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p–q superstrings in anti-de-Sitter spacetime

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Abstract
We study a field theoretical model for p–q superstrings in a fixed anti-de-Sitter background. We find that the presence of the negative cosmological constant tends to decrease the core radius of the strings. Moreover, the binding energy decreases with the increase of the absolute value of the cosmological constant. Studying the effect of the p–q strings on anti-de-Sitter space, we observe that the presence of the negative cosmological constant tends to decrease the deficit angle as compared to asymptotically flat spacetime.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The anti-de-Sitter/conformal field theory (AdS/CFT) correspondence [1] is an explicit and well-tested realization of the holographic principle which connects quantum gravity in a $(d+1)$-dimensional spacetime to a conformal field theory on the d-dimensional boundary of this spacetime. In the case of the AdS/CFT correspondence, the $(d+1)$-dimensional spacetime is a spacetime containing a negative cosmological constant. Often it is easier to deal with the lower-dimensional theory on the boundary than with the full quantum gravity theory. Recently, the Abelian Higgs model has been studied in a four-dimensional AdS spacetime [2]. It was found that this model contains string-like solutions. The holographic description of these solutions via the AdS/CFT correspondence has been studied and it was found that the spacetime around the string contains a deficit angle—very similar to the case in asymptotically flat spacetime.

The interest in string-like solutions has grown again over the past years since it is believed that cosmic strings might be connected to the fundamental strings of string theory. The reason is that mechanisms appear in so-called brane world models [3] that decrease the fundamental Planck scale, i.e. the energy scale of fundamental strings, to much lower energy scales. Moreover, models of brane inflation in which a D-brane and an anti-D-brane collide and annihilate always seem to predict the creation of networks of cosmic strings at the end of inflation [4]. The objects that form in such brane inflation models are so-called D-strings and F-strings (for a review see e.g. [5]). D-strings are one-dimensional D-branes and are charged under the Ramond–Ramond potential, while F-strings are the fundamental strings charged under the Neveu-Schwartz–Neveu-Schwartz potential. Also bound states of p F-strings and q D-strings, so-called p–q strings are possible. The interesting thing about these bound states is that they are supersymmetric and the square of the tension of these bound states is equal to the sum of the squares of the tensions of the p strings and the q strings. Field theoretical models describing these bound states have recently been studied [6].

In this paper, we study a field theoretical model for p–q superstrings in a fixed anti-de-Sitter background. Our paper is organized as follows: in section 2, we give the model, the Ansatz and the equations and discuss the gravitational effects of p–q strings. In section 3, we discuss our numerical results and we conclude in section 4.

2. The model

The metric of the fixed anti-de-Sitter background in static, spherical coordinates (representing the coordinates of an inertial observer) can be parametrized as follows:

$$\mathrm{d}s^2 = -\left(1+\frac{r^2}{l^2}\right)\mathrm{d}t^2 + \left(1+\frac{r^2}{l^2}\right)^{-1}\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi^2) \tag{1}$$

where $l = \sqrt{-3/\Lambda}$ is the anti-de-Sitter radius and Λ is the negative cosmological constant.

In the following, we want to study p–q strings in this fixed anti-de-Sitter background. The Lagrangian describing the p–q strings reads [6]

$$\mathcal{L}_m = -D_\mu\phi(D^\mu\phi)^* - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - D_\mu\xi(D^\mu\xi)^* - \tfrac{1}{4}H_{\mu\nu}H^{\mu\nu} - V(\phi,\xi) \tag{2}$$

with the covariant derivatives $D_\mu\phi = \partial_\mu\phi - \mathrm{i}e_1A_\mu\phi$, $D_\mu\xi = \partial_\mu\xi - \mathrm{i}e_2B_\mu\xi$ and the field strength tensors $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ of the two U(1) gauge potentials A_μ, B_μ with coupling constants e_1 and e_2. The fields ϕ and ξ are complex scalar fields (Higgs fields). The potential reads

$$V(\phi,\xi) = \frac{\lambda_1}{4}\left(\phi\phi^* - \eta_1^2\right)^2 + \frac{\lambda_2}{4}\left(\xi\xi^* - \eta_2^2\right)^2 - \lambda_3\left(\phi\phi^* - \eta_1^2\right)\left(\xi\xi^* - \eta_2^2\right). \tag{3}$$

In order for $\phi = \eta_1$ and $\xi = \eta_2$ to be the global minimum of this potential, we need to require that [7]

$$\lambda_3^2 < \frac{\lambda_1\lambda_2}{4}. \tag{4}$$

When ϕ and ξ attain their vacuum expectation values η_1 and η_2, respectively, the two $U(1)$ symmetries are spontaneously broken to 1 and the particle content of the theory are two massive gauge bosons with masses $M_{W,1} = e_1\eta_1$, $M_{W,2} = e_2\eta_2$ and two massive scalar fields (Higgs fields) with masses $M_{H,1} = \sqrt{2\lambda_1}\eta_1$, $M_{H,2} = \sqrt{2\lambda_2}\eta_2$.

Each of the two strings consists of a scalar core of radius $r_{H,i} \approx M_{H,i}^{-1}$, $i = 1, 2$ and of a magnetic flux tube with radius $r_{W,i} \approx M_{W,i}^{-1}$, $i = 1, 2$.

2.1. The Ansatz and equations of motion

The Ansatz for the matter and gauge fields in spherical coordinates r, θ, φ reads [8]

$$\phi(r,\theta) = \eta_1 h(r,\theta)\,\mathrm{e}^{\mathrm{i}n\varphi}, \qquad \xi(r,\theta) = \eta_1 f(r,\theta)\,\mathrm{e}^{\mathrm{i}m\varphi}, \tag{5}$$

$$A_\mu \, dx^\mu = \frac{1}{e_1}(n - P(r,\theta))\, d\varphi, \qquad B_\mu \, dx^\mu = \frac{1}{e_2}(m - R(r,\theta))\, d\varphi. \tag{6}$$

n and m are integers indexing the vorticity of the two Higgs fields around the z-axis.

Using the above Ansatz for the metric and matter fields, the resulting field equations would be partial differential equations. However, since we want to study cylindrical, vortex-type configurations here we can assume that in the following the matter field functions P, f, R and h depend only on the specific combination $r \sin\theta \equiv \rho$. The partial differential equations then reduce to ordinary differential equations that depend only on the coordinate ρ and in the limit $l \to \infty$ correspond to the equations of the p–q strings studied in [6].

We define the following dimensionless variable:

$$x = e_1 \eta_1 \rho. \tag{7}$$

Then, the total Lagrangian only depends on the following dimensionless coupling constants:

$$L = e_1 \eta_1 l, \qquad g = \frac{e_2}{e_1}, \qquad q = \frac{\eta_2}{\eta_1}, \qquad \beta_i = \frac{\lambda_i}{e_1^2}, \qquad i = 1, 2, 3. \tag{8}$$

Varying the action with respect to the matter fields, we obtain a system of four nonlinear differential equations. The Euler–Lagrange equations for the matter field functions read

$$\left(1 + \frac{x^2}{L^2}\right) P'' = 2Ph^2 + \frac{P'}{x}\left(1 - \frac{2x^2}{L^2}\right) \tag{9}$$

$$\left(1 + \frac{x^2}{L^2}\right) h'' = \frac{\beta_1}{2} h(h^2 - 1) - \beta_3 h(f^2 - q^2) - \frac{h'}{x}\left(1 + \frac{4x^2}{L^2}\right) + \frac{P^2 h}{x^2} \tag{10}$$

$$\left(1 + \frac{x^2}{L^2}\right) R'' = 2g^2 R f^2 + \frac{R'}{x}\left(1 - \frac{2x^2}{L^2}\right) \tag{11}$$

$$\left(1 + \frac{x^2}{L^2}\right) f'' = \frac{\beta_2}{2} f(f^2 - q^2) - \beta_3 f(h^2 - 1) - \frac{f'}{x}\left(1 + \frac{4x^2}{L^2}\right) + \frac{R^2 f}{x^2} \tag{12}$$

where the prime now and in the following denotes the derivative with respect to x.

In the general case, the solutions to the above equations have to be constructed numerically subject to suitable boundary conditions.

The requirement of regularity at the origin leads to the following boundary conditions:

$$h(0) = 0, \qquad f(0) = 0, \qquad P(0) = n, \qquad R(0) = m. \tag{13}$$

In the special case where $n \neq 0$ and $m = 0$ the boundary conditions (13) change according to

$$h(0) = 0, \qquad f'(0) = 0, \qquad P(0) = n, \qquad R(0) = 0 \tag{14}$$

while for a $n = 0$, $m \neq 0$ string, they read

$$h'(0) = 0, \qquad f(0) = 0, \qquad P(0) = 0, \qquad R(0) = m. \tag{15}$$

By letting the derivatives of the non-winding scalar field be zero at the origin instead of imposing the boundary conditions for the fields themselves, the non-winding scalar field can take non-zero values at the origin and a 'condensate' of the non-winding scalar field will form in the core of the winding string.

The finiteness of the energy per unit length requires

$$h(\infty) = 1, \qquad f(\infty) = q, \qquad P(\infty) = 0, \qquad R(\infty) = 0. \tag{16}$$

2.2. Asymptotic behaviour

For $x \ll 1$, we find that

$$P(x \ll 1) = n + p_0 x^2, \qquad R(x \ll 1) = m + r_0 x^2 \tag{17}$$

and

$$h(x \ll 1) = h_0 x^n, \qquad f(x \ll 1) = f_0 x^m \tag{18}$$

for standard p–q strings with $n \neq 0, m \neq 0$. p_0, r_0, h_0 and f_0 are constants that have to be determined numerically.

For a solution with vanishing winding, the behaviour is different. Here, we present the behaviour for $m = 0$ (the $n = 0$ case works in an analogue way). We find that $R(x) \equiv 0$ while the corresponding scalar field $f(x)$ forms a condensate at the core of the string and behaves as

$$f(x \ll 1) = f_0 + C_1 I_0(\sqrt{A}x) - \frac{B}{A} \tag{19}$$

where I_0 is the modified Bessel function of the first kind and

$$A = \frac{\beta_2}{2}\left(3f_0^2 - q^2\right) + \beta_3, \qquad B = \frac{\beta_2}{2}\left(f_0^2 - q^2\right) + \beta_3. \tag{20}$$

C_1 is an integration constant. Note that $f_0 = q$ for $\beta_3 = 0$, while $f_0 \neq q$ for $\beta_3 \neq 0$.

For $x \gg 1$, we find a power-law behaviour for the gauge field functions P and R with

$$P(x \gg 1) = P_0 x^{c_1} \quad \text{with} \quad c_1 = -\tfrac{1}{2}(1 + \sqrt{1 + 8L^2}), \tag{21}$$

$$R(x \gg 1) = R_0 x^{c_2} \quad \text{with} \quad c_2 = -\tfrac{1}{2}(1 + \sqrt{1 + 8g^2L^2}) \tag{22}$$

where P_0 and R_0 are constants.

The scalar functions have the following behaviour

$$h(x \gg 1) = 1 + H_0 x^{\alpha}, \qquad f(x \gg 1) = q + F_0 x^{\alpha} \tag{23}$$

where

$$\frac{F_0}{H_0} = \frac{\beta_1 - \beta_2 q + \sqrt{(\beta_1 - q\beta_2)^2 + 16\, q\beta_3^2}}{4q\beta_3} \tag{24}$$

and

$$\alpha = -\frac{3}{2} - \frac{1}{2}\sqrt{9 + 4\beta_1 L^2 - 8qL^2\beta_3\frac{B}{A}}. \tag{25}$$

For the case where $\beta_1 = \beta_2, q = 1$ (which we will study in this paper) we find—of course—that $H_0 = F_0$. Note also that the positivity of the expression under the square root is guaranteed by the requirement that $\beta_3 < \sqrt{\beta_1\beta_2}/2$.

We define as inertial mass per unit length of the solution the integral of the energy density $\epsilon = -T_0^0$ over a slice of the constant $z = r\sin\theta$. We have

$$\epsilon = \eta_1^4\left[\left(1 + \frac{x^2}{L^2}\right)(h'^2 + f'^2) + \left(1 + \frac{x^2}{L^2}\right)\left(\frac{P'^2}{2x^2} + \frac{R'^2}{2g^2x^2}\right) + \frac{P^2h^2}{x^2} + \frac{R^2f^2}{x^2}\right.$$
$$\left. + \frac{\beta_1}{4}(h^2 - 1)^2 + \frac{\beta_2}{4}(f^2 - q^2)^2 - \beta_3(h^2 - 1)(f^2 - q^2)\right]. \tag{26}$$

The inertial energy per unit length can then be defined by integrating T_0^0 over a section of constant z, leading to

$$E_{in} = 2\pi\int_0^{\infty} \mathrm{d}x\, x\epsilon. \tag{27}$$

We then define the binding energy of a (n, m)-solution as

$$E_{bin}^{(n,m)} := E_{in}^{(n,m)} - nE_{in}^{(1,0)} - mE_{in}^{(0,1)}. \tag{28}$$

2.3. Gravitational effects

If we want to study the gravitational effects of p–q strings on the spacetime, we have to solve the full Einstein equations. This is very difficult since the resulting equations would be partial differential equations. In [2], an approximation for weak gravitational fields and string cores much smaller than the anti-de-Sitter radius was used to study the effects of a single Abelian string on anti-de-Sitter spacetime. In that case, the Einstein equations can be linearized. We employ this method here for p–q strings. The metric reads [2]

$$ds^2 = \exp(2z/l)(-\exp(A)\,d\hat{t}^2 + d\hat{\rho}^2 + F^2\,d\varphi^2) + \exp(C)\,dz^2 \tag{29}$$

where A, F and C are functions of $\hat{\rho}$ and z. For $T_0^0 = 0$, i.e. in the absence of the p–q string, the solution to the corresponding Einstein equation would be $A = C = 0$, $F = \hat{\rho}$. The metric (29) is then just equivalent to the metric of a pure four-dimensional anti-de-Sitter spacetime.

Introducing the rescaled coordinate $\hat{x} = e_1\eta_1\hat{\rho}$ and letting $z \to e_1\eta_1 z$, $t \to e_1\eta_1 t$, the connection to the coordinates used before in this paper is given by the following relations [2]:

$$\begin{aligned} &\hat{x}\exp(z/L) = x\sin\theta, \qquad \exp(z/L) = \frac{x}{L}\cos\theta + \sqrt{1+\frac{x^2}{L^2}}\cos(t/L), \\ &\hat{t}\exp(z/L) = L\sqrt{1+\frac{x^2}{L^2}}\sin(t/L). \end{aligned} \tag{30}$$

Letting the functions depend only on the combination $x = \hat{x}\exp(z/L)$, the linearized Einstein equation for F reads

$$\frac{2}{L^2} - \frac{1}{F}\frac{d}{dx}\left(\left(1+\frac{x^2}{L^2}\right)\frac{dF}{dx}\right) = -\gamma T_0^0 \tag{31}$$

where $\gamma = 8\pi G$ and T_0^0 is the energy–momentum tensor of the p–q string in the fixed AdS background (see (26)). The deficit angle δ of the spacetime is then given by $\delta = 2\pi(1 - F'|_{x=x_0})$, where we assume $T_0^0 = 0$ for $x > x_0$, i.e. outside of the core of the string. The deficit angle defined in this way thus 'measures' the departure of the spacetime from a pure anti-de-Sitter spacetime due to the presence of a p–q string.

3. Numerical results

In all calculations, we will fix $g = q = 1$ and $\beta_1 = \beta_2 = 2$ and study the properties of the solutions depending on the parameters β_3, L and on the windings n and m. In all numerical calculations, we have used the ODE solver COLSYS [9].

3.1. Effect of the negative cosmological constant

The presence of a negative cosmological constant tends to decrease the core radii of particle-like objects as compared to flat spacetime. This is also what we observe here. In figure 1, we plot the energy density $\epsilon = -T_0^0$ of a (1, 1) string for $\beta_3 = 0.9$ and different values of the parameter L as a function of x/L.

We observe that for L decreasing, the maximal value of ϵ increases and at the same time ϵ becomes strongly localized[1]. The presence of the negative cosmological constant thus tends to 'squash' the energy density into a smaller region of spacetime, thus decreasing the core radius of the string. While the effect is small when the anti-de-Sitter radius L is large compared to

[1] Note that we plot the energy density as a function of x/L. If we would plot it as a function of x only, the stronger localization would be apparent.

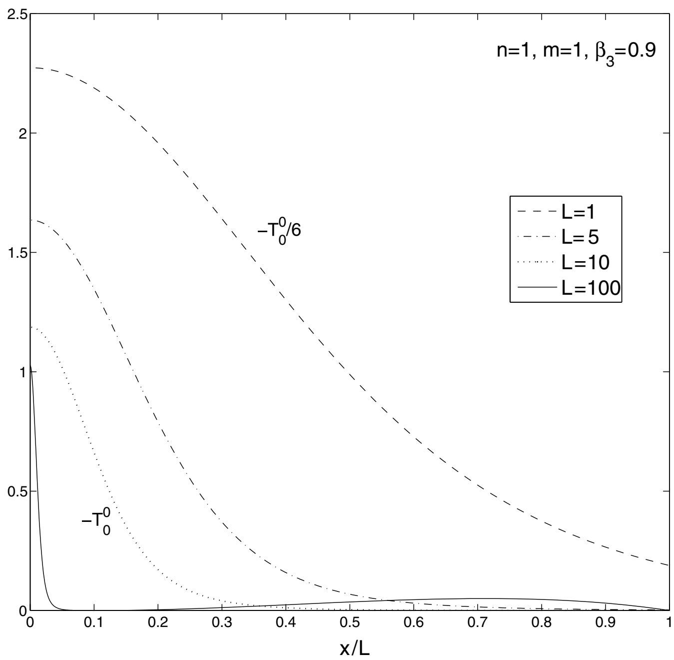

Figure 1. The energy density $\epsilon = -T_0^0$ for $\beta_3 = 0.9, n = m = 1$ and different values of L is shown as a function of x/L. Note that for $L = 1$, we plot $-T_0^0/6$.

the radius of the string core, it becomes much larger when L is comparable to the core radius. Due to our choice of parameters we have (in rescaled coordinates) that $x_{W,i} = x_{H,i} \approx 1$. For $L = 1$, the AdS radius is thus comparable to the core radius of the strings.

In figure 2, we show the dependence of the energy (per unit length) of the strings on L for $\beta_3 = 0.9$ and different choices of (n, m). We observe that the decrease of L leads to an increase of the energy and that this increase is stronger for strings with higher total winding $n + m$. The increase of energy in the case of AdS solitons was already observed in [10] for magnetic monopoles.

In addition, we observe that the energy curves run nearly parallel for $n + m$ constant. The increase in energy due to the negative cosmological constant thus seems—to a good approximation—only to depend on the sum $n + m$ of the two windings.

3.2. The condensate

In the particular case where $m = 0$, the scalar field function $f(x)$ is not forced to zero at the origin and can develop a condensate inside the n-string. The existence of this condensate was observed before in [6], but has not been discussed in detail yet. Since it seems that strings with condensates in their core 'react' in a particular way to the presence of the cosmological constant, we study these condensate solutions in more detail here. First of all note that from the potential (3) we find that a non-zero value of f at $x = 0$ can lower the potential energy inside the string core. The second term in the potential tries to choose a value for f that is close to q, while the third term tries to choose a value of f close to zero. The potential energy related to these two terms is minimized (at the origin) for

$$f(0) = \sqrt{q^2 - 2\beta_3/\beta_2}. \tag{32}$$

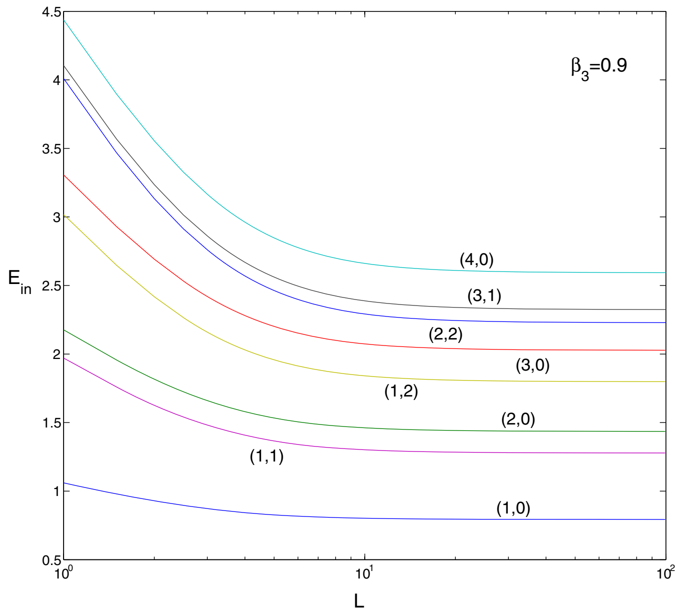

Figure 2. The value of the energy E_{in} is shown as a function of L for $\beta_3 = 0.9$ and different choices of (n, m).

Of course, this is only an approximation at the origin, while in the full system, we would have nonlinear effects. However, we can already see from this approximation that $f(0)$ should be decreasing for β_3 increasing and would be $f(0) = q$ for $\beta_3 = 0$, i.e. in the non-interacting case.

In figure 3, we plot the value of the condensate at the origin, $f(0)$, as a function of β_3 for $L = 5$, that we have determined numerically when solving the system of differential equations. Clearly, the value of $f(0)$ decreases for increasing β_3 and tends to $q = 1$ in the limit $\beta_3 = 0$. Moreover, for a fixed value of β_3 and L, the condensate decreases for increasing n. The reason for this is that for increasing n, the core of the string extends more and more to large x. The condensate does the same and has thus decreasing values of $f(0)$. We also observe that the decrease in the condensate is stronger going from a $n = 1$ solution to a $n = 2$ solution than from a $n = 2$ to a $n = 3$ solution.

In figure 4, we show the value of $f(0)$ as a function of L for $\beta_3 = 0.9$. The curves tend to their non-zero, flat spacetime limits for $L \to \infty$. Apparently, the value of $f(0)$ increases for decreasing L, i.e. increasing absolute value of the cosmological constant. The reason for this is the decrease of the core radius mentioned above. For decreasing L the core radius decreases and the condensate becomes more and more compressed, thus developing a larger maximal value.

3.3. Binding energies

Increasing the parameter β_3 increases the binding between the strings. We clearly observe that this is the same in anti-de-Sitter space. In figure 5, we show the binding energy as a function of β_3 for $L = 10$ and different values of (n, m). The first thing that is apparent from this plot is that the binding energy curves look qualitatively different for condensate strings with $n \neq 0, m = 0$ than those of 'standard' p–q strings with $n \neq 0, m \neq 0$. While the binding

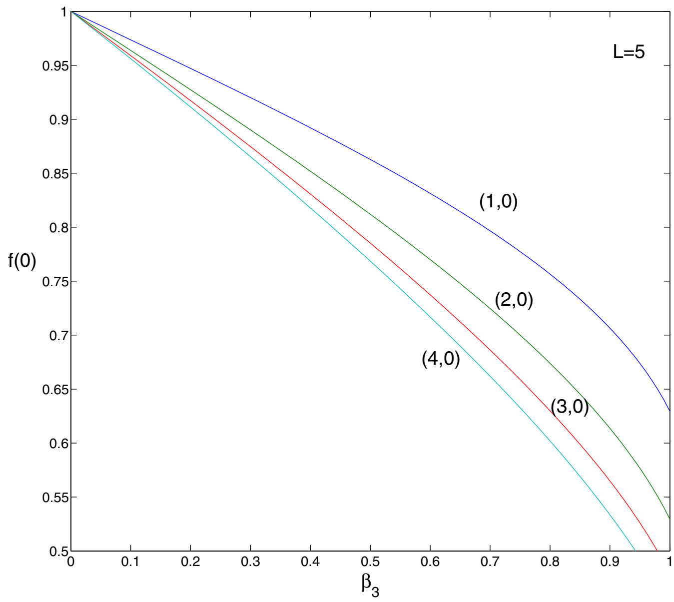

Figure 3. The value of the scalar field function $f(x)$ at the origin, $f(0)$ as a function of β_3 for $L = 5$ and different values of n.

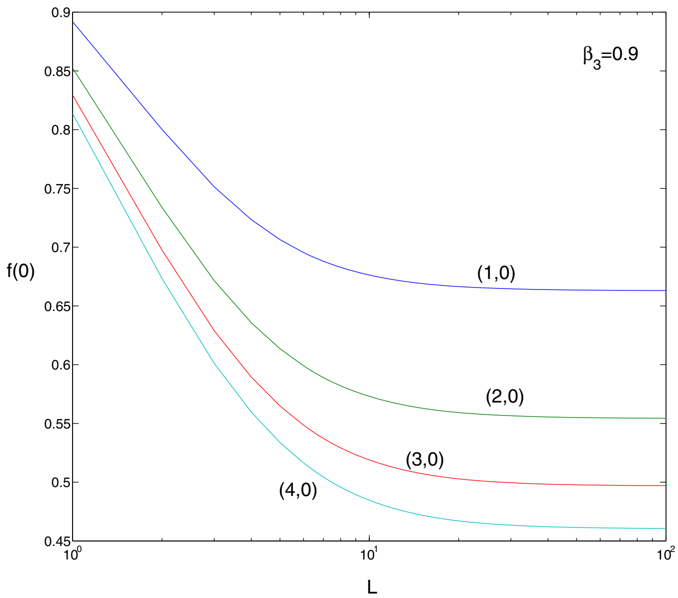

Figure 4. The value of the scalar field function $f(x)$ at the origin, $f(0)$, as a function of L for $\beta_3 = 0.9$ and different values of n.

energy seems to decrease approximately linearly with β_3 for standard p–q strings, it decreases to a good approximation quadratically with β_3 for the condensate strings.

In figure 6, we show the binding energy as a function of L for $\beta_3 = 0.9$ and different values of (n, m). As observed in [11], the absolute value of the binding energy of a $(n+k, n-k)$ string,

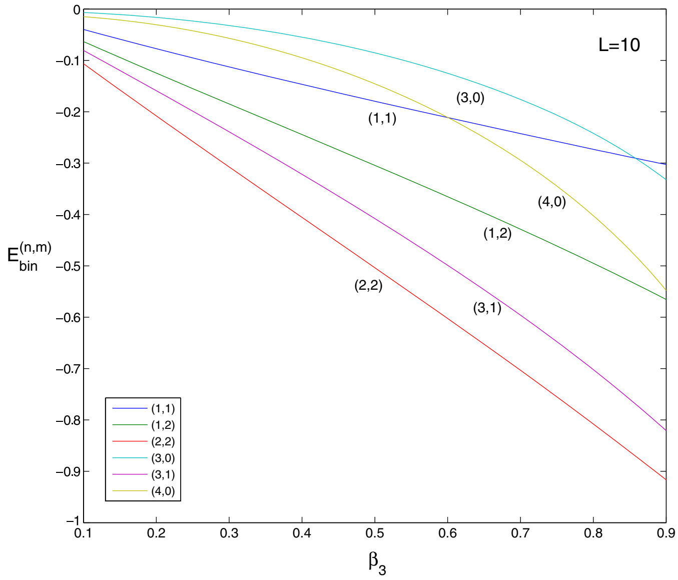

Figure 5. The value of the binding energy $E_{bin}^{(n,m)}$ as a function of β_3 for $L = 10$ and different values of (n, m).

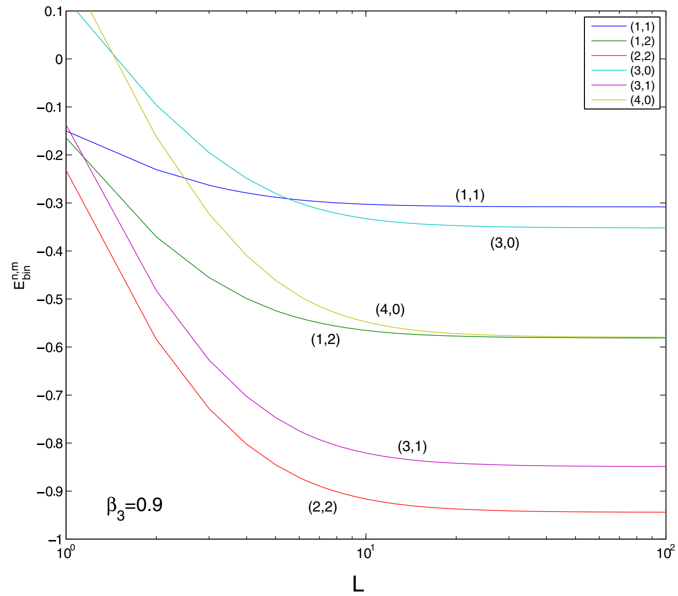

Figure 6. The value of the binding energy $E_{bin}^{(n,m)}$ as a function of L for $\beta_3 = 0.9$ and different values of (n, m).

$k = 0, \ldots, n$ is decreasing for k increasing. We observe the same for anti-de-Sitter space. For example, the curves for the $(2, 2)$, $(3, 1)$ and $(4, 0)$ strings run parallel as L decreases.

The absolute value of the binding energy is decreasing for decreasing L, i.e. the larger the absolute value of the cosmological constant, the weaker bound are the strings. We even observe that if the cosmological constant is large enough, the binding energy becomes

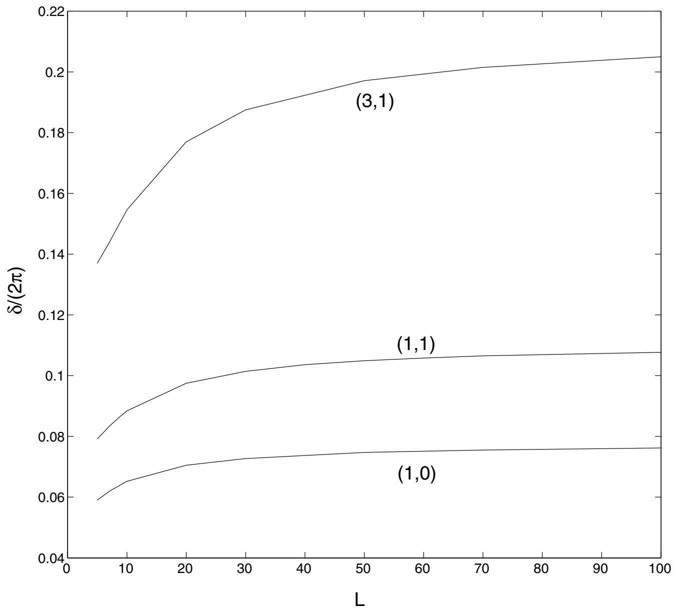

Figure 7. The deficit angle δ in units of 2π is given as a function of L for three different choices of (n, m). Here, the gravitational coupling $\gamma = 0.1$ and $\beta_3 = 0.9$.

Table 1. Values of $\beta_{3,cr}^{(n,m)}$ for $L = 2$ and different choices of (n, m).

$\beta_{3,cr}^{(2,0)}$	$\beta_{3,cr}^{(3,0)}$	$\beta_{3,cr}^{(4,0)}$	$\beta_{3,cr}^{(1,2)}$	$\beta_{3,cr}^{(3,1)}$	$\beta_{3,cr}^{(2,2)}$
0.483	0.434	0.371	0.033	0.049	0.040

positive signalling that the strings become unbound and would—in a dynamical process—likely separate into $n(1, 0)$ and $m(0, 1)$ strings. In table 1, we give $\beta_{3,cr}^{(n,m)}$, the value of β_3 that separates bound from unbound strings for $L = 2$ and different choices of (n, m). For $\beta_3 < \beta_{3,cr}^{(n,m)}$, the strings are unbound, while for larger values of β_3 they become bound states.

Clearly, it is the 'condensate strings' which become unbound for quite high values of β_3 and the value of $\beta_{3,cr}^{(n,0)}$ decreases for increasing n. The standard p–q strings on the other hand become unbound only for very small values of β_3. This can be explained with the different qualitative dependence of the energies of condensate strings and standard p–q strings, respectively, on β_3, which we have discussed before in this paper. It seems that for an AdS radius close to the core radius of the string, condensate strings become unbound for quite large values of β_3.

Moreover, we observe that when L is small enough, a $(1, 1)$ string becomes more strongly bound than a $(3, 0)$ string and for even smaller L more strongly bound than a $(4, 0)$ string. Thus, standard p–q strings with a smaller total winding can become more strongly bound than condensate strings with a larger total winding.

3.4. Deficit angle

We have studied the value of the deficit angle by integrating (31) subject to the boundary conditions $F(0) = 0$ and $F'(0) = 1$. We have chosen $\gamma = 0.1$. Our results for different values of (n, m) are given in figure 7. For $L \to \infty$, the value of δ tends to the value in asymptotically flat spacetime. We observe that the deficit angle decreases with the decrease of L, i.e. with the increase of the absolute value of the cosmological constant. Moreover, the deficit angle decreases stronger for strings with higher total winding $n + m$. Of course, our calculations are only valid as long as the gravitational field is weak and as long as the core radius of the p–q strings is much smaller than the AdS radius, however, we believe that the results of a calculation of the solution of the 'full' nonlinear Einstein equations would be qualitatively the same.

4. Conclusions

We have studied a field theoretical model for p–q superstrings in a fixed anti-de-Sitter background. The negative cosmological constant influences the p–q strings in different ways. As observed before for other localized structures, it tends to decrease the core radius of the strings and increases the energy. We observe in particular for condensate strings which have $n \neq 0$ and $m = 0$ that the cosmological constant increases the value of the condensate in the string core. While the model studied here is different from that describing so-called superconducting strings (in our case, both $U(1)$s are spontaneously broken, while in [12] one $U(1)$ remains unbroken), it would be interesting to see whether the negative cosmological constant has similar effects on these solutions.

We also observe that if the absolute value of the cosmological constant is large enough and the interaction parameter is small enough p–q strings become unbound. It would be interesting to know whether this result remains qualitatively the same when studying the gravitational interaction of p–q strings with a dynamical AdS spacetime and to understand what interpretation this has in the context of the AdS/CFT correspondence.

Studying the gravitational effects within a linearized approximation, we observe that the spacetime has a deficit angle. This deficit angle tends to decrease with increasing absolute value of the cosmological constant.

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