

5. Backward Pass

- o Please calculate $w3^+$

Please also explain how you get the formulas.

Step	Description	Formula	Calculated answer
1	Complete and optimize the formula to calculate w3's impact to E_{total} Expressing the formula with <ul style="list-style-type: none"> o w3's impact to E_{o1} o w3's impact to E_{o2} 		
2	Complete and optimize the formula to calculate w3's impact to E_{o1} Expressing the formula with chain rule		
3	Complete and optimize the formula to calculate w3's impact to E_{o2} Expressing the formula with chain rule		
4	Complete and optimize the formula to calculate w3's impact to net_{h2}		
5	Complete and optimize the formula to calculate net_{h2}'s impact to out_{h2}		
6	Complete and optimize the formula to calculate out_{h2}'s impact to net_{o1}		
7	Complete and optimize the formula to calculate net_{o1} to out_{o1}		
8	Complete and optimize the formula to calculate out_{o1} to E_{o1} Expressed with $target_{o1}$ and out_{o1}		
9	$w3^+$ New W3		

Solution:

Step	Description	Formula	Calculated Answer
1	Complete and optimize the formula to calculate w3's impact to E_{total} Expressing the formula with <ul style="list-style-type: none"> o w3's impact to E_{o1} o w3's impact to E_{o2} 	$\partial E_{total} / \partial w3 = \partial E_{o1} / \partial w3 + \partial E_{o2} / \partial w3$	
2	Complete and optimize the formula to calculate w3's impact to E_{o1} Expressing the formula with chain rule	$(\partial net_{h2} / \partial w3) * (\partial out_{h2} / \partial net_{h2}) * (\partial net_{o1} / \partial out_{h2}) * (\partial out_{o1} / \partial net_{o1}) * (\partial E_{o1} / \partial out_{o1})$	

3	Complete and optimize the formula to calculate w3's impact to E_{o2} Expressing the formula with chain rule	$(\partial \text{neth2} / \partial w3) * (\partial \text{outh2} / \partial \text{neth2}) * (\partial \text{neto2} / \partial \text{outh2}) * (\partial \text{outo2} / \partial \text{neto2}) * (\partial E_{o2} / \partial \text{outo2})$	
4	Complete and optimize the formula to calculate w3's impact to net_{h2}	$\partial(i1 * w3 + i2 * w4 + b1 * 1) / \partial w3 = i1$	
5	Complete and optimize the formula to calculate net_{h2}'s impact to out_{h2}	$\text{outh2} (1 - \text{outh2})$	
6	Complete and optimize the formula to calculate out_{h2}'s impact to net_{o1}	$\partial(w5 * \text{outh1} + w6 * \text{outh2} + b2 * 1) / \partial \text{outh2} = w6$	
7	Complete and optimize the formula to calculate net_{o1} to out_{o1}	$\text{outo1} (1 - \text{outo1})$	
8	Complete and optimize the formula to calculate out_{o1} to E_{o1} Expressed with target _{o1} and out _{o1}	$\partial ((\text{targeto1} - \text{outo1})^2 / 2) / \partial \text{outo1} = \text{outo1} - \text{targeto1}$	
9	w3⁺ New W3	$W3 + = w3 - n * \partial E_{\text{total}} / \partial w3$	

Calculated Result:

Step 1: Expressing the formula with:

w3's impact to E_{o1}

$$\partial E_{o1} / \partial w3 \Rightarrow (\partial \text{neth2} / \partial w3) * (\partial \text{outh2} / \partial \text{neth2}) * (\partial \text{neto1} / \partial \text{outh2}) * (\partial \text{outo1} / \partial \text{neto1}) * (\partial E_{o1} / \partial \text{outo1})$$

$$\partial \text{neth2} / \partial w3 \Rightarrow \partial(i1 * w3 + i2 * w4 + b1 * 1) / \partial w3 = i1 \quad \partial \text{outh2} / \partial \text{neth2} : \text{outh2} (1 - \text{outh2})$$

$$\partial \text{neto1} / \partial \text{outh2} : \partial(w5 * \text{outh1} + w6 * \text{outh2} + b2 * 1) / \partial \text{outh2} = w6$$

$$\partial \text{outo1} / \partial \text{neto1} : \text{outo1} (1 - \text{outo1}) \quad \partial E_{o1} / \partial \text{outo1} : \partial((\text{targeto1} - \text{outo1})^2 / 2) / \partial \text{outo1} = \text{outo1} - \text{targeto1}$$

w3's impact to E_{o2}

$$\partial E_{o2} / \partial w3 : (\partial \text{neth2} / \partial w3) * (\partial \text{outh2} / \partial \text{neth2}) * (\partial \text{neto2} / \partial \text{outh2}) * (\partial \text{outo2} / \partial \text{neto2}) * (\partial E_{o2} / \partial \text{outo2})$$

$$\partial \text{neth2} / \partial w3 : \partial(i1 * w3 + i2 * w4 + b1 * 1) / \partial w3 = i1$$

$$\partial \text{outh2} / \partial \text{neth2}: \text{outh2} (1 - \text{outh2})$$

$$\partial \text{neto2} / \partial \text{outh2}: \partial (w7 * \text{outh1} + w8 * \text{outh2} + b2 * 1) / \partial \text{outh2} = w8$$

$$\partial \text{outo2} / \partial \text{neto2}: \text{outo2} (1 - \text{outo2})$$

$$\partial Eo2 / \partial \text{outo2}: \partial ((\text{targeto2} - \text{outo2})^2 / 2) / \partial \text{outo2} = \text{outo2} - \text{targeto2}$$

Step 2: Complete and optimize the formula to calculate w3's impact to Eo1

Expressing the formula with chain rule $\partial Eo1 / \partial w3 = (\partial \text{neth2} / \partial w3) * (\partial \text{outh2} / \partial \text{neth2}) * (\partial \text{neto1} / \partial \text{outh2}) * (\partial \text{outo1} / \partial \text{neto1}) * (\partial Eo1 / \partial \text{outo1})$

$$\partial E_{\text{total}} / \partial \text{outh2} = \partial Eo1 / \partial \text{outh2} + \partial Eo2 / \partial \text{outh2}$$

$\partial Eo1 / \partial \text{outh2}$:

$$\partial Eo1 / \partial \text{outh2} = \partial Eo1 / \partial \text{neto1} * \partial \text{neto1} / \partial \text{outh2}$$

$$\text{But, } \partial Eo1 / \partial \text{neto1} = \partial Eo1 / \partial \text{outo1} * \partial \text{outo1} / \partial \text{neto1},$$

Hence Substitute $\partial Eo1 / \partial \text{neto1}$ in $\partial Eo1 / \partial \text{outh2}$ formula – $\partial Eo1 / \partial \text{outh2} = \partial Eo1 / \partial \text{outo1} * \partial \text{outo1} / \partial \text{neto1} * \partial \text{neto1} / \partial \text{outh2}$

Also

Substitute $\partial Eo1 / \partial \text{outh2}$ in $\partial E_{\text{total}} / \partial w3$ formula for how w3's impact to Eo1:

$$\partial E_{\text{total}} / \partial w3 = \partial Eo1 / \partial \text{outo1} * \partial \text{outo1} / \partial \text{neto1} * \partial \text{neto1} / \partial \text{outh2} * \partial \text{outh2} / \partial \text{neth2} * \partial \text{neth2} / \partial w3$$

$$\partial Eo1 / \partial w3 = (\partial \text{neth2} / \partial w3) * (\partial \text{outh2} / \partial \text{neth2}) * (\partial \text{neto1} / \partial \text{outh2}) * (\partial \text{outo1} / \partial \text{neto1}) * (\partial Eo1 / \partial \text{outo1})$$

Step 3 : Complete and optimize the formula to calculate w3's impact to Eo2 Expressing the formula with the chain rule

$$\partial E_{\text{total}} / \partial w3 = \partial E_{\text{total}} / \partial \text{outh2} * \partial \text{outh2} / \partial \text{neth2} * \partial \text{neth2} / \partial w3 \quad \partial E_{\text{total}} / \partial \text{outh2} = \partial Eo1 / \partial \text{outh2} + \partial Eo2 / \partial \text{outh2}$$

$$\partial Eo2 / \partial \text{outh2}:$$

$$\partial E_{02} / \partial outh2 = \partial E_{02} / \partial net_{02} * \partial net_{02} / \partial outh2$$

$$\text{But, } \partial E_{02} / \partial net_{02} = \partial E_{o2} / \partial out_{o2} * \partial out_{o2} / \partial net_{o2}$$

Substitute $\partial E_{02} / \partial net_{02}$ in $\partial E_{02} / \partial outh2$ formula,

$$\partial E_{02} / \partial outh2 = \partial E_{o2} / \partial out_{o2} * \partial out_{o2} / \partial net_{o2} * \partial net_{o2} / \partial outh2$$

Substitute $\partial E_{02} / \partial outh2$ in $\partial E_{total} / \partial w3$ formula for how $w3$'s impact to E_{o2} :

$$\partial E_{total} / \partial w3 = \partial E_{o2} / \partial out_{o2} * \partial out_{o2} / \partial net_{o2} * \partial net_{o2} / \partial outh2 * \partial outh2 / \partial neth2 * \partial neth2 / \partial w3$$

$$\partial E_{o2} / \partial w3 = (\partial neth2 / \partial w3) * (\partial outh2 / \partial neth2) * (\partial net_{o2} / \partial outh2) * (\partial out_{o2} / \partial net_{o2}) * (\partial E_{o2} / \partial out_{o2})$$

Step4: Complete and optimize the formula to calculate $w3$'s impact to $neth2$

$$\partial neth2 / \partial w3 = \partial(i1 * w3 + i2 * w4 + b1 * 1) / \partial w3$$

$$\partial(i1 * w3 + 0 + 0) / \partial w3 = i1 = 0.05$$

Step5: Complete and optimize the formula to calculate $neth2$'s impact to $outh2$

$$\partial outh2 / \partial neth2 = outh2 (1 - outh2) = 0.59688437826(1 - 0.59688437826) = 0.24061341724$$

Step6: Complete and optimize the formula to calculate $outh2$'s impact to net_{o1}

$$\partial net_{o1} / \partial outh2 = \partial(w5 * outh1 + w6 * outh2 + b2 * 1) / \partial outh2 = \partial(0 + w6 * outh2 + 0) / \partial outh2 = w6 = 0.45$$

Step7: Complete and optimize the formula to calculate net_{o1} to out_{o1}

$$\partial out_{o1} / \partial net_{o1} = out_{o1} (1 - out_{o1}) = 0.75136507(1 - 0.75136507) = 0.186815602$$

Step8: Complete and optimize the formula to calculate net_{o1} to out_{o1}

$$\partial E_{o1} / \partial out_{o1} = \partial((target_{o1} - out_{o1})^2 / 2) / \partial out_{o1}$$

$$E_{total} = \frac{1}{2}(target_{o1} - out_{o1})^2 + \frac{1}{2}(target_{o2} - out_{o2})^2$$

$$\frac{\partial E_{total}}{\partial out_{o1}} = -(target_{o1} - out_{o1}) = -(0.01 - 0.75136507) = 0.74136507$$

Step 9: $w3$ + New $W3$

$$W3 + = w3 - n * \partial E_{total} / \partial w3 ==> 0.25 - (0.5 * 0.00049771273) ==> 0.24975114363$$