + \nu_k}{3 l_e})$$

$$+ \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} (x^f_{k,j} C^f_{k,j} + \lambda_k x^f_{k,j}\tau^f_{k,j} + \nu_k x^f_{k,j} - \frac{\lambda_k \delta_k + \nu_k}{3 l_f})$$

$$+ \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} (x^c_{k,h} C^c_{k,h} + \lambda_k x^c_{k,h}\tau^c_{k,h} + \nu_k x^c_{k,h} - \frac{\lambda_k \delta_k + \nu_k}{3 l_c})$$

$$L(x^e, x^f, x^c, \lambda, \nu) = \sum_{i=1}^{l_e} L_i^e(x_i^e, \lambda, \nu) \tag{32a}$$

$$+ \sum_{j=1}^{l_f} L_j^f(x_j^f, \lambda, \nu)$$

$$+ \sum_{h=1}^{l_c} L_h^c(x_h^c, \lambda, \nu)$$

$$g(\lambda, \nu) = \inf_{x^e, x^f, x^c} L(x^e, x^f, x^c, \lambda, \nu) \tag{33a}$$

$$= \sum_{i=1}^{l_e} \inf_{x_i^e} L_i^e(x_i^e, \lambda, \nu)$$

$$+ \sum_{j=1}^{l_f} \inf_{x_j^f} L_j^f(x_j^f, \lambda, \nu)$$

$$+ \sum_{h=1}^{l_c} \inf_{x_h^c} L_h^c(x_h^c, \lambda, \nu)$$

$$= \sum_{i=1}^{l_e} g_i^e(\lambda, \nu)$$

$$+ \sum_{j=1}^{l_f} g_j^f(\lambda, \nu)$$

$$+ \sum_{h=1}^{l_c} g_h^c(\lambda, \nu)$$

$$\lambda_k^+ = \lambda_k^- + \alpha(\sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c - \delta_k) \tag{34a}$$

$$\nu_k^+ = \nu_k^- + \alpha(\sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c - 1) \tag{34b}$$

Algorithm 1

while not converged **do**
  1. update x^c, x^f, x^e using (19a)
  2. update lagrangian multipliers using (20a) and (21a)
  $\alpha = \frac{\alpha_0}{k}$ for each iteration k
end while

REFERENCES