Paper Title*

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***Abstract*—**
***Index Terms*—**

I. Introduction

II. System Model

A. Graph Model

C is set of cloud nodes. F is set of fog nodes. E is set of edge nodes. S is set of sensor nodes. R is set of resources in each computational node(cloud, fog or edge node).

$$C = \{v_1^c, v_2^c, ..., v_{|C|}^c\}, c \in C \tag{1a}$$
$$F = \{v_1^f, v_2^f, ..., v_{|F|}^f\}, f \in F \tag{1b}$$
$$E = \{v_1^e, v_2^e, ..., v_{|E|}^e\}, e \in E \tag{1c}$$
$$S = \{v_1^s, v_2^s, ..., v_{|S|}^s\}, s \in S \tag{1d}$$
$$R = \{CPU, RAM, Storage\}, r \in R \tag{1e}$$

σ_c^r is total capacity of resource $r \in R$ on node $c \in C$. and also σ_f^r and σ_e^r are total capacity of resource $r \in R$ on nodes $f \in F$ and $e \in E$ respectively.

T is set of tasks.

$$T = \{t_1, t_2, ..., t_{|T|}\} \tag{2a}$$

Each task expresses as follows:

$$t \in T => t = (w_t, \delta_t, N_t, f_t^r(\lambda_t)) \tag{3}$$

w_t shows computation workload of the task. δ_t is completion deadline of the task and N_t determines the maximum number of instances of task $t \in T$.

π_c is unit price of processing in node $c \in C$ and also π_f and π_e are the related prices in nodes $f \in F$ and $e \in E$ respectively.

Transmission delays that show required time for transmitting packets from sensors to each computational node are defined as follows:
$\tau_{s,c}^{tr}$ = transmission delay between node $s \in S$ and $c \in C$
$\tau_{s,f}^{tr}$ = transmission delay between node $s \in S$ and $f \in F$
$\tau_{s,e}^{tr}$ = transmission delay between node $s \in S$ and $e \in E$

B. Variables

We define three integer variables for allocating tasks between nodes.

$$x_{t,c} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } c \in C \\ 0 & \text{o.w.} \end{cases} \tag{4a}$$

$$x_{t,f} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } f \in F \\ 0 & \text{o.w.} \end{cases} \tag{4b}$$

$$x_{t,e} = \begin{cases} 1 & \text{task } t \in T \text{ is allocated to node } e \in E \\ 0 & \text{o.w.} \end{cases} \tag{4c}$$

there are two continuous variables:

$$\lambda_{t,s} = \text{poisson rate of task } t \in T \text{ generated by node } s \in S \tag{5a}$$

$$0 \leq \beta_{t,s,c} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall c \in C \tag{5b}$$
$$0 \leq \beta_{t,s,f} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall f \in F \tag{5c}$$
$$0 \leq \beta_{t,s,e} \leq \lambda_{t,s} \quad \forall t \in T, \forall s \in S, \forall e \in E \tag{5d}$$

$$\beta_{t,s,c} = \text{size of flow of task } t \in T \text{ from node } s \in S \text{ to node } c \in C \tag{5e}$$

C. Constraints

$$\lambda_{t,c} = \sum_{s \in S} \beta_{t,s,c} \quad \forall t \in T, \forall c \in C \tag{6a}$$
$$\lambda_{t,f} = \sum_{s \in S} \beta_{t,s,f} \quad \forall t \in T, \forall f \in F \tag{6b}$$
$$\lambda_{t,e} = \sum_{s \in S} \beta_{t,s,e} \quad \forall t \in T, \forall e \in E \tag{6c}$$

$$\frac{\lambda_{t,c}}{\sum_{s \in S} \lambda_{t,s}} \leq x_{t,c} \quad \forall t \in T, \forall c \in C \tag{7a}$$
$$\frac{\lambda_{t,f}}{\sum_{s \in S} \lambda_{t,s}} \leq x_{t,f} \quad \forall t \in T, \forall f \in F \tag{7b}$$
$$\frac{\lambda_{t,e}}{\sum_{s \in S} \lambda_{t,s}} \leq x_{t,e} \quad \forall t \in T, \forall e \in E \tag{7c}$$

$$\lambda_{t,s} = \sum_{e\in E}\beta_{t,s,e} + \sum_{f\in F}\beta_{t,s,f} + \sum_{c\in C}\beta_{t,s,c} \quad \forall t\in T, \forall s\in S \tag{8a}$$

$$\lambda_{t,s} \le \sum_{e\in E}\beta_{t,s,e} + \sum_{f\in F}\beta_{t,s,f} + \sum_{c\in C}\beta_{t,s,c} \le \lambda_{t,s} + \epsilon \quad \forall t\in T, \forall s\in S \tag{8b}$$

$$\sum_{t\in T} x_{t,c} f_t^r(\lambda_{t,c}) \le \sigma_c^r \quad \forall r\in R, \forall c\in C \tag{9a}$$
$$x_{t,c} f_t^r(\lambda_{t,c}) = k_1^r x_{t,c}\lambda_{t,c} + k_2^r x_{t,c} \tag{9b}$$
$$\psi_{t,c} \triangleq x_{t,c}\lambda_{t,c} \Rightarrow 0 \le \psi_{t,c} \le \lambda_{t,c} \tag{9c}$$
$$Q(x_{t,c}-1) + \lambda_{t,c} \le \psi_{t,c} \le x_{t,c}Q \tag{9d}$$
$$\begin{aligned} Q &= \max_{t\in T, c\in C} \lambda_{t,c} \\ &= \max_{t\in T, c\in C} \sum_{s\in S}\beta_{t,s,c} \\ &= \sum_{s\in S}\max_{t\in T, c\in C}\beta_{t,s,c} \\ &= \sum_{s\in S}\lambda_{t,s} \end{aligned} \tag{9e}$$

$$0 \le \psi_{t,c} \le \lambda_{t,c} \tag{10a}$$
$$Q(x_{t,c}-1) + \lambda_{t,c} \le \psi_{t,c} \le x_{t,c}Q \quad \forall t\in T, \forall c\in C \tag{10b}$$
$$0 \le \psi_{t,f} \le \lambda_{t,f} \tag{10c}$$
$$Q(x_{t,f}-1) + \lambda_{t,f} \le \psi_{t,f} \le x_{t,f}Q \quad \forall t\in T, \forall f\in F \tag{10d}$$
$$0 \le \psi_{t,e} \le \lambda_{t,e} \tag{10e}$$
$$Q(x_{t,e}-1) + \lambda_{t,e} \le \psi_{t,e} \le x_{t,e}Q \quad \forall t\in T, \forall e\in E \tag{10f}$$

$$\sum_{t\in T} k_1^r\psi_{t,c} + k_2^r x_{t,c} \le \sigma_c^r \quad \forall r\in R, \forall c\in C \tag{11a}$$
$$\sum_{t\in T} k_1^r\psi_{t,f} + k_2^r x_{t,f} \le \sigma_f^r \quad \forall r\in R, \forall f\in F \tag{11b}$$
$$\sum_{t\in T} k_1^r\psi_{t,e} + k_2^r x_{t,e} \le \sigma_e^r \quad \forall r\in R, \forall e\in E \tag{11c}$$

$$\tau_{t,c} = \tau_{t,s,c}^{tr} + \frac{1}{\mu_{t,c} - \lambda_{t,c}} \tag{12a}$$

We have:

$$\frac{1}{\mu_{t,c}} = \frac{w_t}{f_t^{cpu}(\lambda_{t,c})} \tag{12b}$$
$$f_t^{cpu}(\lambda_{t,c}) = k_1^{cpu}\lambda_{t,c} + k_2^{cpu} \tag{12c}$$
$$\begin{aligned} &\Rightarrow x_{t,c}\tau_{t,c} = x_{t,c}(\tau_{t,s,c}^{tr} + \frac{w_t}{(k_1^{cpu} - w_t)\lambda_{t,c} + k_2^{cpu}}) \\ &\le \delta_t \quad \forall t\in T, \forall s\in S, \forall c\in C \end{aligned} \tag{12d}$$

$$\begin{aligned} &x_{t,c}\lambda_{t,c}(k_1^{cpu} - w_t)\tau_{t,s,c}^{tr} + \\ &x_{t,c}k_2^{cpu}\tau_{t,s,c}^{tr} + w_t x_{t,c} - k_2^{cpu}\delta_t \\ &-(k_1^{cpu} - w_t)\delta_t\lambda_{t,c} \le 0 \quad \forall t\in T, \forall s\in S, \forall c\in C \end{aligned} \tag{12e}$$

$$\begin{aligned} &\psi_{t,s,c}(k_1^{cpu} - w_t)\tau_{t,s,c}^{tr} + \\ &x_{t,c}k_2^{cpu}\tau_{t,s,c}^{tr} + w_t x_{t,c} - k_2^{cpu}\delta_t \\ &-(k_1^{cpu} - w_t)\delta_t\lambda_{t,c} \le 0 \end{aligned} \tag{13a}$$

$$\begin{aligned} &\psi_{t,s,f}(k_1^{cpu} - w_t)\tau_{t,s,f}^{tr} + \\ &x_{t,f}k_2^{cpu}\tau_{t,s,f}^{tr} + w_t x_{t,f} - k_2^{cpu}\delta_t \\ &-(k_1^{cpu} - w_t)\delta_t\lambda_{t,f} \le 0 \end{aligned} \tag{13b}$$

$$\begin{aligned} &\psi_{t,s,e}(k_1^{cpu} - w_t)\tau_{t,s,e}^{tr} + \\ &x_{t,e}k_2^{cpu}\tau_{t,s,e}^{tr} + w_t x_{t,e} - k_2^{cpu}\delta_t \\ &-(k_1^{cpu} - w_t)\delta_t\lambda_{t,e} \le 0 \end{aligned} \tag{13c}$$

$$1 \le \sum_{e\in E} x_{t,e} + \sum_{f\in F} x_{t,f} + \sum_{c\in C} x_{t,c} \le N_t \quad \forall t\in T \tag{14a}$$

$$x_{t,c}(\lambda_{t,c} < \mu_{t,c}) => x_{t,c}(\lambda_{t,c} + \epsilon \le \mu_{t,c}) \tag{15a}$$
$$x_{t,c}\lambda_{t,c} = \lambda_{t,c} \tag{15b}$$
$$=> \epsilon x_{t,c} - k_1^{cpu}\lambda_{t,c} - k_2^{cpu} + w_t\lambda_{t,c} \le 0 \quad \forall t\in T, \forall c\in C \tag{15c}$$
$$\epsilon x_{t,f} - k_1^{cpu}\lambda_{t,f} - k_2^{cpu} + w_t\lambda_{t,f} \le 0 \quad \forall t\in T, \forall f\in F \tag{15d}$$
$$\epsilon x_{t,e} - k_1^{cpu}\lambda_{t,e} - k_2^{cpu} + w_t\lambda_{t,e} \le 0 \quad \forall t\in T, \forall e\in E \tag{15e}$$

$$\begin{aligned} \min &\sum_{t\in T}\sum_{e\in E}(x_{t,e}\pi_e\sum_{r\in R} f_t^r(\lambda_{t,e})) \\ &+ \sum_{t\in T}\sum_{f\in F}(x_{t,f}\pi_f\sum_{r\in R} f_t^r(\lambda_{t,f})) \\ &+ \sum_{t\in T}\sum_{c\in C}(x_{t,c}\pi_c\sum_{r\in R} f_t^r(\lambda_{t,c})) \end{aligned} \tag{16a}$$

$$\begin{aligned} \min &\sum_{t\in T}\sum_{e\in E} x_{t,e}\Gamma_{t,e} \\ &+ \sum_{t\in T}\sum_{f\in F} x_{t,f}\Gamma_{t,f} \\ &+ \sum_{t\in T}\sum_{c\in C} x_{t,c}\Gamma_{t,c} \end{aligned} \tag{16b}$$

$$\begin{aligned} \Gamma_{t,e} &= \pi_e((k_1^{cpu} + k_1^{ram} + k_1^{storage})\lambda_{t,e} \\ &+ k_2^{cpu} + k_2^{ram} + k_2^{storage}) \\ &= \pi_e(K_1\lambda_{t,e} + K_2) \end{aligned} \tag{16c}$$

$$\begin{aligned} x_{t,e}\Gamma_{t,e} &= K_1\pi_e x_{t,e}\lambda_{t,e} + K_2\pi_e x_{t,e} \\ &= K_1\pi_e\psi_{t,e} + K_2\pi_e x_{t,e} \end{aligned} \tag{16d}$$

$$\begin{aligned}
&\min \sum_{t\in T}\sum_{e\in E} K_1\pi_e\psi_{t,e} + K_2\pi_e x_{t,e} \\
&\sum_{t\in T}\sum_{f\in F} K_1\pi_f\psi_{t,f} + K_2\pi_f x_{t,f} \\
&\sum_{t\in T}\sum_{c\in C} K_1\pi_c\psi_{t,c} + K_2\pi_c x_{t,c}
\end{aligned} \tag{17a}$$

$$\begin{aligned}
&L(\underline{\underline{x}}, \underline{\underline{\beta}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) = \sum_{t\in T}\sum_{e\in E} x_{t,e}\Gamma_{t,e} \\
&+ \sum_{t\in T}\sum_{f\in F} x_{t,f}\Gamma_{t,f} + \sum_{t\in T}\sum_{c\in C} x_{t,c}\Gamma_{t,c} \\
&+ \sum_{t\in T}\eta_{1,t}(1 - \sum_{e\in E} x_{t,e} + \sum_{f\in F} x_{t,f} + \sum_{c\in C} x_{t,c}) \\
&+ \sum_{t\in T}\eta_{2,t}(\sum_{e\in E} x_{t,e} + \sum_{f\in F} x_{t,f} + \sum_{c\in C} x_{t,c} - N_t) \\
&+ \sum_{t\in T}\sum_{s\in S}\nu_{t,s}(\lambda_{t,s} - \sum_{e\in E}\beta_{t,s,e} + \sum_{f\in F}\beta_{t,s,f} + \sum_{c\in C}\beta_{t,s,c})
\end{aligned} \tag{18a}$$

$$\begin{aligned}
L &= \sum_{e\in E}\sum_{t\in T}(\sum_{s\in S}(\nu_{t,s}\beta_{t,s,e} + \frac{\nu_{t,s}\lambda_{t,s}}{3|E|}) \\
&+ x_{t,e}(\Gamma_{t,e} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t\eta_{2,t}}{3|E|}) \\
&+ \sum_{f\in F}\sum_{t\in T}(\sum_{s\in S}(\nu_{t,s}\beta_{t,s,f} + \frac{\nu_{t,s}\lambda_{t,s}}{3|F|}) \\
&+ x_{t,f}(\Gamma_{t,f} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t\eta_{2,t}}{3|F|}) \\
&+ \sum_{c\in C}\sum_{t\in T}(\sum_{s\in S}(\nu_{t,s}\beta_{t,s,c} + \frac{\nu_{t,s}\lambda_{t,s}}{3|C|}) \\
&+ x_{t,c}(\Gamma_{t,c} - \eta_{1,t} + \eta_{2,t}) + \frac{\eta_{1,t} - N_t\eta_{2,t}}{3|C|})
\end{aligned} \tag{19a}$$

$$L = \sum_{e\in E} L_e + \sum_{f\in F} L_f + \sum_{c\in C} L_c \tag{19b}$$

$$\begin{aligned}
&g(\underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) = \inf_{\underline{\underline{x}}, \underline{\underline{\beta}}} L(\underline{\underline{x}}, \underline{\underline{\beta}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\
&= \sum_{e\in E}\inf_{\underline{\underline{x_e}}, \underline{\underline{\beta_e}}} L_e(\underline{\underline{x_e}}, \underline{\underline{\beta_e}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\
&+ \sum_{f\in F}\inf_{\underline{\underline{x_f}}, \underline{\underline{\beta_f}},} L_f(\underline{\underline{x_f}}, \underline{\underline{\beta_f}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\
&+ \sum_{c\in C}\inf_{\underline{\underline{x_c}}, \underline{\underline{\beta_c}},} L_c(\underline{\underline{x_c}}, \underline{\underline{\beta_c}}, \underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\
&= \sum_{e\in E} g_e(\underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\
&+ \sum_{f\in F} g_f(\underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}}) \\
&+ \sum_{c\in C} g_c(\underline{\eta_1}, \underline{\eta_2}, \underline{\underline{\nu}})
\end{aligned} \tag{20a}$$

$$\underline{\underline{x_e}}^{(k+1)}, \underline{\underline{\beta_e}}^{(k+1)} = arg\min_{\underline{\underline{x_e}}, \underline{\underline{\beta_e}}} L_e(\underline{\underline{x_e}}, \underline{\underline{\beta_e}}, \underline{\eta_1}^{(k)}, \underline{\eta_2}^{(k)}, \underline{\underline{\nu}}^{(k)}) \tag{21a}$$

$$\underline{\underline{x_f}}^{(k+1)}, \underline{\underline{\beta_f}}^{(k+1)} = arg\min_{\underline{\underline{x_f}}, \underline{\underline{\beta_f}}} L_f(\underline{\underline{x_f}}, \underline{\underline{\beta_f}}, \underline{\eta_1}^{(k)}, \underline{\eta_2}^{(k)}, \underline{\underline{\nu}}^{(k)}) \tag{21b}$$

$$\underline{\underline{x_c}}^{(k+1)}, \underline{\underline{\beta_c}}^{(k+1)} = arg\min_{\underline{\underline{x_c}}, \underline{\underline{\beta_c}}} L_c(\underline{\underline{x_c}}, \underline{\underline{\beta_c}}, \underline{\eta_1}^{(k)}, \underline{\eta_2}^{(k)}, \underline{\underline{\nu}}^{(k)}) \tag{21c}$$

$$\eta_{1,t}^{(k+1)} = \eta_{1,t}^{(k)} + \alpha^{(k)}(1 - \sum_{e\in E} x_{t,e}^{(k+1)} + \sum_{f\in F} x_{t,f}^{(k+1)} + \sum_{c\in C} x_{t,c}^{(k+1)}) \tag{22a}$$

$$\eta_{2,t}^{(k+1)} = \eta_{2,t}^{(k)} + \alpha^{(k)}(\sum_{e\in E} x_{t,e}^{(k+1)} + \sum_{f\in F} x_{t,f}^{(k+1)} + \sum_{c\in C} x_{t,c}^{(k+1)} - N_t) \tag{22b}$$

$$\nu_{t,s}^{(k+1)} = \nu_{t,s}^{(k)} + \alpha^{(k)}(\lambda_{t,s} - \sum_{e\in E}\beta_{t,s,e}^{(k+1)} + \sum_{f\in F}\beta_{t,s,f}^{(k+1)} + \sum_{c\in C}\beta_{t,s,c}^{(k+1)}) \tag{22c}$$

D. Objective

$$\begin{aligned}
p_k = &\sum_{i=1}^{l_e} x_{k,i}^e C(v_i^e, t_k) \\
&+ \sum_{j=1}^{l_f} x_{k,j}^f C(v_j^f, t_k) \\
&+ \sum_{h=1}^{l_c} x_{k,h}^c C(v_h^c, t_k)
\end{aligned} \tag{23a}$$

$$\min \sum_{k=1}^{l_t} p_k \tag{24a}$$

subject to: 9

E. Solution

We can reshape main problem as following:

$$\min(\sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x^e_{k,i}C^e_{k,i} \tag{25a}$$
$$+\sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x^f_{k,j}C^f_{k,j}+\sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x^c_{k,h}C^c_{k,h})$$

subject to:

$$\sum_{i=1}^{l_e} x^e_{k,i}\tau^e_{k,i}+\sum_{j=1}^{l_f} x^f_{k,j}\tau^f_{k,j}+\sum_{h=1}^{l_c} x^c_{k,h}\tau^c_{k,h}\le \delta_k \quad \forall k\in\{1,...,l_t\}$$
$$\sum_{k=1}^{l_t} x^c_{k,h}w_k \le c^c_h \qquad \forall h\in\{1,2,...,l_c\}$$
$$\sum_{k=1}^{l_t} x^f_{k,j}w_k \le c^f_j \qquad \forall j\in\{1,2,...,l_f\}$$
$$\sum_{k=1}^{l_t} x^e_{k,i}w_k \le c^e_i \qquad \forall i\in\{1,2,...,l_e\}$$
$$\sum_{i=1}^{l_e} x^e_{k,i}+\sum_{j=1}^{l_f} x^f_{k,j}+\sum_{h=1}^{l_c} x^c_{k,h}=1 \qquad \forall k\in\{1,2,...,l_t\}$$

We define u^m for each computational agent m, that is a matrix with size $l_t*(l_e+l_f+l_c)$. It is the local copy of all variables in agent m. i.e. $u^{e,m}_{k,i}$ is the copy of variable $x^e_{k,i}$ in agent m for $m=1,..,(l_m=l_e+l_f+l_c)$. So we should add new constraint $u^m = z \quad \forall m$ to main problem. We will use admm on this new constraint so:

$$L_p=\sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x^e_{k,i}C^e_{k,i}+\sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x^f_{k,j}C^f_{k,j}+\sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x^c_{k,h}C^c_{k,h} \tag{26a}$$
$$+\sum_{m=1}^{l_m}\nu^m*(u^m-z)+\sum_{m=1}^{l_m}\frac{\rho}{2}\|u^m-z\|^2$$

We can seperate augmented lagrangian for each computational agent m then:

$$L^m_p=\sum_{k=1}^{l_t} u^m_{k,m}C_{k,m}+\nu^m*(u^m-z)+\frac{\rho}{2}\|u^m-z\|^2 \tag{27a}$$
$$\forall m\in\{1,2,...,l_m\}$$

So we can write the algorithm as following:

for each iteration k (28a)

1. $u^{m,(k+1)} = arg\min L^m_p(u^m, z^{(k)}, \nu^{m,(k)}) =$
$$\sum_{k=1}^{l_t} u^m_{k,m}C_{k,m}+\nu^{m,(k)}*(u^m-z^{(k)})+\frac{\rho}{2}\|u^m-z^{(k)}\|^2$$
subject to:
$$\sum_{k=1}^{l_t} u^m_{k,m}w_k\le c^m$$
$$\sum_{i=1}^{l_e} u^{e,m}_{k,i}+\sum_{j=1}^{l_f} u^{f,m}_{k,j}+\sum_{h=1}^{l_c} u^{c,m}_{k,h}=1 \qquad \forall k\in\{1,2,...,l_t\}$$
$$\sum_{i=1}^{l_e} u^{e,m}_{k,i}\tau^e_{k,i}+\sum_{j=1}^{l_f} u^{f,m}_{k,j}\tau^f_{k,j}+\sum_{h=1}^{l_c} u^{c,m}_{k,h}\tau^c_{k,h}\le\delta_k \qquad \forall k\in\{1,2,...,l_t\}$$
2. $z^{(k+1)}=\bar{u}^{(k+1)}+\frac{1}{\rho}\bar{\nu}^{(k)}$
3. $\nu^{m,(k+1)}=\nu^{m,(k)}+\rho(u^{m,(k+1)}-z^{(k+1)})$

F. Solution 2

lagrangian of main problem is as following

$$L(x^e,x^f,x^c,\lambda,\nu)=\sum_{i=1}^{l_e}\sum_{k=1}^{l_t} x^e_{k,i}C^e_{k,i}+\sum_{j=1}^{l_f}\sum_{k=1}^{l_t} x^f_{k,j}C^f_{k,j}+\sum_{h=1}^{l_c}\sum_{k=1}^{l_t} x^c_{k,h}C \tag{29a}$$
$$+\sum_{k=1}^{l_t}\lambda_k(\sum_{i=1}^{l_e} x^e_{k,i}\tau^e_{k,i}+\sum_{j=1}^{l_f} x^f_{k,j}\tau^f_{k,j}+\sum_{h=1}^{l_c} x^c_{k,h}\tau^c_{k,h}-\delta_k)$$
$$+\sum_{k=1}^{l_t}\nu_k(\sum_{i=1}^{l_e} x^e_{k,i}+\sum_{j=1}^{l_f} x^f_{k,j}+\sum_{h=1}^{l_c} x^c_{k,h}-1)$$

So we can decompose the lagrangian as follows

$$L(x^e,x^f,x^c,\lambda,\nu)= \tag{30a}$$
$$\sum_{i=1}^{l_e}\sum_{k=1}^{l_t}(x^e_{k,i}C^e_{k,i}+\lambda_k x^e_{k,i}\tau^e_{k,i}+\nu_k x^e_{k,i}-\frac{\lambda_k\delta_k+\nu_k}{3l_e})$$
$$+\sum_{j=1}^{l_f}\sum_{k=1}^{l_t}(x^f_{k,j}C^f_{k,j}+\lambda_k x^f_{k,j}\tau^f_{k,j}+\nu_k x^f_{k,j}-\frac{\lambda_k\delta_k+\nu_k}{3l_f})$$
$$+\sum_{h=1}^{l_c}\sum_{k=1}^{l_t}(x^c_{k,h}C^c_{k,h}+\lambda_k x^c_{k,h}\tau^c_{k,h}+\nu_k x^c_{k,h}-\frac{\lambda_k\delta_k+\nu_k}{3l_c})$$

$$
\begin{aligned}
L(x^e, x^f, x^c, \lambda, \nu) = \sum_{i=1}^{l_e} L_i^e(x_i^e, \lambda, \nu) \\
+ \sum_{j=1}^{l_f} L_j^f(x_j^f, \lambda, \nu) \\
+ \sum_{h=1}^{l_c} L_h^c(x_h^c, \lambda, \nu)
\end{aligned}
\tag{31a}
$$

$$
\begin{aligned}
g(\lambda, \nu) = \inf_{x^e, x^f, x^c} L(x^e, x^f, x^c, \lambda, \nu) \\
= \sum_{i=1}^{l_e} \inf_{x_i^e} L_i^e(x_i^e, \lambda, \nu) \\
+ \sum_{j=1}^{l_f} \inf_{x_j^f} L_j^f(x_j^f, \lambda, \nu) \\
+ \sum_{h=1}^{l_c} \inf_{x_h^c} L_h^c(x_h^c, \lambda, \nu) \\
= \sum_{i=1}^{l_e} g_i^e(\lambda, \nu) \\
+ \sum_{j=1}^{l_f} g_j^f(\lambda, \nu) \\
+ \sum_{h=1}^{l_c} g_h^c(\lambda, \nu)
\end{aligned}
\tag{32a}
$$

$$
\lambda_k^+ = \lambda_k^- + \alpha(\sum_{i=1}^{l_e} x_{k,i}^e \tau_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f \tau_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c \tau_{k,h}^c - \delta_k) \tag{33a}
$$

$$
\nu_k^+ = \nu_k^- + \alpha(\sum_{i=1}^{l_e} x_{k,i}^e + \sum_{j=1}^{l_f} x_{k,j}^f + \sum_{h=1}^{l_c} x_{k,h}^c - 1) \tag{33b}
$$

Algorithm Test Algorithm

for $n = 1 : L_v$ **do**
    Determine the set of states Z_n
    $RemovedStates =$
    for $j \in Z_n$ **do**
        Determine the index of computational node l and the index of task t and the index of part u
        $XP_n^j = Z_{n-1}$
        if $j \neq 0$ **then**
            for $i \in XP_n^j$ **do**
                if $ServerResource < 0$ **then**
                    $XP_n^j = XP_n^j - \{i\}$
                end if
            end for
        end if
        if $XP_n^j \neq \emptyset$ **then**
            for $i \in XP_n^j$ **do**
                Calculate $T_{n-1,n}^{i,j}$
            end for
            Calculate I_n^j and Λ_n^j
            Calculate ϕ_n^j
            if $j \neq 0$ **then**
                $ServerResource < 0$
            end if
        else
            $RemovedStates = RemovedStates + \{j\}$
        end if
    end for
    if $RemovedStates = Z_n$ **then**
        Set $H = \sum_{m=1}^{t} N_m$ and $ResourceIndicator = 0$
        Determine the Viterbi path P with H
        break
    else
        Remove all states in the $RemovedStates$ from the Z_n
    end if
end for
if $ResourceIndicator = 1$ **then**
    Determine the Viterbi path P with H
end if
Determine the task scheduling using Viterbi path P