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***Abstract*—**

***Index Terms*—**

I. Introduction

II. System Model

A. Graph Model

C is set of cloud nodes. F is set of fog nodes. E is set of edge nodes. S is set of sensor nodes. R is set of resources in each computational node(cloud, fog or edge node).

$$C = \{v_1^c, v_2^c, ..., v_{|C|}^c\}, c \in C \tag{1a}$$
$$F = \{v_1^f, v_2^f, ..., v_{|F|}^f\}, f \in F \tag{1b}$$
$$E = \{v_1^e, v_2^e, ..., v_{|E|}^e\}, e \in E \tag{1c}$$
$$S = \{v_1^s, v_2^s, ..., v_{|S|}^s\}, s \in S \tag{1d}$$
$$R = \{CPU, RAM, Storage\}, r \in R \tag{1e}$$

σ_c^r is total capacity of resource $r \in R$ on node $c \in C$. and also σ_f^r and σ_e^r are total capcity of resource $r \in R$ on nodes $f \in F$ and $e \in E$ respectively.

T is set of tasks.

$$T = \{t_1, t_2, ..., t_{|T|}\} \tag{2a}$$

Each task expresses as follows:

$$t \in T => t = (w_t, \delta_t, N_t, f_t^r(\lambda_t)) \tag{3}$$

w_t shows computation workload of the task. δ_t is completion deadline of the task and N_t determines the maximum number of instaces of task $t \in T$.

π_c is unit price of processing in node $c \in C$ and also π_f and π_e are the ralated prices in nodes $f \in F$ and $e \in E$ respectively.

Transmition delays that show required time for trasmiting packets from sensors to each computational node are defined as follows:

$\tau_{s,c}^{tr}$ = trasmition delay between node $s \in S$ and $c \in C$
$\tau_{s,f}^{tr}$ = trasmition delay between node $s \in S$ and $f \in F$
$\tau_{s,e}^{tr}$ = trasmition delay between node $s \in S$ and $e \in E$

B. Variables

We define three integer variables for allocating tasks between nodes.

$$x_{t,c} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } c \in C \\ 0 & \text{o.w.} \end{cases} \tag{4a}$$

$$x_{t,f} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } f \in F \\ 0 & \text{o.w.} \end{cases} \tag{4b}$$

$$x_{t,e} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } e \in E \\ 0 & \text{o.w.} \end{cases} \tag{4c}$$

there are two continouse variables:

$$\lambda_{t,s} = \text{poisson rate of task } t \in T \text{ generated by node } s \in S \tag{5a}$$

$$0 \le \beta_{t,s,c} \le \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{5b}$$
$$0 \le \beta_{t,s,f} \le \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{5c}$$
$$0 \le \beta_{t,s,e} \le \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{5d}$$

$$\beta_{t,s,c} = \text{size of flow of task } t \in T \text{ from node } s \in S \text{ to node } c \in C \tag{5e}$$

C. Constraints

$$\lambda_{t,c} = \sum_{s \in S} \beta_{t,s,c} \quad \forall t \in T, \forall c \in C \tag{6a}$$
$$\lambda_{t,f} = \sum_{s \in S} \beta_{t,s,f} \quad \forall t \in T, \forall f \in F \tag{6b}$$
$$\lambda_{t,e} = \sum_{s \in S} \beta_{t,s,e} \quad \forall t \in T, \forall e \in E \tag{6c}$$

$$\frac{\lambda_{t,c}}{\sum_{s \in S} \lambda_{t,s}} \le x_{t,c} \quad \forall t \in T, \forall c \in C \tag{7a}$$
$$\frac{\lambda_{t,f}}{\sum_{s \in S} \lambda_{t,s}} \le x_{t,f} \quad \forall t \in T, \forall f \in F \tag{7b}$$
$$\frac{\lambda_{t,e}}{\sum_{s \in S} \lambda_{t,s}} \le x_{t,e} \quad \forall t \in T, \forall e \in E \tag{7c}$$

$$\lambda_{t,s} = \sum_{e\in E}\beta_{t,s,e} + \sum_{f\in F}\beta_{t,s,f} + \sum_{c\in C}\beta_{t,s,c} \quad \forall t\in T, \forall s\in S \tag{8a}$$

$$\lambda_{t,s} \le \sum_{e\in E}\beta_{t,s,e} + \sum_{f\in F}\beta_{t,s,f} + \sum_{c\in C}\beta_{t,s,c} \le \lambda_{t,s} + \epsilon \quad \forall t\in T, \forall s\in S \tag{8b}$$

$$\sum_{t\in T} x_{t,c} f_t^r(\lambda_{t,c}) \le \sigma_c^r \quad \forall r\in R, \forall c\in C \tag{9a}$$

$$x_{t,c} f_t^r(\lambda_{t,c}) = k_1^r x_{t,c}\lambda_{t,c} + k_2^r x_{t,c} \tag{9b}$$

$$\psi_{t,c} \triangleq x_{t,c}\lambda_{t,c} \Rightarrow 0 \le \psi_{t,c} \le \lambda_{t,c} \tag{9c}$$

$$Q(x_{t,c}-1) + \lambda_{t,c} \le \psi_{t,c} \le x_{t,c} Q \tag{9d}$$

$$\begin{aligned} Q &= \max_{t\in T, c\in C}\lambda_{t,c} \\ &= \max_{t\in T, c\in C}\sum_{s\in S}\beta_{t,s,c} \\ &= \sum_{s\in S}\max_{t\in T, c\in C}\beta_{t,s,c} \\ &= \sum_{s\in S}\lambda_{t,s} \end{aligned} \tag{9e}$$

$$0 \le \psi_{t,c} \le \lambda_{t,c} \tag{10a}$$

$$Q(x_{t,c}-1) + \lambda_{t,c} \le \psi_{t,c} \le x_{t,c} Q \quad \forall t\in T, \forall c\in C \tag{10b}$$

$$0 \le \psi_{t,f} \le \lambda_{t,f} \tag{10c}$$

$$Q(x_{t,f}-1) + \lambda_{t,f} \le \psi_{t,f} \le x_{t,f} Q \quad \forall t\in T, \forall f\in F \tag{10d}$$

$$0 \le \psi_{t,e} \le \lambda_{t,e} \tag{10e}$$

$$Q(x_{t,e}-1) + \lambda_{t,e} \le \psi_{t,e} \le x_{t,e} Q \quad \forall t\in T, \forall e\in E \tag{10f}$$

$$\sum_{t\in T} k_1^r\psi_{t,c} + k_2^r x_{t,c} \le \sigma_c^r \quad \forall r\in R, \forall c\in C \tag{11a}$$

$$\sum_{t\in T} k_1^r\psi_{t,f} + k_2^r x_{t,f} \le \sigma_f^r \quad \forall r\in R, \forall f\in F \tag{11b}$$

$$\sum_{t\in T} k_1^r\psi_{t,e} + k_2^r x_{t,e} \le \sigma_e^r \quad \forall r\in R, \forall e\in E \tag{11c}$$

$$\tau_{t,c} = \tau_{t,s,c}^{tr} + \frac{1}{\mu_{t,c} - \lambda_{t,c}} \tag{12a}$$

We have:

$$\frac{1}{\mu_{t,c}} = \frac{w_t}{f_t^{cpu}(\lambda_{t,c})} \tag{12b}$$

$$f_t^{cpu}(\lambda_{t,c}) = k_1^{cpu}\lambda_{t,c} + k_2^{cpu} \tag{12c}$$

$$\begin{aligned} &\Rightarrow x_{t,c}\tau_{t,c} = x_{t,c}(\tau_{t,s,c}^{tr} + \frac{w_t}{(k_1^{cpu} - w_t)\lambda_{t,c} + k_2^{cpu}}) \\ &\le \delta_t \quad \forall t\in T, \forall s\in S, \forall c\in C \end{aligned} \tag{12d}$$

$$\begin{aligned} &x_{t,c}\lambda_{t,c}(k_1^{cpu} - w_t)\tau_{t,s,c}^{tr} + \\ &x_{t,c}k_2^{cpu}\tau_{t,s,c}^{tr} + w_t x_{t,c} - k_2^{cpu}\delta_t \\ &-(k_1^{cpu} - w_t)\delta_t\lambda_{t,c} \le 0 \quad \forall t\in T, \forall s\in S, \forall c\in C \end{aligned} \tag{12e}$$

$$\begin{aligned} &\psi_{t,s,c}(k_1^{cpu} - w_t)\tau_{t,s,c}^{tr} + \\ &x_{t,c}k_2^{cpu}\tau_{t,s,c}^{tr} + w_t x_{t,c} - k_2^{cpu}\delta_t \\ &-(k_1^{cpu} - w_t)\delta_t\lambda_{t,c} \le 0 \end{aligned} \tag{13a}$$

$$\begin{aligned} &\psi_{t,s,f}(k_1^{cpu} - w_t)\tau_{t,s,f}^{tr} + \\ &x_{t,f}k_2^{cpu}\tau_{t,s,f}^{tr} + w_t x_{t,f} - k_2^{cpu}\delta_t \\ &-(k_1^{cpu} - w_t)\delta_t\lambda_{t,f} \le 0 \end{aligned} \tag{13b}$$

$$\begin{aligned} &\psi_{t,s,e}(k_1^{cpu} - w_t)\tau_{t,s,e}^{tr} + \\ &x_{t,e}k_2^{cpu}\tau_{t,s,e}^{tr} + w_t x_{t,e} - k_2^{cpu}\delta_t \\ &-(k_1^{cpu} - w_t)\delta_t\lambda_{t,e} \le 0 \end{aligned} \tag{13c}$$

$$1 \le \sum_{e\in E} x_{t,e} + \sum_{f\in F} x_{t,f} + \sum_{c\in C} x_{t,c} \le N_t \quad \forall t\in T \tag{14a}$$

$$x_{t,c}(\lambda_{t,c} < \mu_{t,c}) => x_{t,c}(\lambda_{t,c} + \epsilon \le \mu_{t,c}) \tag{15a}$$

$$x_{t,c}\lambda_{t,c} = \lambda_{t,c} \tag{15b}$$

$$=> \epsilon x_{t,c} - k_1^{cpu}\lambda_{t,c} - k_2^{cpu} + w_t\lambda_{t,c} \le 0 \quad \forall t\in T, \forall c\in C \tag{15c}$$

$$\epsilon x_{t,f} - k_1^{cpu}\lambda_{t,f} - k_2^{cpu} + w_t\lambda_{t,f} \le 0 \quad \forall t\in T, \forall f\in F \tag{15d}$$

$$\epsilon x_{t,e} - k_1^{cpu}\lambda_{t,e} - k_2^{cpu} + w_t\lambda_{t,e} \le 0 \quad \forall t\in T, \forall e\in E \tag{15e}$$

$$\begin{aligned} \min &\sum_{t\in T}\sum_{e\in E}(x_{t,e}\pi_e\sum_{r\in R} f_t^r(\lambda_{t,e})) \\ &+ \sum_{t\in T}\sum_{f\in F}(x_{t,f}\pi_f\sum_{r\in R} f_t^r(\lambda_{t,f})) \\ &+ \sum_{t\in T}\sum_{c\in C}(x_{t,c}\pi_c\sum_{r\in R} f_t^r(\lambda_{t,c})) \end{aligned} \tag{16a}$$

$$\begin{aligned} \min &\sum_{t\in T}\sum_{e\in E} x_{t,e}\Gamma_{t,e} \\ &+ \sum_{t\in T}\sum_{f\in F} x_{t,f}\Gamma_{t,f} \\ &+ \sum_{t\in T}\sum_{c\in C} x_{t,c}\Gamma_{t,c} \end{aligned} \tag{16b}$$

$$\begin{aligned} \Gamma_{t,e} &= \pi_e((k_1^{cpu} + k_1^{ram} + k_1^{storage})\lambda_{t,e} \\ &+ k_2^{cpu} + k_2^{ram} + k_2^{storage}) \\ &= \pi_e(K_1\lambda_{t,e} + K_2) \end{aligned} \tag{16c}$$

$$\begin{aligned} x_{t,e}\Gamma_{t,e} &= K_1\pi_e x_{t,e}\lambda_{t,e} + K_2\pi_e x_{t,e} \\ &= K_1\pi_e\psi_{t,e} + K_2\pi_e x_{t,e} \end{aligned} \tag{16d}$$

$$
\begin{aligned}
\min \sum_{t\in T}\sum_{e\in E} & K_1\pi_e\psi_{t,e}+K_2\pi_e x_{t,e} \\
\sum_{t\in T}\sum_{f\in F} & K_1\pi_f\psi_{t,f}+K_2\pi_f x_{t,f} \\
\sum_{t\in T}\sum_{c\in C} & K_1\pi_c\psi_{t,c}+K_2\pi_c x_{t,c}
\end{aligned} \tag{17a}
$$

$$
\begin{aligned}
L(\underline{\underline{x}},\underline{\underline{\beta}},\underline{\eta_1},\underline{\eta_2},\underline{\underline{\nu}}) &= \sum_{t\in T}\sum_{e\in E} x_{t,e}\Gamma_{t,e} \\
&+\sum_{t\in T}\sum_{f\in F} x_{t,f}\Gamma_{t,f}+\sum_{t\in T}\sum_{c\in C} x_{t,c}\Gamma_{t,c} \\
&+\sum_{t\in T}\eta_{1,t}(1-\sum_{e\in E}x_{t,e}+\sum_{f\in F}x_{t,f}+\sum_{c\in C}x_{t,c}) \\
&+\sum_{t\in T}\eta_{2,t}(\sum_{e\in E}x_{t,e}+\sum_{f\in F}x_{t,f}+\sum_{c\in C}x_{t,c}-N_t) \\
&+\sum_{t\in T}\sum_{s\in S}\nu_{t,s}(\lambda_{t,s}-\sum_{e\in E}\beta_{t,s,e}+\sum_{f\in F}\beta_{t,s,f}+\sum_{c\in C}\beta_{t,s,c})
\end{aligned} \tag{18a}
$$

$$
\begin{aligned}
L &= \sum_{e\in E}\sum_{t\in T}(\sum_{s\in S}(\nu_{t,s}\beta_{t,s,e}+\frac{\nu_{t,s}\lambda_{t,s}}{3|E|}) \\
&+x_{t,e}(\Gamma_{t,e}-\eta_{1,t}+\eta_{2,t})+\frac{\eta_{1,t}-N_t\eta_{2,t}}{3|E|}) \\
&+\sum_{f\in F}\sum_{t\in T}(\sum_{s\in S}(\nu_{t,s}\beta_{t,s,f}+\frac{\nu_{t,s}\lambda_{t,s}}{3|F|}) \\
&+x_{t,f}(\Gamma_{t,f}-\eta_{1,t}+\eta_{2,t})+\frac{\eta_{1,t}-N_t\eta_{2,t}}{3|F|}) \\
&+\sum_{c\in C}\sum_{t\in T}(\sum_{s\in S}(\nu_{t,s}\beta_{t,s,c}+\frac{\nu_{t,s}\lambda_{t,s}}{3|C|}) \\
&+x_{t,c}(\Gamma_{t,c}-\eta_{1,t}+\eta_{2,t})+\frac{\eta_{1,t}-N_t\eta_{2,t}}{3|C|})
\end{aligned} \tag{19a}
$$

$$
L=\sum_{e\in E}L_e+\sum_{f\in F}L_f+\sum_{c\in C}L_c \tag{19b}
$$

$$
\begin{aligned}
g(\underline{\eta_1},\underline{\eta_2},\underline{\underline{\nu}}) &= \inf_{\underline{\underline{x}},\underline{\underline{\beta}}} L(\underline{\underline{x}},\underline{\underline{\beta}},\underline{\eta_1},\underline{\eta_2},\underline{\underline{\nu}}) \\
&= \sum_{e\in E}\inf_{\underline{\underline{x_e}},\underline{\underline{\beta_e}}} L_e(\underline{\underline{x_e}},\underline{\underline{\beta_e}},\underline{\eta_1},\underline{\eta_2},\underline{\underline{\nu}}) \\
&+\sum_{f\in F}\inf_{\underline{\underline{x_f}},\underline{\underline{\beta_f}},} L_f(\underline{\underline{x_f}},\underline{\underline{\beta_f}},\underline{\eta_1},\underline{\eta_2},\underline{\underline{\nu}}) \\
&+\sum_{c\in C}\inf_{\underline{\underline{x_c}},\underline{\underline{\beta_c}},} L_c(\underline{\underline{x_c}},\underline{\underline{\beta_c}},\underline{\eta_1},\underline{\eta_2},\underline{\underline{\nu}}) \\
&= \sum_{e\in E} g_e(\underline{\eta_1},\underline{\eta_2},\underline{\underline{\nu}}) \\
&+\sum_{f\in F} g_f(\underline{\eta_1},\underline{\eta_2},\underline{\underline{\nu}}) \\
&+\sum_{c\in C} g_c(\underline{\eta_1},\underline{\eta_2},\underline{\underline{\nu}})
\end{aligned} \tag{20a}
$$

$$
\underline{\underline{x_e}}^{(k+1)},\underline{\underline{\beta_e}}^{(k+1)}=arg\min_{\underline{\underline{x_e}},\underline{\underline{\beta_e}}} L_e(\underline{\underline{x_e}},\underline{\underline{\beta_e}},\underline{\eta_1}^{(k)},\underline{\eta_2}^{(k)},\underline{\underline{\nu}}^{(k)}) \tag{21a}
$$

$$
\underline{\underline{x_f}}^{(k+1)},\underline{\underline{\beta_f}}^{(k+1)}=arg\min_{\underline{\underline{x_f}},\underline{\underline{\beta_f}}} L_f(\underline{\underline{x_f}},\underline{\underline{\beta_f}},\underline{\eta_1}^{(k)},\underline{\eta_2}^{(k)},\underline{\underline{\nu}}^{(k)}) \tag{21b}
$$

$$
\underline{\underline{x_c}}^{(k+1)},\underline{\underline{\beta_c}}^{(k+1)}=arg\min_{\underline{\underline{x_c}},\underline{\underline{\beta_c}}} L_c(\underline{\underline{x_c}},\underline{\underline{\beta_c}},\underline{\eta_1}^{(k)},\underline{\eta_2}^{(k)},\underline{\underline{\nu}}^{(k)}) \tag{21c}
$$

$$
\eta_{1,t}^{(k+1)}=\eta_{1,t}^{(k)}+\alpha^{(k)}(1-\sum_{e\in E}x_{t,e}^{(k+1)}+\sum_{f\in F}x_{t,f}^{(k+1)}+\sum_{c\in C}x_{t,c}^{(k+1)}) \tag{22a}
$$

$$
\eta_{2,t}^{(k+1)}=\eta_{2,t}^{(k)}+\alpha^{(k)}(\sum_{e\in E}x_{t,e}^{(k+1)}+\sum_{f\in F}x_{t,f}^{(k+1)}+\sum_{c\in C}x_{t,c}^{(k+1)}-N_t) \tag{22b}
$$

$$
\nu_{t,s}^{(k+1)}=\nu_{t,s}^{(k)}+\alpha^{(k)}(\lambda_{t,s}-\sum_{e\in E}\beta_{t,s,e}^{(k+1)}+\sum_{f\in F}\beta_{t,s,f}^{(k+1)}+\sum_{c\in C}\beta_{t,s,c}^{(k+1)}) \tag{22c}
$$

D. Objective

$$
\begin{aligned}
p_k &= \sum_{i=1}^{l_e} x_{k,i}^{e} C(v_i^e,t_k) \\
&+\sum_{j=1}^{l_f} x_{k,j}^{f} C(v_j^f,t_k) \\
&+\sum_{h=1}^{l_c} x_{k,h}^{c} C(v_h^c,t_k)
\end{aligned} \tag{23a}
$$

$$
\min \sum_{k=1}^{l_t} p_k \tag{24a}
$$

subject to: 9

E. Solution

We can reshape main problem as following:

$$\min(\sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e \tag{25a}$$
$$+\sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c)$$

subject to:

$$\sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c \leq \delta_k \quad \forall k \in \{1,...,l_t\}$$
$$\sum_{k=1}^{l_t} x_{k,h}^c w_k \leq c_h^c \qquad \forall h \in \{1,2,...,l_c\}$$
$$\sum_{k=1}^{l_t} x_{k,j}^f w_k \leq c_j^f \qquad \forall j \in \{1,2,...,l_f\}$$
$$\sum_{k=1}^{l_t} x_{k,i}^e w_k \leq c_i^e \qquad \forall i \in \{1,2,...,l_e\}$$
$$\sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c = 1 \qquad \forall k \in \{1,2,...,l_t\}$$

We define u^m for each computational agent m, that is a matrix with size $l_t * (l_e + l_f + l_c)$. It is the local copy of all variables in agent m. i.e. $u_{k,i}^{e,m}$ is the copy of variable $x_{k,i}^e$ in agent m for $m = 1,..,(l_m = l_e + l_f + l_c)$. So we should add new constraint $u^m = z \quad \forall m$ to main problem. We will use admm on this new constraint so:

$$L_p = \sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e + \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x_{k,h}^c C_{k,h}^c \tag{26a}$$
$$+\sum_{m=1}^{l_m} \nu^m * (u^m - z) + \sum_{m=1}^{l_m} \frac{\rho}{2}\|u^m - z\|^2$$

We can seperate augmented lagrangian for each computational agent m then:

$$L_p^m = \sum_{k=1}^{l_t} u_{k,m}^m C_{k,m} + \nu^m * (u^m - z) + \frac{\rho}{2}\|u^m - z\|^2 \tag{27a}$$
$$\forall m \in \{1,2,...,l_m\}$$

So we can write the algorithm as following:

for each iteration k (28a)

1. $u^{m,(k+1)} = arg\min L_p^m(u^m, z^{(k)}, \nu^{m,(k)}) =$
$$\sum_{k=1}^{l_t} u_{k,m}^m C_{k,m} + \nu^{m,(k)} * (u^m - z^{(k)}) + \frac{\rho}{2}\|u^m - z^{(k)}\|^2$$
subject to:
$$\sum_{k=1}^{l_t} u_{k,m}^m w_k \leq c^m$$
$$\sum_{i=1}^{l_e} u_{k,i}^{e,m} + \sum_{j=1}^{l_f} u_{k,j}^{f,m} + \sum_{h=1}^{l_c} u_{k,h}^{c,m} = 1 \qquad \forall k \in \{1,2,...,l_t\}$$
$$\sum_{i=1}^{l_e} u_{k,i}^{e,m} \tau_{k,i}^e + \sum_{j=1}^{l_f} u_{k,j}^{f,m} \tau_{k,j}^f + \sum_{h=1}^{l_c} u_{k,h}^{c,m} \tau_{k,h}^c \leq \delta_k \qquad \forall k \in \{1,2,...,l_t\}$$
2. $z^{(k+1)} = \bar{u}^{(k+1)} + \frac{1}{\rho}\bar{\nu}^{(k)}$
3. $\nu^{m,(k+1)} = \nu^{m,(k)} + \rho(u^{m,(k+1)} - z^{(k+1)})$

F. Solution 2

lagrangian of main problem is as following

$$L(x^e, x^f, x^c, \lambda, \nu) = \sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x_{k,i}^e C_{k,i}^e + \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x_{k,j}^f C_{k,j}^f + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x_{k,h}^c C \tag{29a}$$
$$+\sum_{k=1}^{l_t} \lambda_k (\sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c - \delta_k)$$
$$+\sum_{k=1}^{l_t} \nu_k (\sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c - 1)$$

So we can decompose the lagrangian as follows

$$L(x^e, x^f, x^c, \lambda, \nu) = \tag{30a}$$
$$\sum_{i=1}^{l_e}\sum_{k=1}^{l_t} (x_{k,i}^e C_{k,i}^e + \lambda_k x_{k,i}^e \tau_{k,i}^e + \nu_k x_{k,i}^e - \frac{\lambda_k \delta_k + \nu_k}{3l_e})$$
$$+\sum_{j=1}^{l_f}\sum_{k=1}^{l_t} (x_{k,j}^f C_{k,j}^f + \lambda_k x_{k,j}^f \tau_{k,j}^f + \nu_k x_{k,j}^f - \frac{\lambda_k \delta_k + \nu_k}{3l_f})$$
$$+\sum_{h=1}^{l_c}\sum_{k=1}^{l_t} (x_{k,h}^c C_{k,h}^c + \lambda_k x_{k,h}^c \tau_{k,h}^c + \nu_k x_{k,h}^c - \frac{\lambda_k \delta_k + \nu_k}{3l_c})$$

$$L(x^e, x^f, x^c, \lambda, \nu) = \sum_{i=1}^{l_e} L_i^e(x_i^e, \lambda, \nu) \tag{31a}$$
$$+ \sum_{j=1}^{l_f} L_j^f(x_j^f, \lambda, \nu)$$
$$+ \sum_{h=1}^{l_c} L_h^c(x_h^c, \lambda, \nu)$$

$$g(\lambda, \nu) = \inf_{x^e, x^f, x^c} L(x^e, x^f, x^c, \lambda, \nu) \tag{32a}$$
$$= \sum_{i=1}^{l_e} \inf_{x_i^e} L_i^e(x_i^e, \lambda, \nu)$$
$$+ \sum_{j=1}^{l_f} \inf_{x_j^f} L_j^f(x_j^f, \lambda, \nu)$$
$$+ \sum_{h=1}^{l_c} \inf_{x_h^c} L_h^c(x_h^c, \lambda, \nu)$$
$$= \sum_{i=1}^{l_e} g_i^e(\lambda, \nu)$$
$$+ \sum_{j=1}^{l_f} g_j^f(\lambda, \nu)$$
$$+ \sum_{h=1}^{l_c} g_h^c(\lambda, \nu)$$

$$\lambda_k^+ = \lambda_k^- + \alpha\Big(\sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c - \delta_k\Big) \tag{33a}$$

$$\nu_k^+ = \nu_k^- + \alpha\Big(\sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c - 1\Big) \tag{33b}$$

REFERENCES

Algorithm Test Algorithm

for $n = 1 : L_v$ **do**
  Determine the set of states Z_n
  $RemovedStates =$
  for $j \in Z_n$ **do**
    Determine the index of computational node l and the index of task t and the index of part u
    $XP_n^j = Z_{n-1}$
    if $j \neq 0$ **then**
      for $i \in XP_n^j$ **do**
        if $ServerResource < 0$ **then**
          $XP_n^j = XP_n^j - \{i\}$
        end if
      end for
    end if
    if $XP_n^j \neq \emptyset$ **then**
      for $i \in XP_n^j$ **do**
        Calculate $T_{n-1,n}^{i,j}$
      end for
      Calculate I_n^j and Λ_n^j
      Calculate ϕ_n^j
      if $j \neq 0$ **then**
        $ServerResource < 0$
      end if
    else
      $RemovedStates = RemovedStates + \{j\}$
    end if
  end for
  if $RemovedStates = Z_n$ **then**
    Set $H = \sum_{m=1}^{t} N_m$ and $ResourceIndicator = 0$
    Determine the Viterbi path P with H
    break
  else
    Remove all states in the $RemovedStates$ from the Z_n
  end if
end for
if $ResourceIndicator = 1$ **then**
  Determine the Viterbi path P with H
end if
Determine the task scheduling using Viterbi path P