# Paper Title*

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***Abstract*—**
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## I. Introduction

## II. System Model

### A. Graph Model

$V^c$ is set of cloud nodes. $V^f$ is set of fog nodes. $V^e$ is set of edge nodes. $V^s$ is set of sensor nodes.

$$V^c = \{v^c_1, v^c_2, ..., v^c_{l_c}\} => |V^c| = l_c \tag{1a}$$
$$V^f = \{v^f_1, v^f_2, ..., v^f_{l_f}\} => |V^f| = l_f \tag{1b}$$
$$V^e = \{v^e_1, v^e_2, ..., v^e_{l_e}\} => |V^e| = l_e \tag{1c}$$
$$V^s = \{v^s_1, v^s_2, ..., v^s_{l_s}\} => |V^s| = l_s \tag{1d}$$

$C^c$ is set of capacity of cloud nodes. $C^f$ is set of capacity of fog nodes. $C^e$ is set of capacity of edge nodes.

$$C^c = \{c^c_1, c^c_2, ..., c^c_{l_c}\} \tag{2a}$$
$$C^f = \{c^f_1, c^f_2, ..., c^f_{l_f}\} \tag{2b}$$
$$C^e = \{c^e_1, c^e_2, ..., c^e_{l_e}\} \tag{2c}$$

$T$ is set of tasks.

$$T = \{t_1, t_2, ..., t_{l_t}\} => |T| = l_t \tag{3a}$$
$$\tag{3b}$$

Each task expresses as follows:

$$t_k \in T => t_k = (w_k, \delta_k, o_k) \tag{4}$$

$w_k$ shows computation workload of task. $\delta_k$ is completion deadline of task and $o_k$ determines the owner of task.

Price unit sets are defined as following:

$$P^c = \{p^c_1, p^c_2, ..., p^c_{l_c}\} \tag{5a}$$
$$P^f = \{p^f_1, p^f_2, ..., p^f_{l_f}\} \tag{5b}$$
$$P^e = \{p^e_1, p^e_2, ..., p^e_{l_e}\} \tag{5c}$$

$P^c$, $P^f$ and $P^e$ are unit price set of using cloud nodes, fog nodes and edge nodes respectively.

Transmition delay sets that show required time for trasmiting packets from sensors to each computational node are defined as follows:

$$T^{c,tr} = \{\tau^{c,tr}_1, \tau^{c,tr}_2, ..., \tau^{c,tr}_{l_c}\} \tag{6a}$$
$$T^{f,tr} = \{\tau^{f,tr}_1, \tau^{f,tr}_2, ..., \tau^{f,tr}_{l_f}\} \tag{6b}$$
$$T^{e,tr} = \{\tau^{e,tr}_1, \tau^{e,tr}_2, ..., \tau^{e,tr}_{l_e}\} \tag{6c}$$

We define execution rate for each computational node as follows:

$$R^c = \{r^c_1, r^c_2, ..., r^c_{l_c}\} \tag{7a}$$
$$R^f = \{r^f_1, r^f_2, ..., r^f_{l_f}\} \tag{7b}$$
$$R^e = \{r^e_1, r^e_2, ..., r^e_{l_e}\} \tag{7c}$$

### B. Variables

We define three integer variables for allocating tasks between nodes.

$$x^c_{k,h} = \begin{cases} 1 & \text{task } k \text{ is allocated to cloud node } h \\ 0 & \text{o.w.} \end{cases} \tag{8a}$$

$$x^f_{k,j} = \begin{cases} 1 & \text{task } k \text{ is allocated to fog node } j \\ 0 & \text{o.w.} \end{cases} \tag{8b}$$

$$x^e_{k,i} = \begin{cases} 1 & \text{task } k \text{ is allocated to edge node } i \\ 0 & \text{o.w.} \end{cases} \tag{8c}$$

$$\begin{aligned} \tau_k = & \sum_{i=1}^{l_e} x^e_{k,i}(\tau^{e,tr}_i + \frac{w_k}{r^e_i}) \\ & + \sum_{j=1}^{l_f} x^f_{k,j}(\tau^{f,tr}_j + \frac{w_k}{r^f_j}) \\ & + \sum_{h=1}^{l_c} x^c_{k,h}(\tau^{c,tr}_k + \frac{w_k}{r^c_h}) \end{aligned} \tag{8d}$$

### C. Constraints

$$\tau_k \leq \delta_k \qquad \forall k \in \{1, 2, ..., l_t\} \tag{9a}$$

$$\sum_{k=1}^{l_t} x^c_{k,h} w_k \le c^c_h \qquad \forall h \in \{1,2,...,l_c\} \tag{9b}$$

$$\sum_{k=1}^{l_t} x^f_{k,j} w_k \le c^f_j \qquad \forall j \in \{1,2,...,l_f\} \tag{9c}$$

$$\sum_{k=1}^{l_t} x^e_{k,i} w_k \le c^e_i \qquad \forall i \in \{1,2,...,l_e\} \tag{9d}$$

$$\sum_{i=1}^{l_e} x^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h} = 1 \qquad \forall k \in \{1,2,...,l_t\} \tag{9e}$$

### D. Objective

$$p_k = \sum_{i=1}^{l_e} x^e_{k,i} C(v^e_i, t_k) \tag{10a}$$
$$+ \sum_{j=1}^{l_f} x^f_{k,j} C(v^f_j, t_k)$$
$$+ \sum_{h=1}^{l_c} x^c_{k,h} C(v^c_h, t_k)$$

$$\min \sum_{k=1}^{l_t} p_k \tag{11a}$$

subject to: 9

### E. Solution

We can reshape main problem as following:

$$\min(\sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x^e_{k,i} C^e_{k,i} \tag{12a}$$
$$+ \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x^f_{k,j} C^f_{k,j} + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x^c_{k,h} C^c_{k,h})$$

subject to:

$$\sum_{i=1}^{l_e} x^e_{k,i}\tau^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j}\tau^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h}\tau^c_{k,h} \le \delta_k \quad \forall k \in \{1,...,l_t\}$$

$$\sum_{k=1}^{l_t} x^c_{k,h} w_k \le c^c_h \qquad \forall h \in \{1,2,...,l_c\}$$

$$\sum_{k=1}^{l_t} x^f_{k,j} w_k \le c^f_j \qquad \forall j \in \{1,2,...,l_f\}$$

$$\sum_{k=1}^{l_t} x^e_{k,i} w_k \le c^e_i \qquad \forall i \in \{1,2,...,l_e\}$$

$$\sum_{i=1}^{l_e} x^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h} = 1 \qquad \forall k \in \{1,2,...,l_t\}$$

We define $u^m$ for each computational agent $m$, that is a matrix with size $l_t * (l_e + l_f + l_c)$. It is the local copy of all variables in agent $m$. i.e. $u^{e,m}_{k,i}$ is the copy of variable $x^e_{k,i}$ in agent $m$ for $m = 1,..,(l_m = l_e + l_f + l_c)$. So we should add new constraint $u^m = z \quad \forall m$ to main problem. We will use admm on this new constraint so:

$$L_p = \sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x^e_{k,i} C^e_{k,i} + \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x^f_{k,j} C^f_{k,j} + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x^c_{k,h} C^c_{k,h} \tag{13a}$$
$$+ \sum_{m=1}^{l_m} \nu^m * (u^m - z) + \sum_{m=1}^{l_m} \frac{\rho}{2}\|u^m - z\|^2$$

We can seperate augmented lagrangian for each computational agent $m$ then:

$$L^m_p = \sum_{k=1}^{l_t} u^m_{k,m} C_{k,m} + \nu^m * (u^m - z) + \frac{\rho}{2}\|u^m - z\|^2 \tag{14a}$$
$$\forall m \in \{1,2,...,l_m\}$$

So we can write the algorithm as following:

for each iteration k (15a)

1. $u^{m,(k+1)} = arg\min L^m_p(u^m, z^{(k)}, \nu^{m,(k)}) =$

$$\sum_{k=1}^{l_t} u^m_{k,m} C_{k,m} + \nu^{m,(k)} * (u^m - z^{(k)}) + \frac{\rho}{2}\|u^m - z^{(k)}\|^2$$

subject to:

$$\sum_{k=1}^{l_t} u^m_{k,m} w_k \le c^m$$

$$\sum_{i=1}^{l_e} u^{e,m}_{k,i} + \sum_{j=1}^{l_f} u^{f,m}_{k,j} + \sum_{h=1}^{l_c} u^{c,m}_{k,h} = 1 \qquad \forall k \in \{1,2,...,l_t\}$$

$$\sum_{i=1}^{l_e} u^{e,m}_{k,i}\tau^e_{k,i} + \sum_{j=1}^{l_f} u^{f,m}_{k,j}\tau^f_{k,j} + \sum_{h=1}^{l_c} u^{c,m}_{k,h}\tau^c_{k,h} \le \delta_k \qquad \forall k \in \{1,2,...,l_t\}$$

2. $z^{(k+1)} = \bar{u}^{(k+1)} + \frac{1}{\rho}\bar{\nu}^{(k)}$

3. $\nu^{m,(k+1)} = \nu^{m,(k)} + \rho(u^{m,(k+1)} - z^{(k+1)})$

### F. Solution 2

lagrangian of main problem is as following

$$L(x^e, x^f, x^c, \lambda, \nu) = \sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x^e_{k,i} C^e_{k,i} + \sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x^f_{k,j} C^f_{k,j} + \sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x^c_{k,h} C \tag{16a}$$
$$+ \sum_{k=1}^{l_t} \lambda_k (\sum_{i=1}^{l_e} x^e_{k,i}\tau^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j}\tau^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h}\tau^c_{k,h} - \delta_k)$$
$$+ \sum_{k=1}^{l_t} \nu_k (\sum_{i=1}^{l_e} x^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h} - 1)$$

So we can decompose the lagrangian as follows

$$L(x^e, x^f, x^c, \lambda, \nu) = \tag{17a}$$

$$\sum_{i=1}^{l_e}\sum_{k=1}^{l_t}(x^e_{k,i}C^e_{k,i} + \lambda_k x^e_{k,i}\tau^e_{k,i} + \nu_k x^e_{k,i} - \frac{\lambda_k\delta_k + \nu_k}{3l_e})$$

$$+\sum_{j=1}^{l_f}\sum_{k=1}^{l_t}(x^f_{k,j}C^f_{k,j} + \lambda_k x^f_{k,j}\tau^f_{k,j} + \nu_k x^f_{k,j} - \frac{\lambda_k\delta_k + \nu_k}{3l_f})$$

$$+\sum_{h=1}^{l_c}\sum_{k=1}^{l_t}(x^c_{k,h}C^c_{k,h} + \lambda_k x^c_{k,h}\tau^c_{k,h} + \nu_k x^c_{k,h} - \frac{\lambda_k\delta_k + \nu_k}{3l_c})$$

$$L(x^e, x^f, x^c, \lambda, \nu) = \sum_{i=1}^{l_e} L^e_i(x^e_i, \lambda, \nu) \tag{18a}$$

$$+\sum_{j=1}^{l_f} L^f_j(x^f_j, \lambda, \nu)$$

$$+\sum_{h=1}^{l_c} L^c_h(x^c_h, \lambda, \nu)$$

$$g(\lambda, \nu) = \inf_{x^e, x^f, x^c} L(x^e, x^f, x^c, \lambda, \nu) \tag{19a}$$

$$= \sum_{i=1}^{l_e} \inf_{x^e_i} L^e_i(x^e_i, \lambda, \nu)$$

$$+\sum_{j=1}^{l_f} \inf_{x^f_j} L^f_j(x^f_j, \lambda, \nu)$$

$$+\sum_{h=1}^{l_c} \inf_{x^c_h} L^c_h(x^c_h, \lambda, \nu)$$

$$= \sum_{i=1}^{l_e} g^e_i(\lambda, \nu)$$

$$+\sum_{j=1}^{l_f} g^f_j(\lambda, \nu)$$

$$+\sum_{h=1}^{l_c} g^c_h(\lambda, \nu)$$

$$\lambda^+_k = \lambda^-_k + \alpha(\sum_{i=1}^{l_e} x^e_{k,i}\tau^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j}\tau^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h}\tau^c_{k,h} - \delta_k) \tag{20a}$$

$$\nu^+_k = \nu^-_k + \alpha(\sum_{i=1}^{l_e} x^e_{k,i} + \sum_{j=1}^{l_f} x^f_{k,j} + \sum_{h=1}^{l_c} x^c_{k,h} - 1) \tag{20b}$$

**Algorithm 1**

**while** not converged **do**
 1. update $x^c, x^f, x^e$ using (19a)
 2. update lagrangian multipliers using (20a) and (21a)
 $\alpha = \frac{\alpha_0}{k}$ for each iteration $k$
**end while**

## REFERENCES