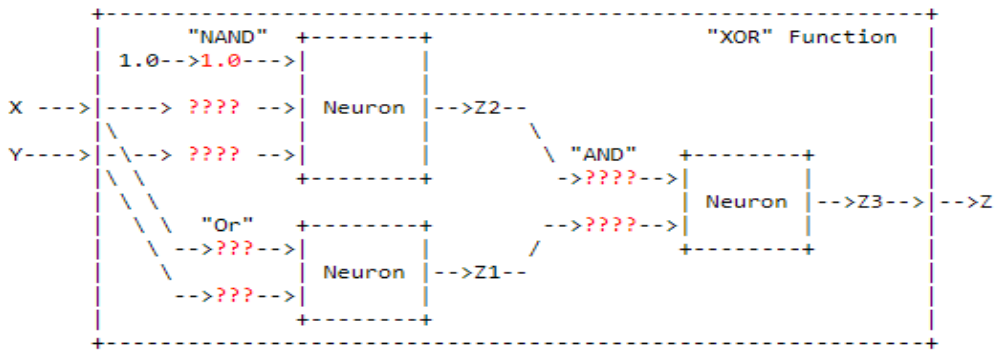
Mahmud Hamed – 19660

Week 10 -Hw1

Q: Project: Design XOR Gate

Please refer to A Neural Network Primer to solve this question.

- Step 1: Study the general idea on how to design XOR Gate
- Step 2: Using the following rules to design your own AND Gate, OR Gate, and NAND Gate



  - The forward/backward process

```
- Forward process
  Calculate the output Z for the given input (X,Y).
- Backward process
  Adjust weights
  + If the output Z is too low, increase the weights by 0.5
    which had inputs that were "1".
  + If the output Z is too high, decrease the weights by 0.5
    which had inputs that were "1".
```

  - Using step activation function

```
Z := ( W0 * C + W1 * X + W2 * Y >= T )
     where T := 1.0

if ( W0 * C + W1 * X + W2 * Y >= T )
then ouput is 1
else output = 0
```

  - The bias C for NAND is 1.0
- Step 3: Please answer
  - What is the formula for

```
Z1 := X  "AND"   Y
```

Hint:

- The formula for AND Gate has this format

```
Z := ( W1 * X + W2 * Y >= T )
   where T := 1.0.
```

But you need to determine the value of W1 and W2.

- In A Neural Network Primer, the increment/decrement of W1 and W2 is 0.375, but these values in this question is 0.5.

- What is the formula for

```
Z1 := X  "OR"   Y
```

  Hint:
  - The formula for OR Gate has this format

```
Z := ( W1 * X + W2 * Y >= T )
   where T := 1.0.
```

    But you need to determine the value of W1 and W2.
    - In A Neural Network Primer, the increment/decrement of W1 and W2 is 0.375, but these values in this question is 0.5.
- What is the formula for

```
Z2 := X  "NAND" Y
```

  Hint:
  - After figuring out the formula for AND Gate, you can refer this process to figure out the formula for NAND Gate.
- What is the formula for

```
Z := Z3 := Z1 "AND" Z2
```

  Hint:
  - The process to create this formula is similar to this process.

- Step 4: Please prove that your designed XOR Gate work
  - X=1, Y=1
  - X=1, Y=0
  - X=0, Y=1
  - X=0, Y=0
- Step 5: Adding the project to your portofolio
  a. Please use Google Slides to document the project
  b. Please link your presentation on GitHub using this structure

```
Machine Learning
  - ChatGPT
    + Use ChatGPT to create customer support website
```

- Step 6: Submit
  1. The URLs of the Google Slides and GitHub web pages related to this project.
  2. A PDF file of your Google Slides

ANS:

Step 1: XOR" function:

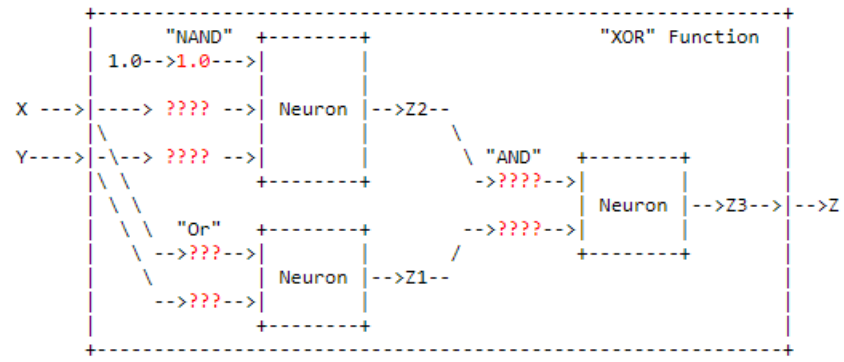
a. Truth table

The truth-table for the "XOR" function.

```
   OR                NAND              XOR

 X  Y | Z1          X  Y | Z2         X  Y | Z3
----------         ----------        ----------
 0  0 | 0           0  0 | 1          0  0 | 0
 0  1 | 1    AND    0  1 | 1    =     0  1 | 1
 1  0 | 1           1  0 | 1          1  0 | 1
 1  1 | 1           1  1 | 0          1  1 | 0
```

b. XOR function neural network structure:



c. The forward/backward process

- Forward process
  Calculate the output Z for the given input (X,Y).
- Backward process
  Adjust weights
  + If the output Z is too low, increase the weights by 0.5
    which had inputs that were "1".
  + If the output Z is too high, decrease the weights by 0.5
    which had inputs that were "1".

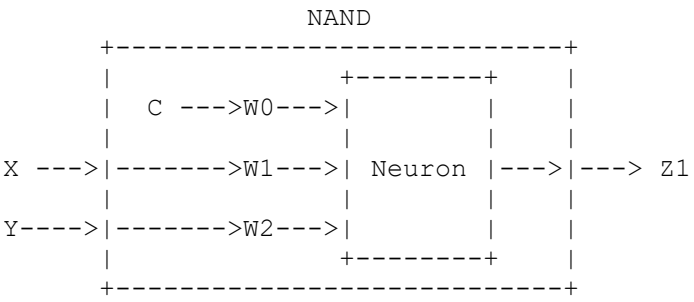
d. step activation function to calculate Z:

Z := (W0 * C + W1 * X + W2 * Y >= T)
where T := 1.0

if (W0 * C + W1 * X + W2 * Y >= T)
then ouput is 1
else output = 0

Step 2: Obtain z1 , z2 and z3

a. Z2:= X "NAND" Y



Z1 := (W0 * C + W1 * X + W2 * Y >= T)

Train NAND gate to get W0,w1, W2

Z:= (W0 * C + W1 * X + W2 * Y >= T)

where T:= 1.0.

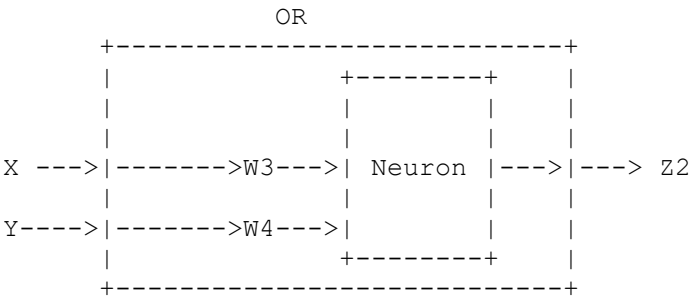
Desired "NAND" Function	Loop 1 W0=0.0 W1=W2=0.5 Function	Loop 2 W0=0 W1=W2=1 Function	Loop 3 W0=0.5 W1=W2=1 Function
C X Y | Z --------- 1 0 0 | 1 1 0 1 | 1 1 1 0 | 1 1 1 1 | 0	C X Y | Z --------- 1 0 0 | 0 1 0 1 | 0 1 1 0 | 0 1 1 1 | 0	C X Y | Z --------- 1 0 0 | 0 1 0 1 | 1 1 1 0 | 1 1 1 1 | 1	C X Y | Z --------- 1 0 0 | 0 1 0 1 | 1 1 1 0 | 1 1 1 1 | 1
Loop 4 W0= 1 W1=W2=1 Function	**Loop 5** W0= 1 W1=W2=0.5 Function	**Loop 6** W0= 1 W1=W2=0 Function	**Loop 7** W0= 1 W1=W2=-0.5 Function
C X Y | Z --------- 1 0 0 | 1 1 0 1 | 1 1 1 0 | 1 1 1 1 | 1	C X Y | Z --------- 1 0 0 | 1 1 0 1 | 1 1 1 0 | 1 1 1 1 | 1	C X Y | Z --------- 1 0 0 | 1 1 0 1 | 1 1 1 0 | 1 1 1 1 | 1	C X Y | Z --------- 1 0 0 | 1 1 0 1 | 0 1 1 0 | 0 1 1 1 | 0
Loop 7 W0= 1.5 W1=W2=-0.5 Function			
C X Y | Z --------- 1 0 0 | 1 1 0 1 | 1 1 1 0 | 1 1 1 1 | 0			

Therefor :

W0= 1.5
W1=W2=-0.5

Z1: = (1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0)

b. Z2:= X "or" Y



Z2 := (W3 * X + W4 * Y >= T)

Train NAND gate to get w3, W4

Z2:= (W3 * X + W4 * Y >= T)

where T:= 1.0.

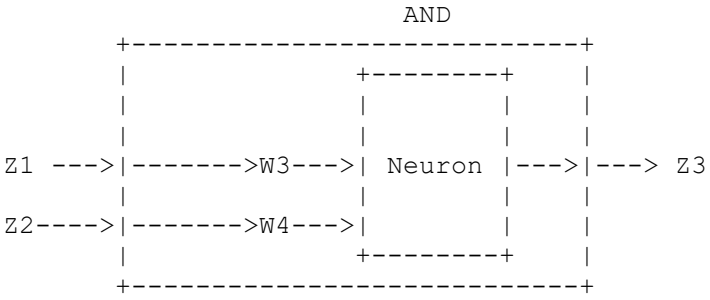
Desired "OR" Function	Loop 1 W3=W4=0.5 Function	Loop 2 W3=W4= 1 Function
X Y | Z --- 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1	X Y | Z --- 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1	X Y | Z --- 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 1

Therefor:

W3=W4= 1

Z2: = (1* X + 1* Y >= 1.0)

c. Z := Z3 := Z1 "AND" Z2



Z3 := (W5 * Z1 + W6 * Z2 >= T)

Train the AND gate to get w5, W6

let Z1 := X, and Z2:= Y

Z3:= (W5 * X + W6* Y >= T)

where T:= 1.0.

Desired "AND" Function	Loop 1 W4=W5=0.5 Function
X Y | Z -------- 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1	X Y | Z ----------- 0 0 | 0 0 1 | 0 1 0 | 0 1 1 | 1

Therefor:

W5=W6= 0.5

Z3: = (0.5* X + 0.5* Y >= 1.0) = (0.5* Z1 + 0.5* Z2 >= 1.0)

Prove result:

Test cases:

- X=1, Y=1
- X=1, Y=0
- X=0, Y=1
- X=0, Y=0

Test 1: X=1, Y=1	Desired Result
Z1: = (1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0) = 1.5-0.5-0.5 := 0 Z2: = (1* X + 1* Y >= 1.0) = 1 + 1 := 1 Z3: = (0.5* Z1 + 0.5* Z2 >= 1.0) = 0.5*0 + 0.5*1 := 0	XOR ----------- X Y | Z ----------- 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0
Test 2: X=1, Y=0	**Desired Result**
Z1: = (1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0) = 1.5-0.5*1-0.5*0 := 1 Z2: = (1* X + 1* Y >= 1.0) = 1*1 + 1*0 := 1 Z3: = (0.5* Z1 + 0.5* Z2 >= 1.0) = 0.5*1 + 0.5*1 := 1	XOR ----------- X Y | Z ----------- 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0

Test 3: X=0, Y=1	Desired Result
Z1: = (1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0) = 1.5*1.0-0.5*0-0.5*1 := 1 Z2: = (1* X + 1* Y >= 1.0) = 1*0 + 1*1 := 1 Z3: = (0.5* Z1 + 0.5* Z2 >= 1.0) = 0.5*1 + 0.5*1 := 1	XOR X Y | Z 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0
Test 2: X=0, Y=0	Desired Result
Z1: = (1.5 * 1.0 + -0.5 * X + -0.5 * Y >= 1.0) = 1.5-0.5*0-0.5*0 := 1 Z2: = (1* X + 1* Y >= 1.0) = 1*0 + 1*0 := 0 Z3: = (0.5* Z1 + 0.5* Z2 >= 1.0) = 0.5*1 + 0.5*0 := 0	XOR X Y | Z 0 0 | 0 0 1 | 1 1 0 | 1 1 1 | 0

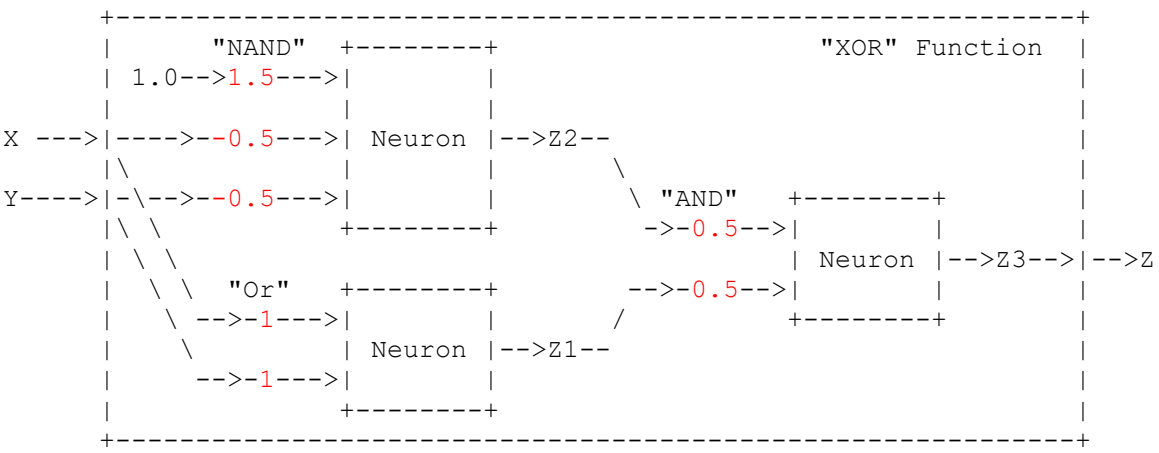
Conclusion:

The test results obtained based on the final parameters agree with the desired values. Hence, for the given XOR gate, below are the final parameters values and the complete ANN circuit diagram.

W0= 1.5
W1=W2=-0.5

W3=W4= 1

W5=W6= 0.5



Google Slides Link :
https://docs.google.com/presentation/d/1O4cUVFGi1j8hAvE3tgBMrJz1H3wAkQZkk9Apo7m36fM/edit?usp=sharing

GitHub Link : https://github.com/momer22/Project-Design-XOR-Gate